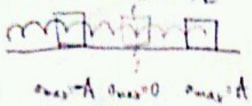
Keywords

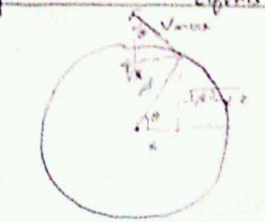
period (T) = the time req to complete one cycle
frequency (f) = the number of complete cycles per second $\Big\rangle$ $T = \frac{1}{f}$ and $f = \frac{1}{T}$
amplitude (A) = the maximum displacement or greatest distant from equilibrium position — a_{max}
Equilibrium position = when the length of the spring exerts no net force on the mass and is at its natural length.
restoring force = the force a spring exerts to return to its equilibrium position.

$x = A\sin\left(\frac{2\pi t}{T}\right)$	Both	$x = A\cos\left(\frac{2\pi t}{T}\right)$
$T=0,\ x=0$ all KE $T=\frac{1}{4}T,\ x=A$ all PE $T=\frac{1}{2}T,\ x=0$ all KE $T=\frac{3}{4}T,\ x=-A$ all PE $T=T,\ x=0$ all KE	period = T max displacement = A approaches 0 twice in a T has 1 max & 1 min in a T	$T=0,\ x=A$ all PE SHM position equation (−) SHM acceleration equation $x = a,\ A = a_{max}$ $T=\frac{1}{4}T,\ x=0$ all KE $T=\frac{1}{2}T,\ x=-A$ all PE $T=\frac{3}{4}T,\ x=0$ all KE $T=T,\ x=A$ all PE

Energy <
spring PE $= \frac{1}{2}kx^2$, when $v=0$, $E = \frac{1}{2}kx^2 = \frac{1}{2}k\,a_{max}^2$
x = amplitude
KE $= \frac{1}{2}mv^2$, when $a_{max} = 0$, $E = \frac{1}{2}mv^2 = \frac{1}{2}mv_{max}^2$

$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$ ← Equals ← Equals





$\boxed{\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2}$

triangles similar; ratios similar:

$\frac{v}{v_{max}} = \frac{\sqrt{A^2 - x^2}}{A}$ or $v = v_{max}\sqrt{1 - \frac{x^2}{A^2}}$

⇓

Same as velocity equation for mass in simple harmonic motion

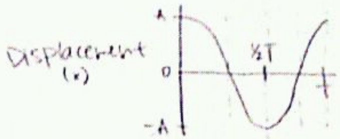
$\frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2$

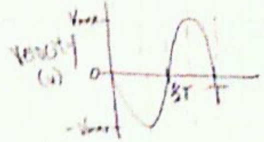
$v_{max}^2 = \frac{kA^2}{m}$ $\quad \frac{k}{m}(A^2 - x^2) = \frac{k}{m}A^2\left(1 - \frac{x^2}{A^2}\right) = v^2$

$v_{max}^2\left(1 - \frac{x^2}{A^2}\right) = v^2$ $\quad \boxed{v = \pm v_{max}\sqrt{1 - \frac{x^2}{A^2}}}$

$v_{max} = \frac{2\pi A}{T} = 2\pi Af$ $\quad T = \frac{2\pi A}{v_{max}}$ $\quad A^2 = \frac{v_{max}^2 m}{k}$

$\boxed{T = \frac{2\pi\, v_{max}\sqrt{\frac{m}{k}}}{v_{max}} = 2\pi\sqrt{\frac{m}{k}}}$ $\quad \boxed{f = \frac{1}{T} = \frac{1}{2\pi\sqrt{\frac{k}{m}}}}$ $\quad \boxed{A = \pm v_{max}\sqrt{\frac{m}{k}}}$

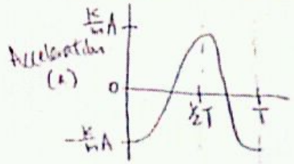




$x = A\cos\theta$ $\quad \theta = \omega t$ $\quad x = A\cos\omega t$ $\quad \omega = 2\pi f$

$\boxed{x = A\cos(2ft\pi) = A\cos\left(\frac{2\pi t}{T}\right)}$

$-V_{max}\sin\theta = \boxed{V_x = -V_{max}\sin\left(\frac{2\pi t}{T}\right)}$



$a = \frac{F}{m} = \frac{-kx}{m}$ $\quad a = a_{max}\cos\theta$

$-a_{max}\cos\theta = \boxed{a = -A\cos\left(\frac{2\pi t}{T}\right)}$

$a_{max} = \frac{kA}{m}$

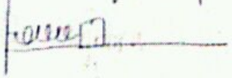
✓+

Clarissa & Howard

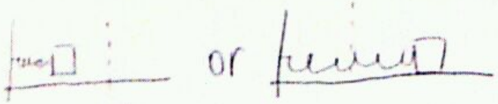
$E = KE + PE = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$ $\quad KE = \frac{1}{2}mv^2 \quad PE = \frac{1}{2}kx^2$

In order to have maximum v, KE has to be largest, thus PE = 0
equilibrium position.

$E = \frac{1}{2}m(v_{max})^2 + 0$



PE is largest when $x = A$, $E = 0 + \frac{1}{2}kA^2$



Since energy is conserved, $\frac{1}{2}mv_{max}^2 = \frac{1}{2}kA^2$

$$\boxed{v_{max} = \pm A\sqrt{\frac{k}{m}}}$$

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

$$v = \pm\sqrt{\frac{kA^2 - kx^2}{m}} = \pm\sqrt{A^2\frac{k}{m}\left(1 - \frac{x^2}{A^2}\right)} = \pm A\sqrt{\frac{k}{m}}\sqrt{1 - \frac{x^2}{A^2}}$$

$$\boxed{v = \pm v_{max}\sqrt{1 - \frac{x^2}{A^2}}}$$

$v(x)$

$$v = \pm v_{max}\sqrt{1 - \frac{A^2\cos^2(\frac{2\pi}{T}t)}{A^2}}$$

$$v = \pm v_{max}\sqrt{1 - \cos^2(\tfrac{2\pi}{T}t)}$$

$$\boxed{v = \pm v_{max}\sin(\tfrac{2\pi}{T}t)}$$

$v(t)$

Energy + Velocity

$F = +kx. \quad x_{max} = A \quad F = ma$

$F_{max} = +kx_{max} = +kA \quad$ substitute $\quad F_{max} = m a_{max}.$

$$kA = m a_{max} \Rightarrow \boxed{a_{max} = \frac{kA}{m}}$$

$F = -kx \quad F = ma$

$$-kx = ma \Rightarrow \boxed{a = \frac{-kx}{m}}$$

$a(x)$

$$\boxed{a = \frac{-kA\cos(\frac{2\pi}{T}t)}{m}}$$

$a(t)$

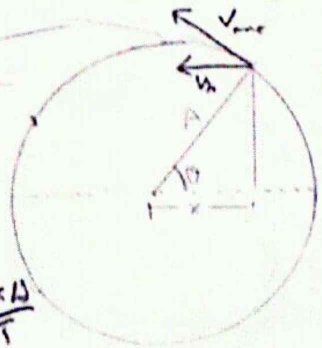
$\cos\theta = \frac{x}{A}$

$x = A\cos\theta$

$\theta = \frac{2\pi}{T}t$

$x = A\cos(\frac{2\pi}{T}t)$

$x(t)$



$v_{max} = \frac{\Delta x}{\Delta t} = \frac{2\pi A}{T}$

$T = \frac{2\pi A}{v_{max}}$

$= \frac{2\pi A}{A\sqrt{\frac{k}{m}}}$

$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

Force + Acceleration + Position

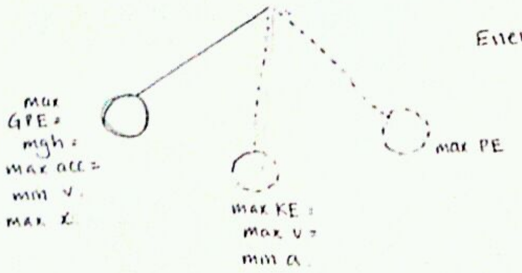
Juiku Shen

$x(t)$ = position at a given time

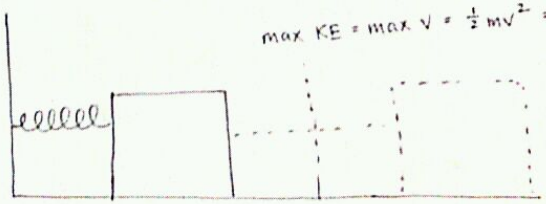
$x'(t) = v(t)$ = change in position in a given time

$x''(t) = v'(t) = a(t)$ = change in velocity in a given time

Energy is conserved



max KE = max v = $\frac{1}{2}mv^2$ = min acc.



max PE = $\frac{1}{2}kx^2$ = max acc = min v = max x.

max PE = $\frac{1}{2}kx^2$

① $F = ma = kx$

$m\,a_{max} = kA$

$$\boxed{a_{max} = \frac{kA}{m}}$$

② PE = KE

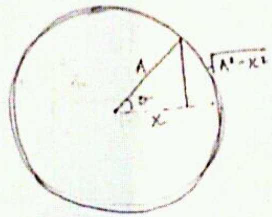
$\frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2$

$v_{max} = \pm\sqrt{\frac{kA^2}{m}}$

$$= \boxed{\pm A\sqrt{\frac{k}{m}}}$$

$\omega = \frac{2\pi}{T}$

$\theta = \frac{2\pi}{T}t$



$x = \cos\theta \times A$

③ $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$

$mv^2 + kx^2 = kA^2$

$mv^2 = kA^2 - kx^2$

$v^2 = \frac{kA^2 - kx^2}{m}$

$v = \sqrt{\frac{k(A^2 - x^2)}{m}}$

$= \sqrt{\frac{k}{m}A^2\left(1 - \frac{x^2}{A^2}\right)}$

$= \sqrt{\frac{kA^2}{m}} \cdot \sqrt{1 - \frac{x^2}{A^2}}$

$$\boxed{v = v_{max}\sqrt{1 - \frac{x^2}{A^2}}}$$

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④ $v_{max} = \frac{d}{t}$

$= \frac{2\pi r}{t}$

$= \frac{2\pi A}{t}$

$v_{max} = \frac{2\pi A}{T}$

$A\sqrt{\frac{k}{m}} = \frac{2\pi A}{T}$

$T = \frac{2\pi}{\sqrt{\frac{k}{m}}} = \frac{2\pi}{\frac{\sqrt{k}}{\sqrt{m}}} = \boxed{2\pi\sqrt{\frac{m}{k}}}$

$v_{max} = \frac{d}{t} = \frac{2\pi r}{t} = \frac{2\pi A}{t} = \frac{2\pi A}{T}$

$f = \frac{1}{2\pi\sqrt{\frac{m}{k}}}$

$= \boxed{\frac{1}{2\pi}\sqrt{\frac{k}{m}}}$

⑤ $x(t) = A\cos\theta$

$= A\cos\omega t$

$= \boxed{A\cos\left(\frac{2\pi}{T}t\right)}$

$x(0) = A$

$v(t) = x'(t) = -A\sin\left(\frac{2\pi}{T}t\right)\frac{2\pi}{T}$

$= \boxed{-\frac{2\pi A}{T}\sin\left(\frac{2\pi}{T}t\right)}$

$v(0) = 0$

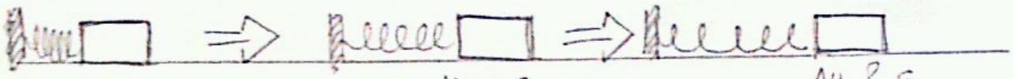
$a(t) = v'(t) = -A\cos\left(\frac{2\pi}{T}t\right)\frac{2\pi}{T}^2$

$= \boxed{-\left(\frac{2\pi A}{T}\right)^2\cos\left(\frac{2\pi}{T}t\right)}$

$a(0) = \frac{-2\pi A}{T} = a_{max} = \frac{kA}{m}$

$\frac{k}{m} \overset{?}{=} \frac{-2\pi}{T}$

Finding V_{max} ($V(x)$)



All P.E. | All K.E. | All P.E.
$x = -A$ | $x = 0$ (equilibrium position) | $x = A$

Mechanical Energy is conserved $\boxed{KE = \frac{1}{2}mv^2 \quad PE = \frac{1}{2}kx^2}$

$ME = \frac{1}{2}k(-A)^2$ $\quad ME = \frac{1}{2}mv_{max}^2$ $\quad ME = \frac{1}{2}k(A)^2$

$$mV_{max}^2 = kA^2$$

$$V_{max}^2 = \frac{k}{m}A^2$$

$$\boxed{V_{max} = \pm\sqrt{\frac{k}{m}A^2}}$$

Finding a_{max}

$$a_{max} = \frac{F_{max}}{m} = \boxed{\frac{kA}{m}}$$

Finding V at any point x

$$\cancel{\frac{1}{2}}mv^2 + \cancel{\frac{1}{2}}kx^2 = \cancel{\frac{1}{2}}mV_{max}^2$$

$$\Rightarrow \quad mv^2 + kx^2 = mV_{max}^2$$

$$\Rightarrow \quad v^2 = V_{max}^2 - \frac{k}{m}x^2$$

$$\Rightarrow \quad \boxed{v = \pm V_{max}\sqrt{1 - \frac{x^2}{A^2}}}$$

Find $x(t)$; $v(t)$; $a(t)$

$$V_{max} = \frac{2\pi A}{T} = 2\pi A \cdot \left(\frac{1}{T}\right) = 2\pi A(f)$$

$$T = \frac{2\pi A}{V_{max}} = 2\pi\left(\frac{A}{V_{max}}\right)$$

$$\rightarrow \frac{1}{2}kA^2 = \frac{1}{2}mV_{max}^2$$

$$= 2\pi\sqrt{\frac{m}{k}} \qquad \frac{A^2}{V_{max}^2} = \frac{m}{k} \Rightarrow \frac{A}{V_{max}} = \sqrt{\frac{m}{k}}$$

$\Downarrow$

- To find $x(t)$

A, θ, x

$$\cos\theta = \frac{x}{A}$$

$$x(t) = A\cos\theta \qquad V_\theta = \frac{2\pi}{T} = 2\pi f$$

$$= A\cos V_\theta t$$

$$= A\cos(2\pi f t) \quad \text{or} \quad A\cos\frac{2\pi t}{T}$$

- To find $v(t)$

$$v(t) = x'(t)$$

$$= -A\sin\frac{2\pi t}{T} \cdot \frac{2\pi}{T}$$

$$= -\sin\frac{2\pi t}{T} \cdot V_{max}$$

- To find $a(t)$

$$a(t) = v'(t)$$

$$= -\cos\frac{2\pi t}{T} \cdot \frac{2\pi V_{max}}{T}$$

$$= -\cos\frac{2\pi t}{T} \cdot a_{max}$$

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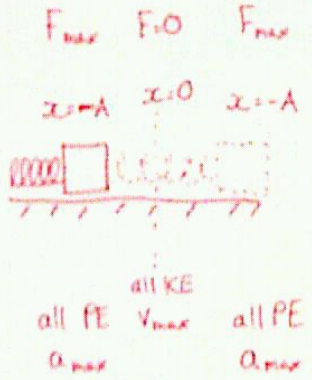


$$\frac{1}{2}kA^2 = \frac{1}{2}m v_{max}^2$$

$$\boxed{v_{max} = \sqrt{\frac{k}{m}} \cdot A}$$

$$F_{max} = m a_{max} = kA$$

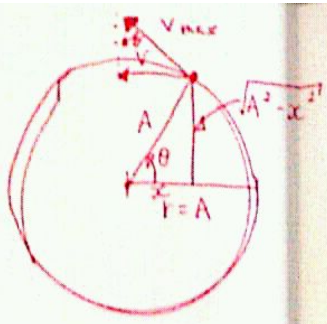
$$\boxed{a_{max} = \frac{k}{m} A}$$

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

$$v = \sqrt{\frac{k(A^2 - x^2)}{m}}$$

$$v = \sqrt{\frac{k}{m}} A \sqrt{1 - x^2/A^2}$$

$$\boxed{v = \pm v_{max} \sqrt{1 - x^2/A^2}}$$



$$\frac{v_{max}}{v} = \frac{A}{\sqrt{A^2 - x^2}}$$

$$v = v_{max}\sqrt{1 - x^2/A^2}$$

$$v_{max} = \frac{C}{t} = \frac{2\pi r}{t} = \frac{2\pi A}{T}$$

$$T = \frac{2\pi A}{v_{max}} = \frac{2\pi \cancel{A}}{\sqrt{\frac{k}{m}}\cancel{A}} = 2\pi\sqrt{\frac{m}{k}}$$

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad T = 2\pi\sqrt{\frac{m}{k}}}$$

$$x = A \cdot \cos\theta$$

$$\omega = \frac{\theta}{t} = \frac{\theta}{T} = \frac{2\pi}{T}$$

$$x = A \cdot \cos \frac{\theta}{T} t$$

$$\boxed{x(t) = A \cdot \cos\left(\frac{2\pi}{T} t\right)}$$

$t = 0,\ x = A$

$$\boxed{x(t) = A \cdot \sin\left(\frac{2\pi}{T} t\right)}$$

$t = 0,\ x = 0$