Projectile Motion

How to solve projectile motion questions:

1) read carefully
2) draw a diagram
3) choose origin and xy coordinate system
4) decide on the time interval you are going to analyze
5) examine the x and y components separately
6) list the knowns and unknowns
7) think for a minute, then apply relevant equations

Keywords:

projectile motion - objecting moving through air in two dimensions

launch - hint that this is a projectile motion question

cliff/initial height - initial y-displacement

Equations:

$v = v_0 + at$ - useful when you are not provided with displacement

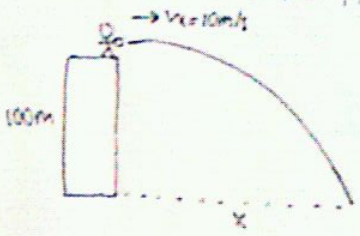
$x = x_0 + v_0 t + \frac{1}{2}at^2$ - for everything except final velocity

$v^2 = v_0^2 + 2(x - x_0)$ - for everything except time

$x = v_0 t$ - usually used for horizontal displacement

$v_{x_0} = v_0 \cos\theta \quad v_{y_0} = v_0 \sin\theta$

A ball is thrown from a 100 meter high cliff with an initial velocity of 10m/s parallel to the cliff. How far from the cliff does it land?



1) $x = v_x t$ - horizontal displacement
$= 10(\cos 0)t$
$= 10t$ find t.

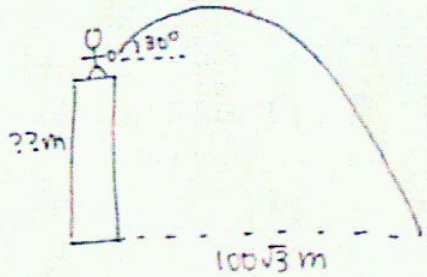
2) $x = v_{y_0} t + \frac{1}{2} a_y t^2$ - vertical displacement

$100 = 10(\sin 0)t + \frac{1}{2}(9.81)t^2$

$t = 4.52\,s$ (rounded; do not round when plugging in)

3) $x = v_x t$
$= 10(4.52)$
$\boxed{= 45.2\,m}$

A ball is thrown from a cliff with an initial velocity of 10m/s at an angle 30° to the horizontal. It travels $100\sqrt{3}$ m before landing. How high is the cliff?



1) $v_x = v_0 \cos\theta$
$= 10(\cos 30)$
$= 5\sqrt{3}$ m/s

2) $x = v_x t$ - horizontal displacement
$100\sqrt{3} = 5\sqrt{3}(t)$
$t = 20\,s$

3) $x = v_y t + \frac{1}{2} a t^2$ - vertical displacement
$x = 10(\sin 30)(20) + \frac{1}{2}(-9.81)(20)^2$
→ gravity is in opposite direction of velocity

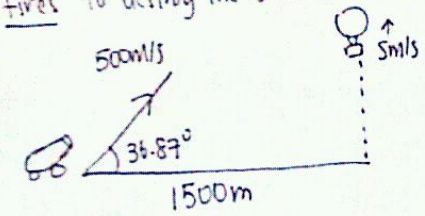
$x = -1860$m

The cliff is 1860m high.

Projectile Motion Qs

1. A ball is thrown from a 100m high cliff. The ball reaches a maximum height of 120m and lands $100\sqrt{3}$ m away from the cliff. With what velocity was it launched?

A gun crew observes a remotely controlled balloon launching an instrumented spy package in enemy territory. When first noticed the balloon is at an altitude of 500m and moving vertically upward at a constant velocity of 5m/s. Shells fired from the gun have an initial velocity of 500m/s at a fixed angle 36.87°. The gun crew waits and fires to destroy the balloon. The balloon is 1500m away.



2. How long does it take for the cannon to hit the balloon?
   a) 5 s
   b) 1.67s
   c) 1.25s
   d) 3.75s
   e) It will never hit the balloon.

3. At what height will they collide?
   a) 1431.09 m
   b) 1125.00 m
   c) 1500.00m
   d) 1056.10m
   e) They will never collide

4. How long did the gun crew wait?
   a) 111.21s
   b) 107.47s
   c) 186.22s
   d) 182.47s
   e) Doesn't matter; they will not be able to hit, no matter how long they wait.

Projectile Motion As

1. $v^2 = v_{y0}^2 + 2a(x - x_0)$

$0^2 = v_{y0}^2 + 2(-9.81)(20)$

$v_{y0}^2 = 392.4$

$v_{y0} = 19.81 \text{ m/s}$

$x = v_{y0} t + \frac{1}{2} a_y t^2$

$-100 = 19.81(t) - \frac{1}{2}(9.81)t^2$

$t = 6.97\text{s}$

$x = v_x t$

$100\sqrt{3} = v_0 \cos\theta \, t$

$100\sqrt{3} = v_{y0}(\cot\theta)(6.97)$

$100\sqrt{3} = 19.81(\cot\theta)(6.97)$

$\theta = 38.56^\circ$

$v_0 \sin\theta = v_y$

$v_0 = \frac{v_y}{\sin\theta}$

$v_0 \sin\theta = v_y$

$v_0(\sin 38.56) = 19.81$

$v_0 = 31.78 \text{ m/s}$

31.78 m/s at angle 38.56° to horizontal

2. $x = v_x t$

$1500 = 500 \cos(36.87)\, t$

$t = 3.75\text{s}$

3. $x = v_y t - \frac{1}{2} a_y t^2$

$= 500 \sin(36.87)(3.75) - 4.9(3.75)^2$

$= 1056.10\text{m}$

4. $x = v_y t + x_0$

$1056.10 = 5(t - 3.75) + 500$

$t = 107.47\text{s}$