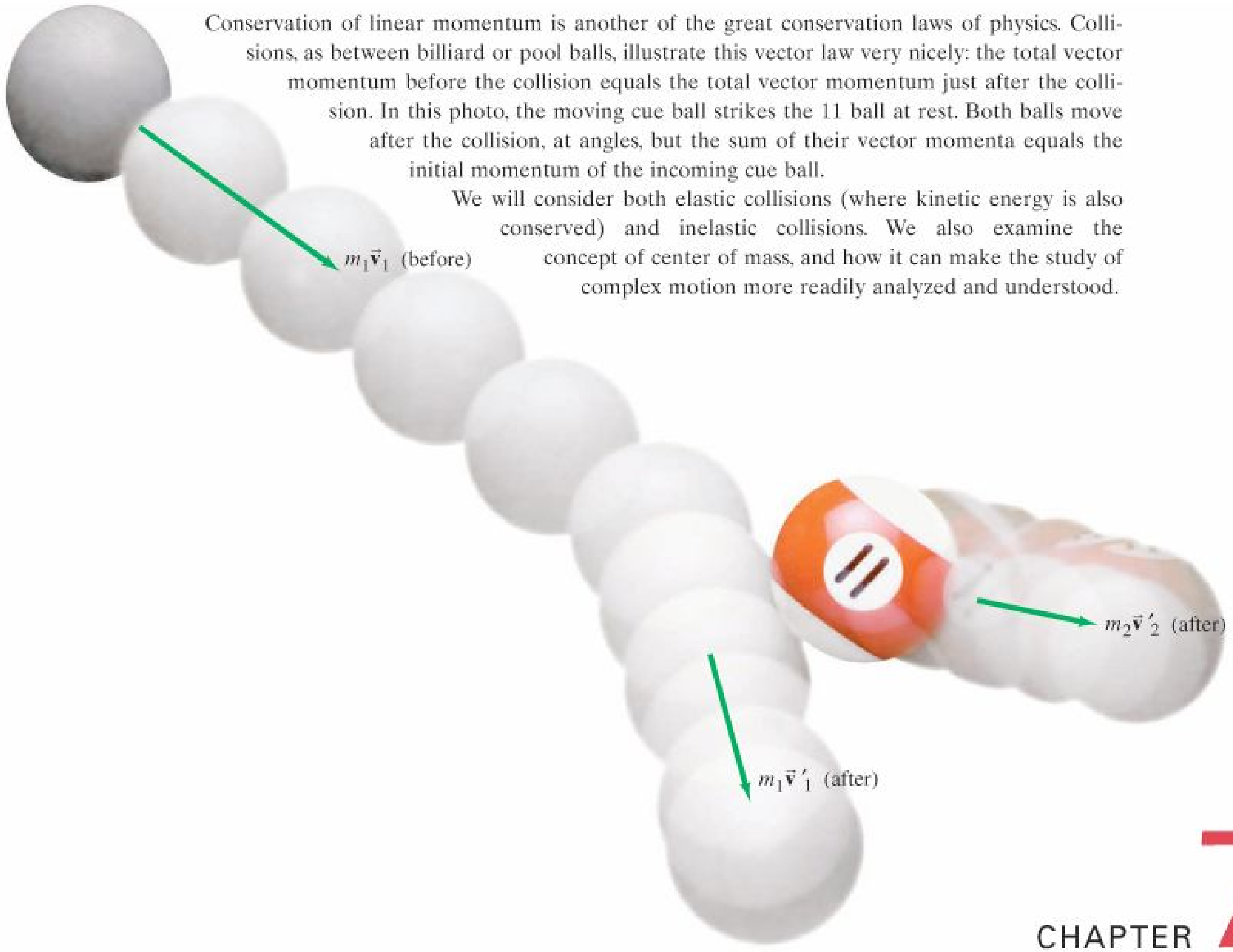


Conservation of linear momentum is another of the great conservation laws of physics. Collisions, as between billiard or pool balls, illustrate this vector law very nicely: the total vector momentum before the collision equals the total vector momentum just after the collision. In this photo, the moving cue ball strikes the 11 ball at rest. Both balls move after the collision, at angles, but the sum of their vector momenta equals the initial momentum of the incoming cue ball.

We will consider both elastic collisions (where kinetic energy is also conserved) and inelastic collisions. We also examine the concept of center of mass, and how it can make the study of complex motion more readily analyzed and understood.

CHAPTER 7

Linear Momentum

The law of conservation of energy, which we discussed in the previous Chapter, is one of several great conservation laws in physics. Among the other quantities found to be conserved are linear momentum, angular momentum, and electric charge. We will eventually discuss all of these because the conservation laws are among the most important ideas in science. In this Chapter, we discuss linear momentum, and its conservation. The law of conservation of momentum is essentially a reworking of Newton's laws that gives us tremendous physical insight and problem-solving power.

We make use of the laws of conservation of linear momentum and of energy to analyze collisions. Indeed, the law of conservation of momentum is particularly useful when dealing with a system of two or more objects that interact with each other, such as in collisions.

Our focus up to now has been mainly on the motion of a single object, often thought of as a "particle" in the sense that we have ignored any rotation or internal motion. In this Chapter we will deal with systems of two or more objects, and toward the end of the Chapter, the concept of center of mass.

7–1 Momentum and Its Relation to Force

The **linear momentum** (or "momentum" for short) of an object is defined as the product of its mass and its velocity. Momentum (plural is *momenta*) is represented by the symbol $\vec{\mathbf{p}}$. If we let m represent the mass of an object and $\vec{\mathbf{v}}$ represent its velocity, then its momentum $\vec{\mathbf{p}}$ is defined as

Linear momentum defined

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}. \tag{7–1}$$

Velocity is a vector, so momentum too is a vector. The direction of the momentum is the direction of the velocity, and the magnitude of the momentum is $p = mv$. Because velocity depends on the reference frame, so does momentum; thus the reference frame must be specified. The unit of momentum is that of mass × velocity, which in SI units is kg·m/s. There is no special name for this unit.

Units of momentum

Everyday usage of the term *momentum* is in accord with the definition above. According to Eq. 7–1, a fast-moving car has more momentum than a slow-moving car of the same mass; a heavy truck has more momentum than a small car moving with the same speed. The more momentum an object has, the harder it is to stop it, and the greater effect it will have if it is brought to rest by striking another object. A football player is more likely to be stunned if tackled by a heavy opponent running at top speed than by a lighter or slower-moving tackler. A heavy, fast-moving truck can do more damage than a slow-moving motorcycle.

EXERCISE A Can a small sports car ever have the same momentum as a large sport-utility vehicle with three times the sports car's mass? Explain.

A force is required to change the momentum of an object, whether it is to increase the momentum, to decrease it, or to change its direction. Newton originally stated his second law in terms of momentum (although he called the product mv the "quantity of motion"). Newton's statement of the **second law of motion**, translated into modern language, is as follows:

NEWTON'S SECOND LAW

The rate of change of momentum of an object is equal to the net force applied to it.

We can write this as an equation,

NEWTON'S SECOND LAW

$$\Sigma\vec{\mathbf{F}} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t}, \tag{7–2}$$

CAUTION
The change in the momentum vector is in the direction of the net force

where $\Sigma\vec{\mathbf{F}}$ is the net force applied to the object (the vector sum of all forces acting on it) and $\Delta\vec{\mathbf{p}}$ is the resulting momentum change that occurs during the time interval† Δt.

We can readily derive the familiar form of the second law, $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$, from Eq. 7–2 for the case of constant mass. If $\vec{\mathbf{v}}_1$ is the initial velocity of an object and $\vec{\mathbf{v}}_2$ is its velocity after a time interval Δt has elapsed, then

$$\begin{aligned}\Sigma\vec{\mathbf{F}} &= \frac{\Delta\vec{\mathbf{p}}}{\Delta t} = \frac{m\vec{\mathbf{v}}_2 - m\vec{\mathbf{v}}_1}{\Delta t} = \frac{m(\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1)}{\Delta t} \\ &= m\frac{\Delta\vec{\mathbf{v}}}{\Delta t}.\end{aligned}$$

By definition, $\vec{\mathbf{a}} = \Delta\vec{\mathbf{v}}/\Delta t$, so

Newton's second law for constant mass

$$\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}. \qquad \text{[constant mass]}$$

Newton's statement, Eq. 7–2, is more general than the more familiar version because it includes the situation in which the mass may change. A change in mass occurs in certain circumstances, such as for rockets which lose mass as they burn fuel, and also in the theory of relativity (Chapter 26).

†Normally we think of Δt as being a small time interval. If it is not small, then Eq. 7–2 is valid if $\Sigma\vec{\mathbf{F}}$ is constant during that time interval, or if $\Sigma\vec{\mathbf{F}}$ is the average net force during that time interval.

EXAMPLE 7–1 **ESTIMATE** **Force of a tennis serve.** For a top player, a tennis ball may leave the racket on the serve with a speed of 55 m/s (about 120 mi/h), Fig. 7–1. If the ball has a mass of 0.060 kg and is in contact with the racket for about 4 ms $(4 \times 10^{-3}\,\mathrm{s})$, estimate the average force on the ball. Would this force be large enough to lift a 60-kg person?

APPROACH The tennis ball is hit when its initial velocity is very nearly zero at the top of the throw, so we take $v_1 = 0$. We use Newton's second law, Eq. 7–2, to calculate the force, ignoring all other forces such as gravity in comparison to that exerted by the tennis racket.

SOLUTION The force exerted on the ball by the racket is

$$F = \frac{\Delta p}{\Delta t} = \frac{mv_2 - mv_1}{\Delta t}$$

where $v_2 = 55\,\mathrm{m/s}$, $v_1 = 0$, and $\Delta t = 0.004\,\mathrm{s}$. Thus

$$F = \frac{\Delta p}{\Delta t} = \frac{(0.060\,\mathrm{kg})(55\,\mathrm{m/s}) - 0}{0.004\,\mathrm{s}} \approx 800\,\mathrm{N}.$$

This is a large force, larger than the weight of a 60-kg person, which would require a force $mg = (60\,\mathrm{kg})(9.8\,\mathrm{m/s^2}) \approx 600\,\mathrm{N}$ to lift.

NOTE The force of gravity acting on the tennis ball is $mg = (0.060\,\mathrm{kg})(9.8\,\mathrm{m/s^2}) = 0.59\,\mathrm{N}$, which justifies our ignoring it compared to the enormous force the racket exerts.

Measuring force

NOTE High-speed photography and radar can give us an estimate of the contact time and the velocity of the ball leaving the racket. But a direct measurement of the force is not practical. Our calculation shows a handy technique for determining an unknown force in the real world.

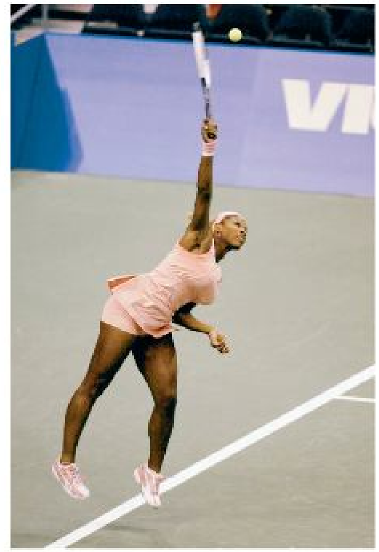


FIGURE 7–1 Example 7–1.

EXAMPLE 7–2 **Washing a car: momentum change and force.** Water leaves a hose at a rate of 1.5 kg/s with a speed of 20 m/s and is aimed at the side of a car, which stops it, Fig. 7–2. (That is, we ignore any splashing back.) What is the force exerted by the water on the car?

APPROACH The water leaving the hose has mass and velocity, so it has a momentum p_{initial}. When the water hits the car, the water loses this momentum $(p_{\mathrm{final}} = 0)$. We use Newton's second law in the momentum form, Eq. 7–2, to find the force that the car exerts on the water to stop it. By Newton's third law, the force exerted by the water on the car is equal and opposite. We have a continuing process: 1.5 kg of water leaves the hose in each 1.0-s time interval. So let us choose $\Delta t = 1.0\,\mathrm{s}$, and $m = 1.5\,\mathrm{kg}$ in Eq. 7–2.

SOLUTION We take the x direction positive to the right. In each 1.0-s time interval, water with a momentum of $p_x = mv_x = (1.5\,\mathrm{kg})(20\,\mathrm{m/s}) = 30\,\mathrm{kg\cdot m/s}$ is brought to rest when it hits the car. The magnitude of the force (assumed constant) that the car must exert to change the momentum of the water by this amount is

$$F = \frac{\Delta p}{\Delta t} = \frac{p_{\mathrm{final}} - p_{\mathrm{initial}}}{\Delta t} = \frac{0 - 30\,\mathrm{kg\cdot m/s}}{1.0\,\mathrm{s}} = -30\,\mathrm{N}.$$

The minus sign indicates that the force on the water is opposite to the water's original velocity. The car exerts a force of 30 N to the left to stop the water, so by Newton's third law, the water exerts a force of 30 N to the right on the car.

NOTE Keep track of signs, although common sense helps too. The water is moving to the right, so common sense tells us the force on the car must be to the right.

FIGURE 7–2 Example 7–2.

EXERCISE B If the water splashes back from the car in Example 7–2, would the force on the car be larger or smaller?

7–2 Conservation of Momentum

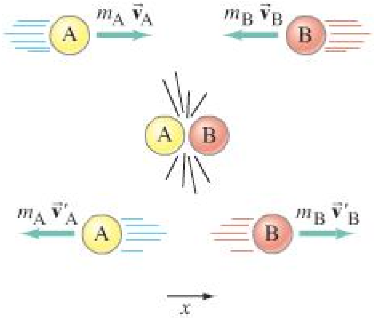


FIGURE 7–3 Momentum is conserved in a collision of two balls, labelled A and B.

The concept of momentum is particularly important because, under certain circumstances, momentum is a conserved quantity. Consider, for example, the head-on collision of two billiard balls, as shown in Fig. 7–3. We assume the net external force on this system of two balls is zero—that is, the only significant forces during the collision are the forces that each ball exerts on the other. Although the momentum of each of the two balls changes as a result of the collision, the *sum* of their momenta is found to be the same before as after the collision. If $m_A \vec{\mathbf{v}}_A$ is the momentum of ball A and $m_B \vec{\mathbf{v}}_B$ the momentum of ball B, both measured just before the collision, then the total momentum of the two balls before the collision is the vector sum $m_A \vec{\mathbf{v}}_A + m_B \vec{\mathbf{v}}_B$. Immediately after the collision, the balls each have a different velocity and momentum, which we designate by a "prime" on the velocity: $m_A \vec{\mathbf{v}}'_A$ and $m_B \vec{\mathbf{v}}'_B$. The total momentum after the collision is the vector sum $m_A \vec{\mathbf{v}}'_A + m_B \vec{\mathbf{v}}'_B$. No matter what the velocities and masses are, experiments show that the total momentum before the collision is the same as afterward, whether the collision is head-on or not, as long as no net external force acts:

CONSERVATION OF MOMENTUM (for two objects colliding)

$$\text{momentum before} = \text{momentum after}$$

$$m_A \vec{\mathbf{v}}_A + m_B \vec{\mathbf{v}}_B = m_A \vec{\mathbf{v}}'_A + m_B \vec{\mathbf{v}}'_B. \qquad \textbf{(7–3)}$$

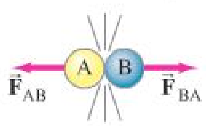
That is, the total vector momentum of the system of two colliding balls is conserved: it stays constant.

Momentum conservation related to Newton's laws

Although the law of conservation of momentum was discovered experimentally, it is closely connected to Newton's laws of motion and they can be shown to be equivalent. We will do a derivation for the head–on collision illustrated in Fig. 7–3. We assume the force F that one ball exerts on the other during the collision is constant over the brief time interval of the collision Δt. We use Newton's second law as expressed in Eq. 7–2, and rewrite it by multiplying both sides by Δt:

$$\Delta \vec{\mathbf{p}} = \vec{\mathbf{F}}\,\Delta t. \qquad \textbf{(7–4)}$$

FIGURE 7–4 Forces on the balls during the collision of Fig. 7–3.



We apply this to ball B alone, noting that the force $\vec{\mathbf{F}}_{BA}$ on ball B exerted by ball A during the collision is to the right ($+x$ direction—see Fig. 7–4):

$$\Delta \vec{\mathbf{p}}_B = \vec{\mathbf{F}}_{BA}\,\Delta t$$

$$m_B \vec{\mathbf{v}}'_B - m_B \vec{\mathbf{v}}_B = \vec{\mathbf{F}}_{BA}\,\Delta t.$$

By Newton's third law, the force $\vec{\mathbf{F}}_{AB}$ on ball A due to ball B is $\vec{\mathbf{F}}_{AB} = -\vec{\mathbf{F}}_{BA}$ and acts to the left. Then applying Newton's second law in the same way to ball A yields

$$\Delta \vec{\mathbf{p}}_A = \vec{\mathbf{F}}_{AB}\,\Delta t$$

or

$$m_A \vec{\mathbf{v}}'_A - m_A \vec{\mathbf{v}}_A = \vec{\mathbf{F}}_{AB}\,\Delta t = -\vec{\mathbf{F}}_{BA}\,\Delta t.$$

We combine these two $\Delta \vec{\mathbf{p}}$ equations (their right sides differ only by a minus sign):

$$m_A \vec{\mathbf{v}}'_A - m_A \vec{\mathbf{v}}_A = -(m_B \vec{\mathbf{v}}'_B - m_B \vec{\mathbf{v}}_B)$$

or

$$m_A \vec{\mathbf{v}}_A + m_B \vec{\mathbf{v}}_B = m_A \vec{\mathbf{v}}'_A + m_B \vec{\mathbf{v}}'_B$$

which is Eq. 7–3, the conservation of momentum.

The above derivation can be extended to include any number of interacting objects. To show this, we let $\vec{\mathbf{p}}$ in Eq. 7–2 represent the total momentum of a system—that is, the vector sum of the momenta of all objects in the system. (For our two-object system above, $\vec{\mathbf{p}} = m_A \vec{\mathbf{v}}_A + m_B \vec{\mathbf{v}}_B$.) If the net force $\Sigma \vec{\mathbf{F}}$ on the system is zero [as it was above for our two-object system, $\vec{\mathbf{F}} + (-\vec{\mathbf{F}}) = 0$,] then from Eq. 7–2, $\Delta \vec{\mathbf{p}} = \vec{\mathbf{F}}\,\Delta t = 0$, so the total momentum doesn't change. Thus the general statement of the **law of conservation of momentum** is

LAW OF CONSERVATION OF MOMENTUM

The total momentum of an isolated system of objects remains constant.

Systems

Isolated system

By a **system**, we simply mean a set of objects that we choose, and which may interact with each other. An **isolated system** is one in which the only (significant) forces are those between the objects in the system. The sum of all these "internal" forces within the system will be zero because of Newton's third law. If there are *external forces*—by which we mean forces exerted by objects outside the system—and they don't add up to zero (vectorially), then the total momentum of the system won't be conserved. However, if the system can be redefined so as to include the other objects exerting these forces, then the conservation of momentum principle can apply. For example, if we take as our system a rock falling under gravity, the momentum of this system (the rock) is not conserved: an external force, the force of gravity exerted by the Earth, is acting on it and changes its momentum. However, if we include the Earth in the system, the total momentum of rock plus Earth is conserved. (This means that the Earth comes up to meet the rock. But the Earth's mass is so great, its upward velocity is very tiny.)

EXAMPLE 7–3 Railroad cars collide: momentum conserved. A 10,000-kg railroad car, A, traveling at a speed of 24.0 m/s strikes an identical car, B, at rest. If the cars lock together as a result of the collision, what is their common speed just afterward? See Fig. 7–5.

APPROACH We choose our system to be the two railroad cars. We consider a very brief time interval, from just before the collision until just after, so that external forces such as friction can be ignored. Then we apply conservation of momentum.

SOLUTION The initial total momentum is

$$p_{\text{initial}} = m_A v_A + m_B v_B = m_A v_A$$

because car B is at rest initially $(v_B = 0)$. The direction is to the right in the $+x$ direction. After the collision, the two cars become attached, so they will have the same speed, call it v'. Then the total momentum after the collision is

$$p_{\text{final}} = (m_A + m_B)v'.$$

We have assumed there are no external forces, so momentum is conserved:

$$p_{\text{initial}} = p_{\text{final}}$$
$$m_A v_A = (m_A + m_B)v'.$$

Solving for v', we obtain

$$v' = \frac{m_A}{m_A + m_B} v_A = \left(\frac{10{,}000\text{ kg}}{10{,}000\text{ kg} + 10{,}000\text{ kg}}\right)(24.0\text{ m/s}) = 12.0\text{ m/s},$$

to the right. Their mutual speed after collision is half the initial speed of car A.

NOTE We kept symbols until the very end, so we have an equation we can use in other (related) situations.

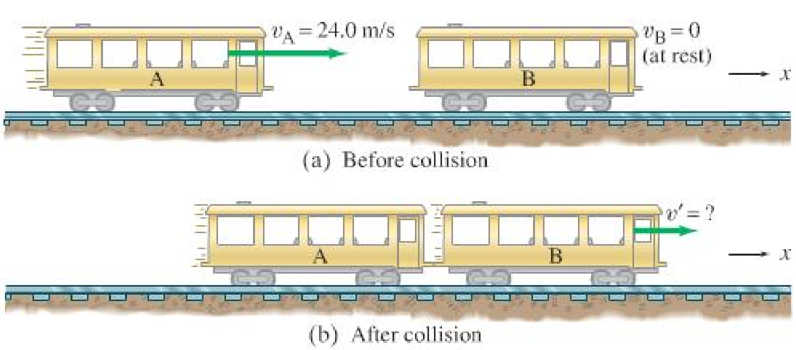


FIGURE 7–5 Example 7–3.

EXERCISE C In Example 7–3, $m_A = m_B$, so in the last equation, $m_A/(m_A + m_B) = \frac{1}{2}$. Hence $v' = \frac{1}{2}v_A$. What result do you get if (*a*) $m_B = 3m_A$, (*b*) m_B is much larger than m_A ($m_B \gg m_A$), and (*c*) $m_B \ll m_A$?

As long as no external forces act on our chosen system, conservation of momentum is valid. In the real world, external forces do act: friction on billiard balls, gravity acting on a baseball, and so on. So it may seem that conservation of momentum cannot be applied. Or can it? In a collision, the force each object exerts on the other acts only over a very brief time interval, and is very strong. When a racket hits a tennis ball (or a bat hits a baseball), both before and after the "collision" the ball moves as a projectile under the action of gravity and air resistance. During the brief time of the collision, however, when the racket hits the ball, external forces (gravity, air resistance) are insignificant compared to the collision forces that the racket and ball exert on each other. So if we measure the momenta just before and just after the collision, we can apply momentum conservation with high accuracy.

PHYSICS APPLIED
Rocket propulsion

CAUTION
A rocket pushes on the gases released by the fuel, not on the Earth or other objects

The law of conservation of momentum is particularly useful when we are dealing with fairly simple systems such as colliding objects and certain types of "explosions". For example, *rocket propulsion*, which we saw in Chapter 4 can be understood on the basis of action and reaction, can also be explained on the basis of the conservation of momentum. We can consider the rocket and fuel as an isolated system if it is far out in space (no external forces). In the reference frame of the rocket, the total momentum of rocket plus fuel is zero. When the fuel burns, the total momentum remains unchanged: the backward momentum of the expelled gases is just balanced by the forward momentum gained by the rocket itself (see Fig. 7–6). Thus, a rocket can accelerate in empty space. There is no need for the expelled gases to push against the Earth or the air (as is sometimes erroneously thought). Similar examples of (nearly) isolated systems where momentum is conserved are the recoil of a gun when a bullet is fired, and the movement of a rowboat just after a package is thrown from it.

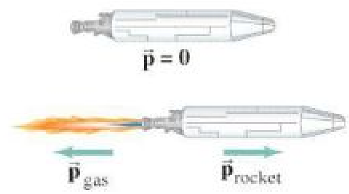


FIGURE 7–6 (a) A rocket, containing fuel, at rest in some reference frame. (b) In the same reference frame, the rocket fires and gases are expelled at high speed out the rear. The total vector momentum, $\vec{\mathbf{p}}_{gas} + \vec{\mathbf{p}}_{rocket}$, remains zero.

EXAMPLE 7–4 **Rifle recoil.** Calculate the recoil velocity of a 5.0-kg rifle that shoots a 0.020-kg bullet at a speed of 620 m/s, Fig. 7–7.

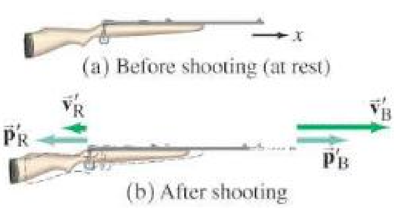


FIGURE 7–7 Example 7–4.

APPROACH Our system is the rifle and the bullet, both at rest initially, just before the trigger is pulled. The trigger is pulled, an explosion occurs, and we look at the rifle and bullet just as the bullet leaves the barrel. The bullet moves to the right ($+x$), and the gun recoils to the left. During the very short time interval of the explosion, we can assume the external forces are small compared to the forces exerted by the exploding gunpowder. Thus we can apply conservation of momentum, at least approximately.

SOLUTION Let subscript B represent the bullet and R the rifle; the final velocities are indicated by primes. Then momentum conservation in the x direction gives

$$\begin{aligned} \text{momentum before} &= \text{momentum after} \\ m_B v_B + m_R v_R &= m_B v'_B + m_R v'_R \\ 0 + 0 &= m_B v'_B + m_R v'_R \end{aligned}$$

so

$$v'_R = -\frac{m_B v'_B}{m_R} = -\frac{(0.020\,\text{kg})(620\,\text{m/s})}{(5.0\,\text{kg})} = -2.5\,\text{m/s}.$$

Since the rifle has a much larger mass, its (recoil) velocity is much less than that of the bullet. The minus sign indicates that the velocity (and momentum) of the rifle is in the negative x direction, opposite to that of the bullet.

CONCEPTUAL EXAMPLE 7–5 **Falling on or off a sled.** (*a*) An empty sled is sliding on frictionless ice when Susan drops vertically from a tree above onto the sled. When she lands, does the sled speed up, slow down, or keep the same speed? (*b*) Later, Susan falls sideways off the sled. When she drops off, does the sled speed up, slow down, or keep the same speed?

RESPONSE (*a*) Because Susan falls vertically onto the sled, she has no initial horizontal momentum. Thus the total horizontal momentum afterward equals the momentum of the sled initially. Since the mass of the system (sled + person) has increased, the speed must decrease.
(*b*) At the instant Susan falls off, she is moving with the same horizontal speed as she was while on the sled. At the moment she leaves the sled, she has the same momentum she had an instant before. Because momentum is conserved, the sled keeps the same speed.

7–3 Collisions and Impulse

Collisions are a common occurrence in everyday life: a tennis racket or a baseball bat striking a ball, billiard balls colliding, a hammer hitting a nail. When a collision occurs, the interaction between the objects involved is usually far stronger than any interaction between our system of objects and their environment. We can then ignore the effects of any other forces during the brief time interval of the collision.

During a collision of two ordinary objects, both objects are deformed, often considerably, because of the large forces involved (Fig. 7–8). When the collision occurs, the force usually jumps from zero at the moment of contact to a very large force within a very short time, and then rapidly returns to zero again. A graph of the magnitude of the force that one object exerts on the other during a collision, as a function of time, is something like the red curve in Fig. 7–9. The time interval Δt is usually very distinct and very small.

FIGURE 7–8 Tennis racket striking a ball. Both the ball and the racket strings are deformed due to the large force each exerts on the other.

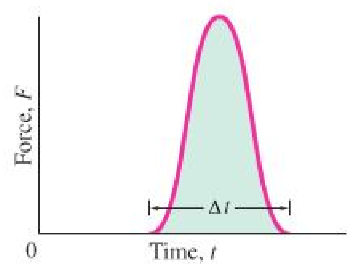
From Newton's second law, Eq. 7–2, the *net* force on one object is equal to the rate of change of its momentum:

$$\vec{\mathbf{F}} = \frac{\Delta \vec{\mathbf{p}}}{\Delta t}.$$

(We have written $\vec{\mathbf{F}}$ instead of $\Sigma\vec{\mathbf{F}}$ for the net force, which we assume is entirely due to the brief but large average force that acts during the collision.) This equation applies to *each* of the two objects in a collision. We multiply both sides of this equation by the time interval Δt, and obtain

$$\vec{\mathbf{F}}\,\Delta t = \Delta \vec{\mathbf{p}}. \qquad \textbf{(7–5)}$$

FIGURE 7–9 Force as a function of time during a typical collision.

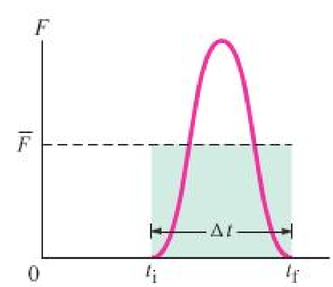


The quantity on the left, the product of the force $\vec{\mathbf{F}}$ times the time Δt over which the force acts, is called the **impulse**:

$$\text{Impulse} = \vec{\mathbf{F}}\,\Delta t.$$

We see that the total change in momentum is equal to the impulse. The concept of impulse is useful mainly when dealing with forces that act during a short time interval, as when a bat hits a baseball. The force is generally not constant, and often its variation in time is like that graphed in Figs. 7–9 and 7–10. We can often approximate such a varying force as an average force $\overline{F}$ acting during a time interval Δt, as indicated by the dashed line in Fig. 7–10. $\overline{F}$ is chosen so that the area shown shaded in Fig. 7–10 (equal to $\overline{F} \times \Delta t$) is equal to the area under the actual curve of F vs. t, Fig. 7–9 (which represents the actual impulse).

FIGURE 7–10 The average force $\overline{F}$ acting over an interval of time Δt gives the same impulse ($\overline{F}\,\Delta t$) as the actual force.



EXERCISE D Suppose Fig. 7–9 illustrates the force on a golf ball vs. the time when the ball hits a wall. How would the shape of this curve change if a softer rubber ball with the same mass and speed hit the same wall?

EXAMPLE 7–6 **Bend your knees when landing.** (*a*) Calculate the impulse experienced when a 70-kg person lands on firm ground after jumping from a height of 3.0 m. (*b*) Estimate the average force exerted on the person's feet by the ground if the landing is stiff-legged, and again (*c*) with bent legs. With stiff legs, assume the body moves 1.0 cm during impact, and when the legs are bent, about 50 cm.

PHYSICS APPLIED
How not to break a leg

APPROACH We consider the short time interval that starts just before the person hits the ground and ends when he is brought to rest. During this time interval, the ground exerts a force on him and gives him an impulse which equals his change in momentum (Eq. 7–5). For part (*a*) we know his final speed (zero, when he comes to rest), but we need to calculate his "initial" speed just before impact with the ground. The latter is found using kinematics and his drop from a height of 3.0 m. Then Eq. 7–5 gives us $F\Delta t$. In parts (*b*) and (*c*) we calculate how long, Δt, it takes him to slow down as he hits the ground, using kinematics, and then obtain F because we know $F\Delta t$.

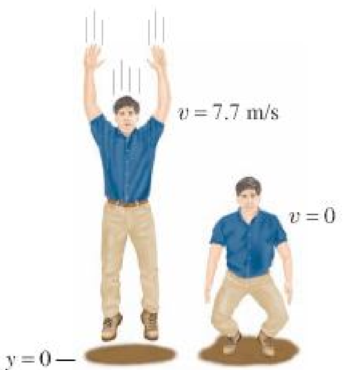


FIGURE 7–11 Example 7–6. Time interval Δt during which the impulse acts.

SOLUTION (*a*) First we need to determine the velocity of the person just before striking the ground, which we do by considering the earlier time period between the initial jump from a height of 3.0 m until just before he touches the ground. The person falls under gravity, so we can use the kinematic Eq. 2–11c, $v^2 = v_0^2 + 2a(y - y_0)$ with $a = -g$ and $v_0 = 0$, so

$$v^2 = 2g(y_0 - y)$$

or

$$v = \sqrt{2g(y_0 - y)} = \sqrt{2(9.8\,\mathrm{m/s^2})(3.0\,\mathrm{m})} = 7.7\,\mathrm{m/s}.$$

This $v = 7.7\,\mathrm{m/s}$ is his speed just before hitting the ground, and so it is the initial speed for the short time interval of the impact with the ground, Δt. Now we can determine the impulse by examining this brief time interval as the person hits the ground and is brought to rest (Fig. 7–11). We don't know F and thus can't calculate the impulse $F\Delta t$ directly; but we can use Eq. 7–5: the impulse equals the change in momentum of the object

$$\begin{aligned}\overline{F}\,\Delta t &= \Delta p = m\,\Delta v \\ &= (70\,\mathrm{kg})(0 - 7.7\,\mathrm{m/s}) = -540\,\mathrm{N\cdot s}.\end{aligned}$$

The negative sign tells us that the force is opposed to the original (downward) momentum; that is, the force acts upward.

(*b*) In coming to rest, the person decelerates from 7.7 m/s to zero in a distance $d = 1.0\,\mathrm{cm} = 1.0 \times 10^{-2}\,\mathrm{m}$. If we assume the upward force exerted on him by the ground is constant, then the average speed during this brief period is

$$\overline{v} = \frac{(7.7\,\mathrm{m/s} + 0\,\mathrm{m/s})}{2} = 3.9\,\mathrm{m/s}.$$

Thus the collision with the ground lasts for a time interval (recall the definition of speed, $\overline{v} = d/\Delta t$):

$$\Delta t = \frac{d}{\overline{v}} = \frac{(1.0 \times 10^{-2}\,\mathrm{m})}{(3.9\,\mathrm{m/s})} = 2.6 \times 10^{-3}\,\mathrm{s}.$$

Since the magnitude of the impulse is $\overline{F}\,\Delta t = 540\,\mathrm{N\cdot s}$, and $\Delta t = 2.6 \times 10^{-3}\,\mathrm{s}$, the average net force $\overline{F}$ on the person has magnitude

$$\overline{F} = \frac{540\,\mathrm{N\cdot s}}{2.6 \times 10^{-3}\,\mathrm{s}} = 2.1 \times 10^5\,\mathrm{N}.$$

PROBLEM SOLVING
Free-body diagrams are always useful!

We are almost there. $\overline{F}$ equals the vector sum of the average force upward on the legs exerted by the ground, F_{grd}, which we take as positive, plus the downward force of gravity, $-mg$ (see Fig. 7–12):

$$\overline{F} = F_{\mathrm{grd}} - mg.$$

Since $mg = (70\,\mathrm{kg})(9.8\,\mathrm{m/s^2}) = 690\,\mathrm{N}$, then

$$F_{\mathrm{grd}} = \overline{F} + mg = (2.1 \times 10^5\,\mathrm{N}) + (0.690 \times 10^3\,\mathrm{N}) \approx 2.1 \times 10^5\,\mathrm{N}.$$

(c) This is just like part (b), except $d = 0.50\text{ m}$, so

$$\Delta t = \frac{d}{\bar{v}} = \frac{0.50\text{ m}}{3.9\text{ m/s}} = 0.13\text{ s}$$

and

$$\bar{F} = \frac{540\text{ N}\cdot\text{s}}{0.13\text{ s}} = 4.2 \times 10^3\text{ N}.$$

The upward force exerted on the person's feet by the ground is, as in part (b):

$$F_{\text{grd}} = \bar{F} + mg = (4.2 \times 10^3\text{ N}) + (0.69 \times 10^3\text{ N}) = 4.9 \times 10^3\text{ N}.$$

Clearly, the force on the feet and legs is much less now with the knees bent, and the impulse occurs over a longer time interval. In fact, the ultimate strength of the leg bone (see Chapter 9, Table 9–2) is not great enough to support the force calculated in part (b), so the leg would likely break in such a stiff landing, whereas it probably wouldn't in part (c) with bent legs.

FIGURE 7–12 Example 7–6. When the person lands on the ground, the average net force during impact is $\bar{F} = F_{\text{grd}} - mg$, where F_{grd} is the force the ground exerts upward on the person.

EXERCISE E In part (b) of Example 7–6, we calculated the force exerted by the ground on the person during the collision, F_{grd}. Was F_{grd} much greater than the "external" force of gravity on the person? By what factor?

7–4 Conservation of Energy and Momentum in Collisions

During most collisions, we usually don't know how the collision force varies over time, and so analysis using Newton's second law becomes difficult or impossible. But by making use of the conservation laws for momentum and energy, we can still determine a lot about the motion after a collision, given the motion before the collision. We saw in Section 7–2 that in the collision of two objects such as billiard balls, the total momentum is conserved. If the two objects are very hard and no heat or other form of energy is produced in the collision, then kinetic energy is conserved as well. By this we mean that the sum of the kinetic energies of the two objects is the same after the collision as before. For the brief moment during which the two objects are in contact, some (or all) of the energy is stored momentarily in the form of elastic potential energy. But if we compare the total kinetic energy just before the collision with the total kinetic energy just after the collision, they are found to be the same. Such a collision, in which the total kinetic energy is conserved, is called an **elastic collision**. If we use the subscripts A and B to represent the two objects, we can write the equation for conservation of total kinetic energy as

$$\text{total KE before} = \text{total KE after}$$

$$\tfrac{1}{2}m_A v_A^2 + \tfrac{1}{2}m_B v_B^2 = \tfrac{1}{2}m_A v_A'^2 + \tfrac{1}{2}m_B v_B'^2. \quad \text{[elastic collision]} \quad \textbf{(7–6)}$$

Here, primed quantities (′) mean after the collision and unprimed mean before the collision, just as in Eq. 7–3 for conservation of momentum.

At the atomic level the collisions of atoms and molecules are often elastic. But in the "macroscopic" world of ordinary objects, an elastic collision is an ideal that is never quite reached, since at least a little thermal energy (and perhaps sound and other forms of energy) is always produced during a collision. The collision of two hard elastic balls, such as billiard balls, however, is very close to being perfectly elastic, and we often treat it as such.

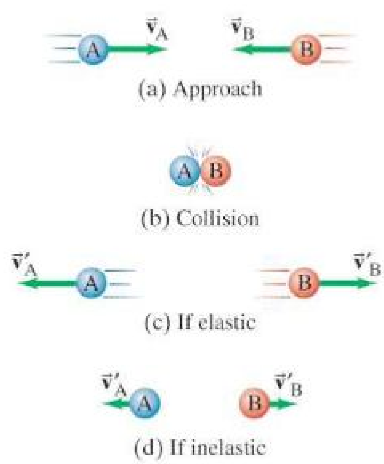
We do need to remember that even when the kinetic energy is not conserved, the *total* energy is always conserved.

Collisions in which kinetic energy is not conserved are said to be **inelastic collisions**. The kinetic energy that is lost is changed into other forms of energy, often thermal energy, so that the total energy (as always) is conserved. In this case,

$$\text{KE}_A + \text{KE}_B = \text{KE}_A' + \text{KE}_B' + \text{thermal and other forms of energy.}$$

See Fig. 7–13, and the details in its caption.

FIGURE 7–13 Two equal-mass objects (a) approach each other with equal speeds, (b) collide, and then (c) bounce off with equal speeds in the opposite directions if the collision is elastic, or (d) bounce back much less or not at all if the collision is inelastic.



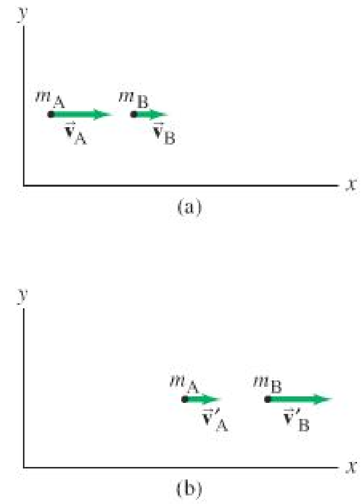


FIGURE 7–14 Two small objects of masses m_A and m_B. (a) before the collision and (b) after the collision.

7–5 Elastic Collisions in One Dimension

We now apply the conservation laws for momentum and kinetic energy to an elastic collision between two small objects that collide head-on, so all the motion is along a line. Let us assume that the two objects are moving with velocities v_A and v_B along the x axis before the collision, Fig. 7–14a. After the collision, their velocities are v'_A and v'_B, Fig. 7–14b. For any $v > 0$, the object is moving to the right (increasing x), whereas for $v < 0$, the object is moving to the left (toward decreasing values of x).

From conservation of momentum, we have

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B.$$

Because the collision is assumed to be elastic, kinetic energy is also conserved:

$$\tfrac{1}{2} m_A v_A^2 + \tfrac{1}{2} m_B v_B^2 = \tfrac{1}{2} m_A v'^2_A + \tfrac{1}{2} m_B v'^2_B.$$

We have two equations, so we can solve for two unknowns. If we know the masses and velocities before the collision, then we can solve these two equations for the velocities after the collision, v'_A and v'_B. We derive a helpful result by rewriting the momentum equation as

$$m_A(v_A - v'_A) = m_B(v'_B - v_B), \qquad \textbf{(i)}$$

and we rewrite the kinetic energy equation as

$$m_A(v_A^2 - v'^2_A) = m_B(v'^2_B - v_B^2).$$

Noting that algebraically $(a - b)(a + b) = a^2 - b^2$, we write this last equation as

$$m_A(v_A - v'_A)(v_A + v'_A) = m_B(v'_B - v_B)(v'_B + v_B). \qquad \textbf{(ii)}$$

We divide Eq. (ii) by Eq. (i), and (assuming $v_A \neq v'_A$ and $v_B \neq v'_B$) obtain

$$v_A + v'_A = v'_B + v_B.$$

We can rewrite this equation as

$$v_A - v_B = v'_B - v'_A$$

or

Relative speeds (one dimension only)

$$v_A - v_B = -(v'_A - v'_B). \qquad \text{[head-on elastic collision]} \quad \textbf{(7–7)}$$

This is an interesting result: it tells us that for any elastic head-on collision, the relative speed of the two objects after the collision has the same magnitude (but opposite direction) as before the collision, no matter what the masses are.

Equation 7–7 was derived from conservation of kinetic energy for elastic collisions, and can be used in place of it. Because the v's are not squared in Eq. 7–7, it is simpler to use in calculations than the conservation of kinetic energy equation (Eq. 7–6) directly.

EXAMPLE 7–7 **Pool or billiards.** Billiard ball A of mass m moving with speed v collides head-on with ball B of equal mass at rest $(v_B = 0)$. What are the speeds of the two balls after the collision, assuming it is elastic?

APPROACH There are two unknowns, v'_A and v'_B, so we need two independent equations. We focus on the time interval from just before the collision until just after. No net external force acts on our system of two balls (mg and the normal force cancel), so momentum is conserved. Conservation of kinetic energy applies as well because the collision is elastic.

SOLUTION Given $v_A = v$ and $v_B = 0$, and $m_A = m_B = m$, then conservation of momentum gives

$$mv = mv'_A + mv'_B$$

or, since the m's cancel out,

$$v = v'_A + v'_B.$$

We have two unknowns (v'_A and v'_B) and need a second equation, which could be the conservation of kinetic energy or the simpler Eq. 7–7 we derived from it:

$$v_A - v_B = v'_B - v'_A, \qquad \text{or} \qquad v = v'_B - v'_A$$

since $v_A = v$ and $v_B = 0$. We subtract $v = v'_B - v'_A$ from our momentum equation $(v = v'_A + v'_B)$ and obtain

$$0 = 2v'_A.$$

Hence $v'_A = 0$. We can now solve for the other unknown (v'_B) since $v = v'_B - v'_A$:

$$v'_B = v + v'_A = v + 0 = v.$$

To summarize, before the collision we have

$$v_A = v, \qquad v_B = 0$$

and after the collision

$$v'_A = 0, \qquad v'_B = v.$$

That is, ball A is brought to rest by the collision, whereas ball B acquires the original velocity of ball A. See Fig. 7–15.

NOTE Our result is often observed by billiard and pool players, and is valid only if the two balls have equal masses (and no spin is given to the balls).

FIGURE 7–15 In this multiflash photo of a head-on collision between two balls of equal mass, the white cue ball is accelerated from rest by the cue stick and then strikes the red ball, initially at rest. The white ball stops in its tracks, and the (equal mass) red ball moves off with the same speed as the white ball had before the collision. See Example 7–7.

EXAMPLE 7–8 **A nuclear collision.** A proton (p) of mass 1.01 u (unified atomic mass units) traveling with a speed of $3.60 \times 10^4\,\text{m/s}$ has an elastic head-on collision with a helium (He) nucleus $(m_{He} = 4.00\,\text{u})$ initially at rest. What are the velocities of the proton and helium nucleus after the collision? (As mentioned in Chapter 1, $1\,\text{u} = 1.66 \times 10^{-27}\,\text{kg}$, but we won't need this fact.) Assume the collision takes place in nearly empty space.

APPROACH Like Example 7–7, this is an elastic head-on collision, but now the masses of our two-particle system are not equal. The only external force is Earth's gravity, but it is insignificant compared to the strong force during the collision. So again we use the conservation laws of momentum and of kinetic energy, and apply them to our system of two particles.

SOLUTION Let the proton (p) be particle A and the helium nucleus (He) be particle B. We have $v_B = v_{He} = 0$ and $v_A = v_p = 3.60 \times 10^4\,\text{m/s}$. We want to find the velocities v'_p and v'_{He} after the collision. From conservation of momentum,

$$m_p v_p + 0 = m_p v'_p + m_{He} v'_{He}.$$

Because the collision is elastic, the kinetic energy of our system of two particles is conserved and we can use Eq. 7–7, which becomes

$$v_p - 0 = v'_{He} - v'_p.$$

Thus

$$v'_p = v'_{He} - v_p,$$

and substituting this into our momentum equation displayed above, we get

$$m_p v_p = m_p v'_{He} - m_p v_p + m_{He} v'_{He}.$$

Solving for v'_{He}, we obtain

$$v'_{He} = \frac{2m_p v_p}{m_p + m_{He}} = \frac{2(1.01\,\text{u})(3.60 \times 10^4\,\text{m/s})}{5.01\,\text{u}} = 1.45 \times 10^4\,\text{m/s}.$$

The other unknown is v'_p, which we can now obtain from

$$v'_p = v'_{He} - v_p = (1.45 \times 10^4\,\text{m/s}) - (3.60 \times 10^4\,\text{m/s}) = -2.15 \times 10^4\,\text{m/s}.$$

The minus sign for v'_p tells us that the proton reverses direction upon collision, and we see that its speed is less than its initial speed (see Fig. 7–16).

NOTE This result makes sense: the lighter proton would be expected to "bounce back" from the more massive helium nucleus, but not with its full original velocity as from a rigid wall (which corresponds to extremely large, or infinite, mass).

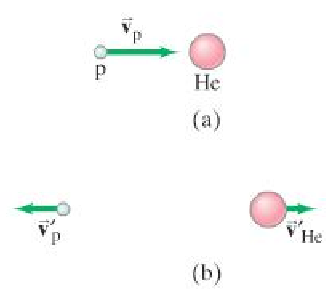


FIGURE 7–16 Example 7–8: (a) before collision, (b) after collision.

7–6 Inelastic Collisions

Collisions in which kinetic energy is not conserved are called *inelastic collisions*. Some of the initial kinetic energy is transformed into other types of energy, such as thermal or potential energy, so the total kinetic energy after the collision is less than the total kinetic energy before the collision. The inverse can also happen when potential energy (such as chemical or nuclear) is released, in which case the total kinetic energy after the interaction can be greater than the initial kinetic energy. Explosions are examples of this type.

Completely inelastic collision

Typical macroscopic collisions are inelastic, at least to some extent, and often to a large extent. If two objects stick together as a result of a collision, the collision is said to be **completely inelastic**. Two colliding balls of putty that stick together or two railroad cars that couple together when they collide are examples of completely inelastic collisions. The kinetic energy in some cases is all transformed to other forms of energy in an inelastic collision, but in other cases only part of it is. In Example 7–3, for instance, we saw that when a traveling railroad car collided with a stationary one, the coupled cars traveled off with some kinetic energy. In a completely inelastic collision, the maximum amount of kinetic energy is transformed to other forms consistent with conservation of momentum. Even though kinetic energy is not conserved in inelastic collisions, the total energy is always conserved, and the total vector momentum is also conserved.

EXAMPLE 7–9 Railroad cars again. For the completely inelastic collision of two railroad cars that we considered in Example 7–3, calculate how much of the initial kinetic energy is transformed to thermal or other forms of energy.

APPROACH The railroad cars stick together after the collision, so this is a completely inelastic collision. By subtracting the total kinetic energy after the collision from the total initial kinetic energy, we can find how much energy is transformed to other types of energy.

SOLUTION Before the collision, only car A is moving, so the total initial kinetic energy is

$$\tfrac{1}{2}m_A v_A^2 = \tfrac{1}{2}(10{,}000\,\text{kg})(24.0\,\text{m/s})^2 = 2.88 \times 10^6\,\text{J}.$$

After the collision, both cars are moving with a speed of 12.0 m/s, by conservation of momentum (Example 7–3). So the total kinetic energy afterward is

$$\tfrac{1}{2}(20{,}000\,\text{kg})(12.0\,\text{m/s})^2 = 1.44 \times 10^6\,\text{J}.$$

Hence the energy transformed to other forms is

$$\left(2.88 \times 10^6\,\text{J}\right) - \left(1.44 \times 10^6\,\text{J}\right) = 1.44 \times 10^6\,\text{J},$$

which is just half the original kinetic energy.

Ballistic pendulum

EXAMPLE 7–10 Ballistic pendulum. The *ballistic pendulum* is a device used to measure the speed of a projectile, such as a bullet. The projectile, of mass m, is fired into a large block (of wood or other material) of mass M, which is suspended like a pendulum. (Usually, M is somewhat greater than m.) As a result of the collision, the pendulum and projectile together swing up to a maximum height h, Fig. 7–17. Determine the relationship between the initial horizontal speed of the projectile, v, and the maximum height h.

APPROACH We can analyze the process by dividing it into two parts or two time intervals: (1) the time interval from just before to just after the collision itself, and (2) the subsequent time interval in which the pendulum moves from the vertical hanging position to the maximum height h.

In part (1), Fig. 7–17a, we assume the collision time is very short, so the projectile comes to rest in the block before the block has moved significantly from its position directly below its support. Thus there is effectively no net external force, and we can apply conservation of momentum to this completely inelastic collision. In part (2), Fig. 7–17b, the pendulum begins to move, subject to a net external force (gravity, tending to pull it back to the vertical position); so for part (2), we cannot use conservation of momentum. But we can use conservation of mechanical energy because gravity is a conservative force (Chapter 6). The kinetic energy immediately after the collision is changed entirely to gravitational potential energy when the pendulum reaches its maximum height, h.

SOLUTION In part (1) momentum is conserved:

$$\text{total } p \text{ before} = \text{total } p \text{ after}$$

$$mv = (m + M)v', \qquad \textbf{(i)}$$

where v' is the speed of the block and embedded projectile just after the collision, before they have moved significantly.

In part (2), mechanical energy is conserved. We choose $y = 0$ when the pendulum hangs vertically, and then $y = h$ when the pendulum–projectile system reaches its maximum height. Thus we write

$$(\text{KE} + \text{PE}) \text{ just after collision} = (\text{KE} + \text{PE}) \text{ at pendulum's maximum height}$$

or

$$\tfrac{1}{2}(m + M)v'^2 + 0 = 0 + (m + M)gh. \qquad \textbf{(ii)}$$

We solve for v':

$$v' = \sqrt{2gh}.$$

Inserting this result for v' into Eq. (i) above, and solving for v, gives

$$v = \frac{m + M}{m}v' = \frac{m + M}{m}\sqrt{2gh},$$

which is our final result.

NOTE The separation of the process into two parts was crucial. Such an analysis is a powerful problem-solving tool. But how do you decide how to make such a division? Think about the conservation laws. They are your *tools*. Start a problem by asking yourself whether the conservation laws apply in the given situation. Here, we determined that momentum is conserved only during the brief collision, which we called part (1). But in part (1), because the collision is inelastic, the conservation of mechanical energy is not valid. Then in part (2), conservation of mechanical energy is valid, but not conservation of momentum.

Note, however, that if there had been significant motion of the pendulum during the deceleration of the projectile in the block, then there *would* have been an external force (gravity) during the collision, so conservation of momentum would not have been valid in part (1).

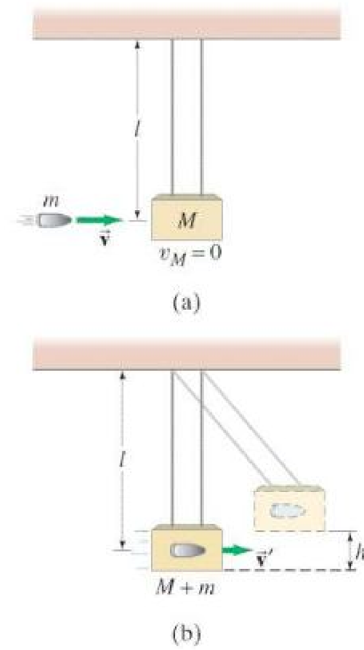


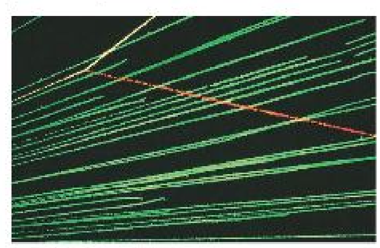
FIGURE 7–17 Ballistic pendulum. Example 7–10.

PROBLEM SOLVING
Use the conservation laws to analyze a problem

* 7–7 Collisions in Two or Three Dimensions

Conservation of momentum and energy can also be applied to collisions in two or three dimensions, where the vector nature of momentum is especially important. One common type of non-head-on collision is that in which a moving object (called the "projectile") strikes a second object initially at rest (the "target"). This is the common situation in games such as billiards and pool, and for experiments in atomic and nuclear physics (the projectiles, from radioactive decay or a high-energy accelerator, strike a stationary target nucleus; Fig. 7–18).

FIGURE 7–18 A recent color-enhanced version of a cloud-chamber photograph made in the early days (1920s) of nuclear physics. Green lines are paths of helium nuclei (He) coming from the left. One He, highlighted in yellow, strikes a proton of the hydrogen gas in the chamber, and both scatter at an angle; the scattered proton's path is shown in red.



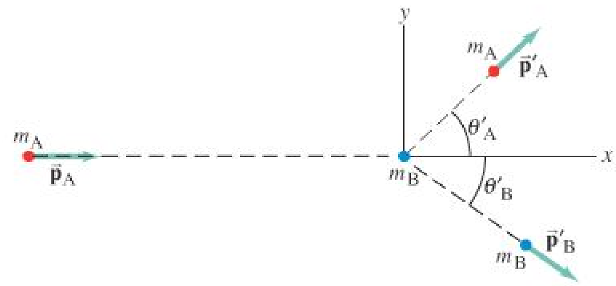


FIGURE 7–19 Object A, the projectile, collides with object B, the target. After the collision, they move off with momenta $\vec{p}'_A$ and $\vec{p}'_B$ at angles θ'_A and θ'_B.

Figure 7–19 shows the incoming projectile, m_A, heading along the x axis toward the target object, m_B, which is initially at rest. If these are billiard balls, m_A strikes m_B and they go off at the angles θ'_A and θ'_B, respectively, which are measured relative to m_A's initial direction (the x axis).†

Let us apply the law of conservation of momentum to a collision like that of Fig. 7–19. We choose the xy plane to be the plane in which the initial and final momenta lie. Momentum is a vector, and because the total momentum is conserved, its components in the x and y directions also are conserved. The x component of momentum conservation gives

$$p_{Ax} + p_{Bx} = p'_{Ax} + p'_{Bx}$$

or, with $p_{Bx} = m_B v_{Bx} = 0$,

p_x conserved

$$m_A v_A = m_A v'_A \cos\theta'_A + m_B v'_B \cos\theta'_B, \qquad \textbf{(7–8a)}$$

where the primes (′) refer to quantities *after* the collision. Because there is no motion in the y direction initially, the y component of the total momentum is zero before the collision. The y component equation of momentum conservation is then

$$p_{Ay} + p_{By} = p'_{Ay} + p'_{By}$$

or

p_y conserved

$$0 = m_A v'_A \sin\theta'_A + m_B v'_B \sin\theta'_B. \qquad \textbf{(7–8b)}$$

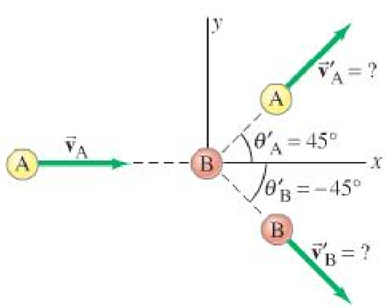


FIGURE 7–20 Example 7–11.

EXAMPLE 7–11 Billiard ball collision in 2-D. Billiard ball A moving with speed $v_A = 3.0\,\text{m/s}$ in the $+x$ direction (Fig. 7–20) strikes an equal-mass ball B initially at rest. The two balls are observed to move off at 45° to the x axis, ball A above the x axis and ball B below. That is, $\theta'_A = 45°$ and $\theta'_B = -45°$ in Fig. 7–20. What are the speeds of the two balls after the collision?

APPROACH There is no net external force on our system of two balls, assuming the table is level (the normal force balances gravity). Thus momentum conservation applies, and we apply it to both the x and y components using the xy coordinate system shown in Fig. 7–20. We get two equations, and we have two unknowns, v'_A and v'_B. From symmetry we might guess that the two balls have the same speed. But let us not assume that now. Even though we aren't told whether the collision is elastic or inelastic, we can still use conservation of momentum.

SOLUTION We apply conservation of momentum, Eqs. 7–8a and b, and we solve for v'_A and v'_B. We are given $m_A = m_B(=m)$, so

$$(\text{for } x) \qquad mv_A = mv'_A \cos(45°) + mv'_B \cos(-45°)$$

and

$$(\text{for } y) \qquad 0 = mv'_A \sin(45°) + mv'_B \sin(-45°).$$

The m's cancel out in both equations (the masses are equal).

†The objects may begin to deflect even before they touch if electric, magnetic, or nuclear forces act between them. You might think, for example, of two magnets oriented so that they repel each other: when one moves toward the other, the second moves away before the first one touches it.

The second equation yields [recall that $\sin(-\theta) = -\sin\theta$]:

$$v'_B = -v'_A \frac{\sin(45°)}{\sin(-45°)} = -v'_A\left(\frac{\sin 45°}{-\sin 45°}\right) = v'_A.$$

So they do have equal speeds as we guessed at first. The x component equation gives [recall that $\cos(-\theta) = \cos\theta$]:

$$v_A = v'_A\cos(45°) + v'_B\cos(45°) = 2v'_A\cos(45°),$$

so

$$v'_A = v'_B = \frac{v_A}{2\cos(45°)} = \frac{3.0\text{ m/s}}{2(0.707)} = 2.1\text{ m/s}.$$

NOTE When we have two independent equations, we can solve for, at most, two unknowns.

EXERCISE F Make a calculation to see if kinetic energy was conserved in the collision of Example 7–11.

If we know that a collision is elastic, we can also apply conservation of kinetic energy and obtain a third equation in addition to Eqs. 7–8a and b:

$$\text{KE}_A + \text{KE}_B = \text{KE}'_A + \text{KE}'_B$$

or, for the collision shown in Fig. 7–20,

$$\tfrac{1}{2}m_A v_A^2 = \tfrac{1}{2}m_A v_A'^2 + \tfrac{1}{2}m_B v_B'^2. \qquad \text{[elastic collision]} \quad \textbf{(7–8c)}$$

KE conserved

If the collision is elastic, we have three independent equations and can solve for three unknowns. If we are given m_A, m_B, v_A (and v_B, if it is not zero), we cannot, for example, predict the final variables, v'_A, v'_B, θ'_A, and θ'_B, because there are four of them. However, if we measure one of these variables, say θ'_A, then the other three variables (v'_A, v'_B, and θ'_B) are uniquely determined, and we can determine them using Eqs. 7–8a, b, c.

A note of caution: Eq. 7–7 does *not* apply for two-dimensional collisions. It works only when a collision occurs along a line.

CAUTION
Equation 7–7 applies only in 1-D

PROBLEM SOLVING Momentum Conservation and Collisions

1. Choose your **system**. If the situation is complex, think about how you might break it up into separate parts when one or more conservation laws apply.
2. Consider whether a significant **net external force** acts on your chosen system; if it does, be sure the time interval Δt is so short that the effect on momentum is neglible. That is, the forces that act between the interacting objects must be the significant ones if momentum conservation is to be used. [Note: If this is valid for a portion of the problem, you can use momentum conservation only for that portion.]
3. Draw a **diagram** of the initial situation, just before the interaction (collision, explosion) takes place, and represent the momentum of each object with an arrow and a label. Do the same for the final situation, just after the interaction.
4. Choose a **coordinate system** and "+" and "−" directions. (For a head-on collision, you will need only an x axis.) It is often convenient to choose the $+x$ axis in the direction of one object's initial velocity.
5. Apply the **momentum conservation** equation(s):

   total initial momentum = total final momentum.

   You have one equation for each component (x, y, z): only one equation for a head-on collision. [Don't forget that it is the *total* momentum of the system that is conserved, not the momenta of individual objects.]
6. If the collision is elastic, you can also write down a **conservation of kinetic energy** equation:

   total initial KE = total final KE.

   [Alternately, you could use Eq. 7–7: $v_A - v_B = v'_B - v'_A$, if the collision is one dimensional (head-on).]
7. Solve for the **unknown(s)**.
8. **Check** your work, check the units, and ask yourself whether the results are reasonable.

7–8 Center of Mass (CM)

Momentum is a powerful concept not only for analyzing collisions but also for analyzing the translational motion of real extended objects. Until now, whenever we have dealt with the motion of an extended object (that is, an object that has size), we have assumed that it could be approximated as a point particle or that it undergoes only translational motion. Real extended objects, however, can undergo rotational and other types of motion as well. For example, the diver in Fig. 7–21a undergoes only translational motion (all parts of the object follow the same path), whereas the diver in Fig. 7–21b undergoes both translational and rotational motion. We will refer to motion that is not pure translation as *general motion*.

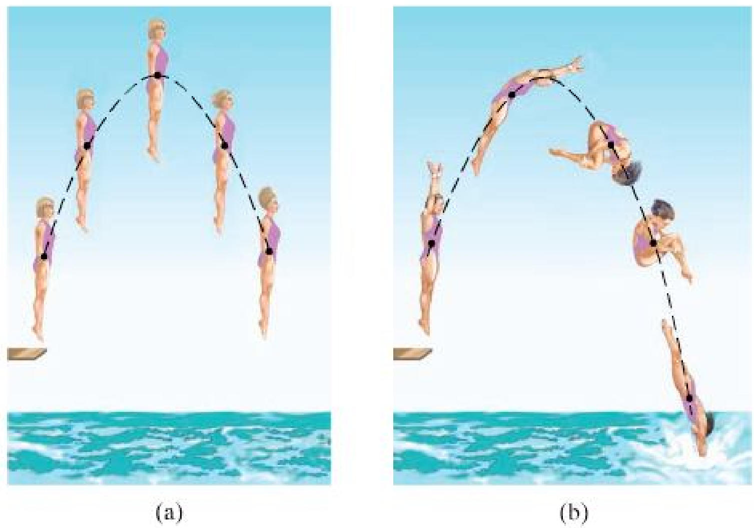


FIGURE 7–21 The motion of the diver is pure translation in (a), but is translation plus rotation in (b). The black dot represents the diver's CM at each moment.

Observations indicate that even if an object rotates, or several parts of a system of objects move relative to one another, there is one point that moves in the same path that a particle would move if subjected to the same net force. This point is called the **center of mass** (abbreviated CM). The general motion of an extended object (or system of objects) can be considered as *the sum of the translational motion of the CM, plus rotational, vibrational, or other types of motion about the CM.*

Center of mass
General motion

As an example, consider the motion of the center of mass of the diver in Fig. 7–21; the CM follows a parabolic path even when the diver rotates, as shown in Fig. 7–21b. This is the same parabolic path that a projected particle follows when acted on only by the force of gravity (that is, projectile motion). Other points in the rotating diver's body, such as her feet or head, follow more complicated paths.

Figure 7–22 shows a wrench acted on by zero net force, translating and rotating along a horizontal surface. Note that its CM, marked by a red cross, moves in a straight line, as shown by the dashed white line.

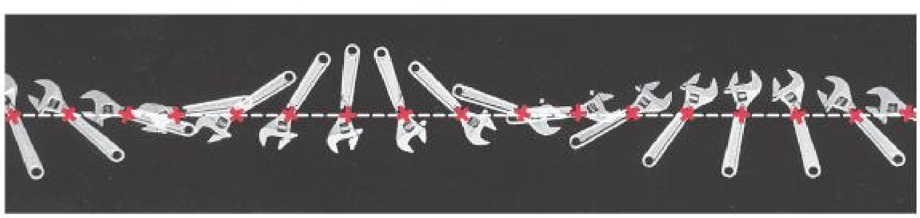


FIGURE 7–22 Translation plus rotation: a wrench moving over a horizontal surface. The CM, marked with a red cross, moves in a straight line.

We will show in Section 7–10 that the important properties of the CM follow from Newton's laws if the CM is defined in the following way. We can consider any extended object as being made up of many tiny particles. But first we consider a system made up of only two particles (or small objects), of masses m_A and m_B. We choose a coordinate system so that both particles lie on the x axis at positions x_A and x_B, Fig. 7–23. The center of mass of this system is defined to be at the position x_{CM}, given by

$$x_{CM} = \frac{m_A x_A + m_B x_B}{m_A + m_B} = \frac{m_A x_A + m_B x_B}{M},$$

x coordinate of CM (2 particles)

where $M = m_A + m_B$ is the total mass of the system. The center of mass lies on the line joining m_A and m_B. If the two masses are equal $(m_A = m_B = m)$, then x_{CM} is midway between them, since in this case

$$x_{CM} = \frac{m(x_A + x_B)}{2m} = \frac{(x_A + x_B)}{2}.$$

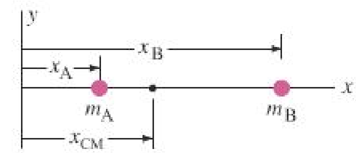


FIGURE 7–23 The center of mass of a two-particle system lies on the line joining the two masses. Here $m_A > m_B$, so the CM is closer to m_A than to m_B.

If one mass is greater than the other, say, $m_A > m_B$, then the CM is closer to the larger mass. If all the mass is concentrated at x_B, say, so $m_A = 0$, then $x_{CM} = (0x_A + m_B x_B)/(0 + m_B) = x_B$, as we would expect.

If there are more than two particles along a line, there will be additional terms:

$$x_{CM} = \frac{m_A x_A + m_B x_B + m_C x_C + \cdots}{m_A + m_B + m_C + \cdots} = \frac{m_A x_A + m_B x_B + m_C x_C + \cdots}{M}, \quad \text{(7–9a)}$$

x coordinate of CM (many particles)

where M is the total mass of all the particles.

EXAMPLE 7–12 **CM of three guys on a raft.** Three people of roughly equal masses m on a lightweight (air-filled) banana boat sit along the x axis at positions $x_A = 1.0$ m, $x_B = 5.0$ m, and $x_C = 6.0$ m, measured from the left-hand end as shown in Fig. 7–24. Find the position of the CM. Ignore mass of boat.

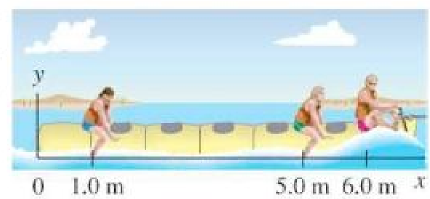


FIGURE 7–24 Example 7–12.

APPROACH We are given the mass and location of the three people, so we use three terms in Eq. 7–9a. We approximate each person as a point particle. Equivalently, the location of each person is the position of that person's own CM.

SOLUTION We use Eq. 7–9a with three terms:

$$x_{CM} = \frac{mx_A + mx_B + mx_C}{m + m + m} = \frac{m(x_A + x_B + x_C)}{3m}$$
$$= \frac{(1.0\text{ m} + 5.0\text{ m} + 6.0\text{ m})}{3} = \frac{12.0\text{ m}}{3} = 4.0\text{ m}.$$

The CM is 4.0 m from the left-hand end of the boat.

NOTE The coordinates of the CM depend on the reference frame or coordinate system chosen. But the physical location of the CM is independent of that choice.

EXERCISE G Calculate the CM of the three people in Example 7–12 taking the origin at the driver $(x_C = 0)$ on the right. Is the physical location of the CM the same?

If the particles are spread out in two or three dimensions, then we must specify not only the x coordinate of the CM (x_{CM}), but also the y and z coordinates, which will be given by formulas like Eq. 7–9a. For example, the y coordinate of the CM will be:

$$y_{CM} = \frac{m_A y_A + m_B y_B + \cdots}{m_A + m_B + \cdots} = \frac{m_A y_A + m_B y_B + \cdots}{M}. \quad \text{(7–9b)}$$

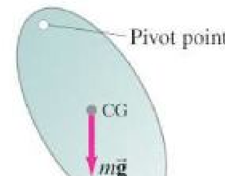
y coordinate of CM

Center of gravity

A concept similar to *center of mass* is **center of gravity** (CG). The CG of an object is that point at which the force of gravity can be considered to act. The force of gravity actually acts on *all* the different parts or particles of an object, but for purposes of determining the translational motion of an object as a whole, we can assume that the entire weight of the object (which is the sum of the weights of all its parts) acts at the CG. There is a conceptual difference between the center of gravity and the center of mass, but for nearly all practical purposes, they are at the same point.†

It is often easier to determine the CM or CG of an extended object experimentally rather than analytically. If an object is suspended from any point, it will swing (Fig. 7–25) unless it is placed so its CG lies on a vertical line directly below the point from which it is suspended. If the object is two dimensional, or has a plane of symmetry, it need only be hung from two different pivot points and the respective vertical (plumb) lines drawn. Then the center of gravity will be at the intersection

FIGURE 7–25 The force of gravity, considered to act at the CG, causes this object to rotate about the pivot point; if the CG were on a vertical line directly below the pivot, the object would remain at rest.



†There would be a difference between the CM and CG only in the unusual case of an object so large that the acceleration due to gravity, g, was different at different parts of the object.

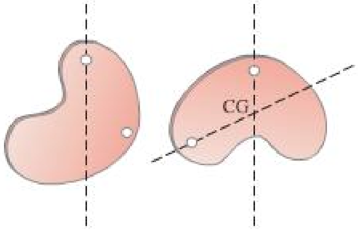


FIGURE 7–26 Finding the CG.

of the two lines, as in Fig. 7–26. If the object doesn't have a plane of symmetry, the CG with respect to the third dimension is found by suspending the object from at least three points whose plumb lines do not lie in the same plane. For symmetrically shaped objects such as uniform cylinders (wheels), spheres, and rectangular solids, the CM is located at the geometric center of the object.

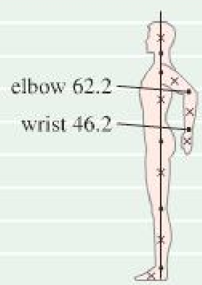
For some objects, the CM may actually lie outside the object. The CM of a donut, for example, lies at the center of the hole.

* 7–9 CM for the Human Body

If we have a group of extended objects, each of whose CM is known, we can find the CM of the group using Eqs. 7–9a and b. As an example, we consider the human body. Table 7–1 indicates the CM and hinge points (joints) for the different components of a "representative" person. Of course, there are wide variations among people, so these data represent only a very rough average. The numbers represent a percentage of the total height, which is regarded as 100 units; similarly, the total mass is 100 units. For example, if a person is 1.70 m tall, his or her shoulder joint would be $(1.70\,\text{m})(81.2/100) = 1.38\,\text{m}$ above the floor.

TABLE 7–1 Center of Mass of Parts of Typical Human Body
(full height and mass = 100 units)

Distance Above Floor of Hinge Points (%)	Hinge Points (•) (Joints)	Center of Mass (×)	(% Height Above Floor)	Percent Mass
91.2	Base of skull	Head	93.5	6.9
81.2	Shoulder joint	Trunk and neck	71.1	46.1
		Upper arms	71.7	6.6
		Lower arms	55.3	4.2
52.1	Hip joint	Hands	43.1	1.7
		Upper legs (thighs)	42.5	21.5
28.5	Knee joint			
		Lower legs	18.2	9.6
4.0	Ankle joint	Feet	1.8	3.4
			Body CM = 58.0	100.0



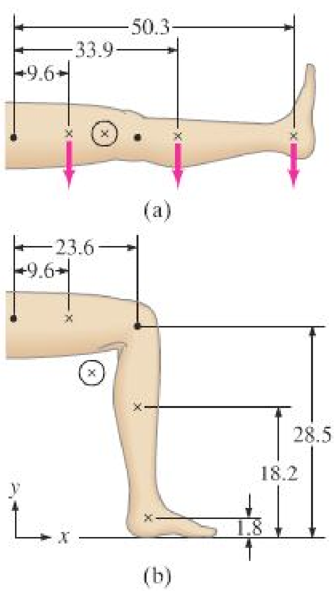


FIGURE 7–27 Example 7–13: finding the CM of a leg in two different positions using percentages from Table 7–1. (⊗ represents the calculated CM).

EXAMPLE 7–13 A leg's CM. Determine the position of the CM of a whole leg (*a*) when stretched out, and (*b*) when bent at 90°. See Fig. 7–27. Assume the person is 1.70 m tall.

APPROACH Our system consists of three objects: upper leg, lower leg, and foot. The location of the CM of each object, as well as the mass of each, is given in Table 7–1, where they are expressed in percentage units. To express the results in meters, these percentage values need to be multiplied by $(1.70\,\text{m}/100)$. When the leg is stretched out, the problem is one dimensional and we can solve for the x coordinate of the CM. When the leg is bent, the problem is two dimensional and we need to find both the x and y coordinates.

SOLUTION (*a*) We determine the distances from the hip joint using Table 7–1 and obtain the numbers (%) shown in Fig. 7–27a. Using Eq. 7–9a, we obtain

$$x_{\text{CM}} = \frac{(21.5)(9.6) + (9.6)(33.9) + (3.4)(50.3)}{21.5 + 9.6 + 3.4} = 20.4 \text{ units.}$$

Thus, the center of mass of the leg and foot is 20.4 units from the hip joint, or $52.1 - 20.4 = 31.7$ units from the base of the foot. Since the person is 1.70 m tall, this is $(1.70\,\text{m})(31.7/100) = 0.54\,\text{m}$ above the bottom of the foot.

(*b*) We use an xy coordinate system, as shown in Fig. 7–27b. First, we calculate how far to the right of the hip joint the CM lies, accounting for all three parts:

$$x_{\text{CM}} = \frac{(21.5)(9.6) + (9.6)(23.6) + (3.4)(23.6)}{21.5 + 9.6 + 3.4} = 14.9 \text{ units.}$$

For our 1.70-m-tall person, this is $(1.70\,\text{m})(14.9/100) = 0.25\,\text{m}$ from the hip joint. Next, we calculate the distance, y_{CM}, of the CM above the floor:

$$y_{\text{CM}} = \frac{(3.4)(1.8) + (9.6)(18.2) + (21.5)(28.5)}{21.5 + 9.6 + 3.4} = 23.0\,\text{units},$$

or $(1.70\,\text{m})(23.0/100) = 0.39\,\text{m}$. Thus, the CM is located 39 cm above the floor and 25 cm to the right of the hip joint.

NOTE The CM actually lies *outside* the body in (*b*).

FIGURE 7–28 A high jumper's CM may actually pass beneath the bar.

Knowing the CM of the body when it is in various positions is of great use in studying body mechanics. One simple example from athletics is shown in Fig. 7–28. If high jumpers can get into the position shown, their CM can pass below the bar which their bodies go over, meaning that for a particular takeoff speed, they can clear a higher bar. This is indeed what they try to do.

PHYSICS APPLIED
High jumping

* 7–10 Center of Mass and Translational Motion

As mentioned in Section 7–8, a major reason for the importance of the concept of center of mass is that the motion of the CM for a system of particles (or an extended object) is directly related to the net force acting on the system as a whole. We now show this, taking the simple case of one-dimensional motion (x direction) and only three particles, but the extension to more objects and to three dimensions follows the same lines.

Suppose the three particles lie on the x axis and have masses $m_{\text{A}}, m_{\text{B}}, m_{\text{C}}$, and positions $x_{\text{A}}, x_{\text{B}}, x_{\text{C}}$. From Eq. 7–9a for the center of mass, we can write

$$Mx_{\text{CM}} = m_{\text{A}}x_{\text{A}} + m_{\text{B}}x_{\text{B}} + m_{\text{C}}x_{\text{C}}, \qquad \textbf{(i)}$$

where $M = m_{\text{A}} + m_{\text{B}} + m_{\text{C}}$ is the total mass of the system. If these particles are in motion (say, along the x axis with velocities $v_{\text{A}}, v_{\text{B}}$, and v_{C}, respectively), then in a short time interval Δt they each will have traveled a distance

$$\begin{aligned}\Delta x_{\text{A}} &= x'_{\text{A}} - x_{\text{A}} = v_{\text{A}}\,\Delta t\\ \Delta x_{\text{B}} &= x'_{\text{B}} - x_{\text{B}} = v_{\text{B}}\,\Delta t\\ \Delta x_{\text{C}} &= x'_{\text{C}} - x_{\text{C}} = v_{\text{C}}\,\Delta t,\end{aligned}$$

where $x'_{\text{A}}, x'_{\text{B}}$, and x'_{C} represent their new positions after time interval Δt. The position of the new CM is given by

$$Mx'_{\text{CM}} = m_{\text{A}}x'_{\text{A}} + m_{\text{B}}x'_{\text{B}} + m_{\text{C}}x'_{\text{C}}. \qquad \textbf{(ii)}$$

If we subtract from this equation (ii) the previous CM equation (i), we get

$$M\,\Delta x_{\text{CM}} = m_{\text{A}}\,\Delta x_{\text{A}} + m_{\text{B}}\,\Delta x_{\text{B}} + m_{\text{C}}\,\Delta x_{\text{C}}.$$

During time interval Δt, the center of mass will have moved a distance

$$\Delta x_{\text{CM}} = x'_{\text{CM}} - x_{\text{CM}} = v_{\text{CM}}\,\Delta t,$$

where v_{CM} is the velocity of the center of mass. We now substitute the relations for all the Δx's into the equation just before the last one:

$$Mv_{\text{CM}}\,\Delta t = m_{\text{A}}v_{\text{A}}\,\Delta t + m_{\text{B}}v_{\text{B}}\,\Delta t + m_{\text{C}}v_{\text{C}}\,\Delta t.$$

We cancel Δt and get

$$Mv_{\text{CM}} = m_{\text{A}}v_{\text{A}} + m_{\text{B}}v_{\text{B}} + m_{\text{C}}v_{\text{C}}. \qquad \textbf{(7–10)}$$

Since $m_{\text{A}}v_{\text{A}} + m_{\text{B}}v_{\text{B}} + m_{\text{C}}v_{\text{C}}$ is the sum of the momenta of the particles of the system, it represents the *total momentum* of the system. Thus we see from Eq. 7–10 that *the total (linear) momentum of a system of particles is equal to the product of the total mass M and the velocity of the center of mass of the system.* Or, *the linear momentum of an extended object is the product of the object's mass and the velocity of its CM.*

Total momentum, and velocity of CM

If forces are acting on the particles, then the particles may be accelerating. In a short time interval Δt, each particle's velocity will change by an amount

$$\Delta v_A = a_A\,\Delta t, \qquad \Delta v_B = a_B\,\Delta t, \qquad \Delta v_C = a_C\,\Delta t.$$

If we now use the same reasoning as we did to derive Eq. 7–10, we obtain

$$Ma_{CM} = m_A a_A + m_B a_B + m_C a_C.$$

According to Newton's second law, $m_A a_A = F_A$, $m_B a_B = F_B$, and $m_C a_C = F_C$, where F_A, F_B, and F_C are the net forces on the three particles, respectively. Thus we get for the system as a whole $Ma_{CM} = F_A + F_B + F_C$, or

Newton's second law for a system of particles or an extended object

$$Ma_{CM} = F_{net}. \qquad \textbf{(7–11)}$$

That is, *the sum of all the forces acting on the system is equal to the total mass of the system times the acceleration of its center of mass.* This is **Newton's second law** for a system of particles, and it also applies to an extended object (which can be thought of as a collection of particles). Thus we conclude that the *center of mass of a system of particles (or of an extended object) with total mass M moves like a single particle of mass M acted on by the same net external force.* That is, the system moves as if all its mass were concentrated at the center of mass and all the external forces acted at that point. We can thus treat the translational motion of any object or system of objects as the motion of a particle (see Figs. 7–21 and 7–22). This result simplifies our analysis of the motion of complex systems and extended objects. Although the motion of various parts of the system may be complicated, we may often be satisfied with knowing the motion of the center of mass. This result also allows us to solve certain types of problems very easily, as illustrated by the following Example.

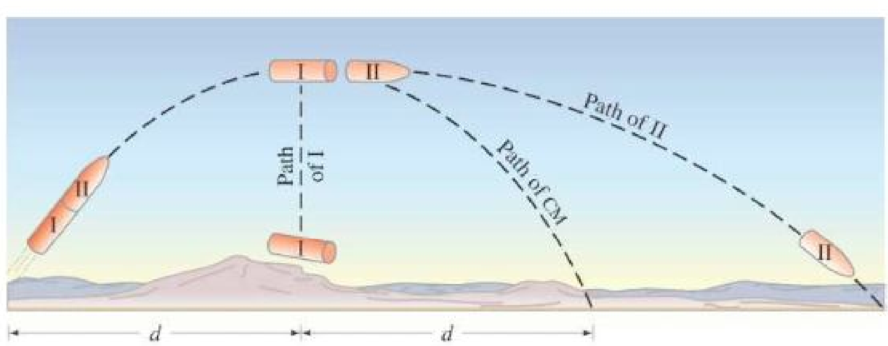
CONCEPTUAL EXAMPLE 7–14 **A two-stage rocket.** A rocket is shot into the air as shown in Fig. 7–29. At the moment the rocket reaches its highest point, a horizontal distance d from its starting point, a prearranged explosion separates it into two parts of equal mass. Part I is stopped in midair by the explosion, and it falls vertically to Earth. Where does part II land? Assume $\vec{g}$ = constant.

RESPONSE After the rocket is fired, the path of the CM of the system continues to follow the parabolic trajectory of a projectile acted on by only a constant gravitational force. The CM will thus land at a point $2d$ from the starting point. Since the masses of I and II are equal, the CM must be midway between them at any time. Therefore, part II lands a distance $3d$ from the starting point.

NOTE If part I had been given a kick up or down, instead of merely falling, the solution would have been more complicated.

EXERCISE H A woman stands up in a rowboat and walks from one end of the boat to the other. How does the boat move, as seen from the shore?

FIGURE 7–29 Example 7–14.



Summary

The **momentum**, $\vec{\mathbf{p}}$, of an object is defined as the product of its mass times its velocity,

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}. \quad \textbf{(7–1)}$$

In terms of momentum, **Newton's second law** can be written as

$$\Sigma\vec{\mathbf{F}} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t}. \quad \textbf{(7–2)}$$

That is, the rate of change of momentum equals the net applied force.

The **law of conservation of momentum** states that the total momentum of an isolated system of objects remains constant. An **isolated system** is one on which the net external force is zero.

The law of conservation of momentum is very useful in dealing with **collisions**. In a collision, two (or more) objects interact with each other over a very short time interval, and the forces between them during this time interval are very large.

The **impulse** of a force on an object is defined as $\vec{\mathbf{F}}\,\Delta t$, where $\vec{\mathbf{F}}$ is the average force acting during the (usually short) time interval Δt. The impulse is equal to the change in momentum of the object:

$$\text{Impulse} = \vec{\mathbf{F}}\,\Delta t = \Delta\vec{\mathbf{p}}. \quad \textbf{(7–5)}$$

Total momentum is conserved in *any* collision as long as any net external force is zero or negligible. If $m_A\vec{\mathbf{v}}_A$ and $m_B\vec{\mathbf{v}}_B$ are the momenta of two objects before the collision and $m_A\vec{\mathbf{v}}'_A$ and $m_B\vec{\mathbf{v}}'_B$ are their momenta after, then momentum conservation tell us that

$$m_A\vec{\mathbf{v}}_A + m_B\vec{\mathbf{v}}_B = m_A\vec{\mathbf{v}}'_A + m_B\vec{\mathbf{v}}'_B \quad \textbf{(7–3)}$$

for this two-object system.

Total energy is also conserved, but this may not be helpful in problem solving unless the only type of energy transformation involves kinetic energy. In that case kinetic energy is conserved and the collision is called an **elastic collision**, and we can write

$$\tfrac{1}{2}m_A v_A^2 + \tfrac{1}{2}m_B v_B^2 = \tfrac{1}{2}m_A v_A'^2 + \tfrac{1}{2}m_B v_B'^2. \quad \textbf{(7–6)}$$

If kinetic energy is not conserved, the collision is called **inelastic**. A **completely inelastic** collision is one in which the colliding objects stick together after the collision.

The **center of mass** (CM) of an extended object (or group of objects) is that point at which the net force can be considered to act, for purposes of determining the translational motion of the object as a whole. The x component of the CM for objects with mass $m_A, m_B, \ldots,$ is given by

$$x_{\text{CM}} = \frac{m_A x_A + m_B x_B + \cdots}{m_A + m_B + \cdots}. \quad \textbf{(7–9a)}$$

[*The complete motion of an object can be described as the translational motion of its center of mass plus rotation (or other internal motion) about its center of mass.]

Questions

1. We claim that momentum is conserved, yet most moving objects eventually slow down and stop. Explain.

2. When a person jumps from a tree to the ground, what happens to the momentum of the person upon striking the ground?

3. When you release an inflated but untied balloon, why does it fly across the room?

4. It is said that in ancient times a rich man with a bag of gold coins froze to death while stranded on a frozen lake. Because the ice was frictionless, he could not push himself to shore. What could he have done to save himself had he not been so miserly?

5. How can a rocket change direction when it is far out in space and is essentially in a vacuum?

6. According to Eq. 7–5, the longer the impact time of an impulse, the smaller the force can be for the same momentum change, and hence the smaller the deformation of the object on which the force acts. On this basis, explain the value of air bags, which are intended to inflate during an automobile collision and reduce the possibility of fracture or death.

7. Cars used to be built as rigid as possible to withstand collisions. Today, though, cars are designed to have "crumple zones" that collapse upon impact. What is the advantage of this new design?

8. Why can a batter hit a pitched baseball further than a ball tossed in the air by the batter?

9. Is it possible for an object to receive a larger impulse from a small force than from a large force? Explain.

10. A light object and a heavy object have the same kinetic energy. Which has the greater momentum? Explain.

11. Describe a collision in which all kinetic energy is lost.

12. At a hydroelectric power plant, water is directed at high speed against turbine blades on an axle that turns an electric generator. For maximum power generation, should the turbine blades be designed so that the water is brought to a dead stop, or so that the water rebounds?

13. A squash ball hits a wall at a 45° angle as shown in Fig. 7–30. What is the direction (*a*) of the change in momentum of the ball, (*b*) of the force on the wall?

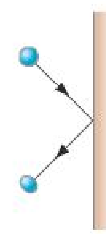


FIGURE 7–30 Question 13.

14. A Superball is dropped from a height h onto a hard steel plate (fixed to the Earth), from which it rebounds at very nearly its original speed. (*a*) Is the momentum of the ball conserved during any part of this process? (*b*) If we consider the ball and Earth as our system, during what parts of the process is momentum conserved? (*c*) Answer part (*b*) for a piece of putty that falls and sticks to the steel plate.

15. Why do you tend to lean backward when carrying a heavy load in your arms?

16. Why is the CM of a 1-m length of pipe at its mid-point, whereas this is not true for your arm or leg?

* **17.** Show on a diagram how your CM shifts when you change from a lying position to a sitting position.

* **18.** If only an external force can change the momentum of the center of mass of an object, how can the internal force of an engine accelerate a car?

* **19.** A rocket following a parabolic path through the air suddenly explodes into many pieces. What can you say about the motion of this system of pieces?

Problems

7–1 and 7–2 Momentum and Its Conservation

1. (I) What is the magnitude of the momentum of a 28-g sparrow flying with a speed of 8.4 m/s?
2. (I) A constant friction force of 25 N acts on a 65-kg skier for 20 s. What is the skier's change in velocity?
3. (II) A 0.145-kg baseball pitched at 39.0 m/s is hit on a horizontal line drive straight back toward the pitcher at 52.0 m/s. If the contact time between bat and ball is 3.00×10^{-3} s, calculate the average force between the ball and bat during contact.
4. (II) A child in a boat throws a 6.40-kg package out horizontally with a speed of 10.0 m/s, Fig. 7–31. Calculate the velocity of the boat immediately after, assuming it was initially at rest. The mass of the child is 26.0 kg, and that of the boat is 45.0 kg. Ignore water resistance.

FIGURE 7–31 Problem 4.

5. (II) Calculate the force exerted on a rocket, given that the propelling gases are expelled at a rate of 1500 kg/s with a speed of 4.0×10^4 m/s (at the moment of takeoff).
6. (II) A 95-kg halfback moving at 4.1 m/s on an apparent breakaway for a touchdown is tackled from behind. When he was tackled by an 85-kg cornerback running at 5.5 m/s in the same direction, what was their mutual speed immediately after the tackle?
7. (II) A 12,600-kg railroad car travels alone on a level frictionless track with a constant speed of 18.0 m/s. A 5350-kg load, initially at rest, is dropped onto the car. What will be the car's new speed?
8. (II) A 9300-kg boxcar traveling at 15.0 m/s strikes a second boxcar at rest. The two stick together and move off with a speed of 6.0 m/s. What is the mass of the second car?
9. (II) During a Chicago storm, winds can whip horizontally at speeds of 100 km/h. If the air strikes a person at the rate of 40 kg/s per square meter and is brought to rest, estimate the force of the wind on a person. Assume the person is 1.50 m high and 0.50 m wide. Compare to the typical maximum force of friction ($\mu \approx 1.0$) between the person and the ground, if the person has a mass of 70 kg.
10. (II) A 3800-kg open railroad car coasts along with a constant speed of 8.60 m/s on a level track. Snow begins to fall vertically and fills the car at a rate of 3.50 kg/min. Ignoring friction with the tracks, what is the speed of the car after 90.0 min?
11. (II) An atomic nucleus initially moving at 420 m/s emits an alpha particle in the direction of its velocity, and the remaining nucleus slows to 350 m/s. If the alpha particle has a mass of 4.0 u and the original nucleus has a mass of 222 u, what speed does the alpha particle have when it is emitted?
12. (II) A 23-g bullet traveling 230 m/s penetrates a 2.0-kg block of wood and emerges cleanly at 170 m/s. If the block is stationary on a frictionless surface when hit, how fast does it move after the bullet emerges?
13. (III) A 975-kg two-stage rocket is traveling at a speed of 5.80×10^3 m/s with respect to Earth when a pre-designed explosion separates the rocket into two sections of equal mass that then move at a speed of 2.20×10^3 m/s relative to each other along the original line of motion. (*a*) What are the speed and direction of each section (relative to Earth) after the explosion? (*b*) How much energy was supplied by the explosion? [*Hint*: What is the change in KE as a result of the explosion?]
14. (III) A rocket of total mass 3180 kg is traveling in outer space with a velocity of 115 m/s. To alter its course by 35.0°, its rockets can be fired briefly in a direction perpendicular to its original motion. If the rocket gases are expelled at a speed of 1750 m/s, how much mass must be expelled?

7–3 Collisions and Impulse

15. (II) A golf ball of mass 0.045 kg is hit off the tee at a speed of 45 m/s. The golf club was in contact with the ball for 3.5×10^{-3} s. Find (*a*) the impulse imparted to the golf ball, and (*b*) the average force exerted on the ball by the golf club.
16. (II) A 12-kg hammer strikes a nail at a velocity of 8.5 m/s and comes to rest in a time interval of 8.0 ms. (*a*) What is the impulse given to the nail? (*b*) What is the average force acting on the nail?
17. (II) A tennis ball of mass $m = 0.060$ kg and speed $v = 25$ m/s strikes a wall at a 45° angle and rebounds with the same speed at 45° (Fig. 7–32). What is the impulse (magnitude and direction) given to the ball?

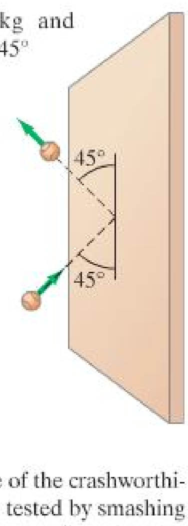


FIGURE 7–32 Problem 17.

18. (II) You are the design engineer in charge of the crashworthiness of new automobile models. Cars are tested by smashing them into fixed, massive barriers at 50 km/h (30 mph). A new model of mass 1500 kg takes 0.15 s from the time of impact until it is brought to rest. (*a*) Calculate the average force exerted on the car by the barrier. (*b*) Calculate the average deceleration of the car.

19. (II) A 95-kg fullback is running at 4.0 m/s to the east and is stopped in 0.75 s by a head-on tackle by a tackler running due west. Calculate (*a*) the original momentum of the fullback, (*b*) the impulse exerted on the fullback, (*c*) the impulse exerted on the tackler, and (*d*) the average force exerted on the tackler.

20. (II) Suppose the force acting on a tennis ball (mass 0.060 kg) points in the $+x$ direction and is given by the graph of Fig. 7–33 as a function of time. Use graphical methods to estimate (*a*) the total impulse given the ball, and (*b*) the velocity of the ball after being struck, assuming the ball is being served so it is nearly at rest initially.

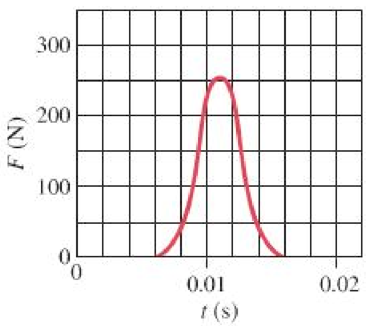


FIGURE 7–33 Problem 20.

21. (III) From what maximum height can a 75-kg person jump without breaking the lower leg bone of either leg? Ignore air resistance and assume the CM of the person moves a distance of 0.60 m from the standing to the seated position (that is, in breaking the fall). Assume the breaking strength (force per unit area) of bone is $170 \times 10^6\ \text{N/m}^2$, and its smallest cross-sectional area is $2.5 \times 10^{-4}\ \text{m}^2$. [*Hint*: Do not try this experimentally.]

7–4 and 7–5 Elastic Collisions

22. (II) A ball of mass 0.440 kg moving east ($+x$ direction) with a speed of 3.30 m/s collides head-on with a 0.220-kg ball at rest. If the collision is perfectly elastic, what will be the speed and direction of each ball after the collision?

23. (II) A 0.450-kg ice puck, moving east with a speed of 3.00 m/s, has a head-on collision with a 0.900-kg puck initially at rest. Assuming a perfectly elastic collision, what will be the speed and direction of each object after the collision?

24. (II) Two billiard balls of equal mass undergo a perfectly elastic head-on collision. If one ball's initial speed was 2.00 m/s, and the other's was 3.00 m/s in the opposite direction, what will be their speeds after the collision?

25. (II) A 0.060-kg tennis ball, moving with a speed of 2.50 m/s, collides head-on with a 0.090-kg ball initially moving away from it at a speed of 1.15 m/s. Assuming a perfectly elastic collision, what are the speed and direction of each ball after the collision?

26. (II) A softball of mass 0.220 kg that is moving with a speed of 8.5 m/s collides head-on and elastically with another ball initially at rest. Afterward the incoming softball bounces backward with a speed of 3.7 m/s. Calculate (*a*) the velocity of the target ball after the collision, and (*b*) the mass of the target ball.

27. (II) Two bumper cars in an amusement park ride collide elastically as one approaches the other directly from the rear (Fig. 7–34). Car A has a mass of 450 kg and car B 550 kg, owing to differences in passenger mass. If car A approaches at 4.50 m/s and car B is moving at 3.70 m/s, calculate (*a*) their velocities after the collision, and (*b*) the change in momentum of each.

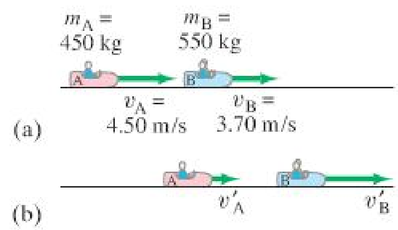


FIGURE 7–34 Problem 27: (a) before collision, (b) after collision.

28. (II) A 0.280-kg croquet ball makes an elastic head-on collision with a second ball initially at rest. The second ball moves off with half the original speed of the first ball. (*a*) What is the mass of the second ball? (*b*) What fraction of the original kinetic energy ($\Delta\text{KE}/\text{KE}$) gets transferred to the second ball?

29. (III) In a physics lab, a cube slides down a frictionless incline as shown in Fig. 7–35, and elastically strikes another cube at the bottom that is only one-half its mass. If the incline is 30 cm high and the table is 90 cm off the floor, where does each cube land? [*Hint*: Both leave the incline moving horizontally.]

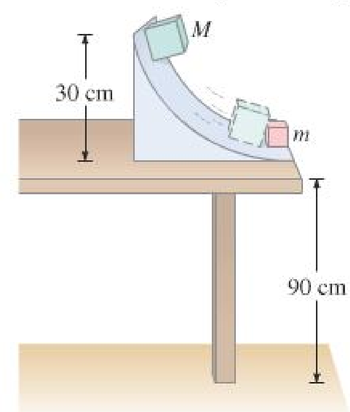


FIGURE 7–35 Problem 29.

30. (III) Take the general case of an object of mass m_A and velocity v_A elastically striking a stationary ($v_B = 0$) object of mass m_B head-on. (*a*) Show that the final velocities v'_A and v'_B are given by

$$v'_A = \left(\frac{m_A - m_B}{m_A + m_B}\right)v_A,$$

$$v'_B = \left(\frac{2m_A}{m_A + m_B}\right)v_A.$$

(*b*) What happens in the extreme case when m_A is much smaller than m_B? Cite a common example of this. (*c*) What happens in the extreme case when m_A is much larger than m_B? Cite a common example of this. (*d*) What happens in the case when $m_A = m_B$? Cite a common example.

7–6 Inelastic Collisions

31. (I) In a ballistic pendulum experiment, projectile 1 results in a maximum height h of the pendulum equal to 2.6 cm. A second projectile causes the the pendulum to swing twice as high, $h_2 = 5.2$ cm. The second projectile was how many times faster than the first?

32. (II) A 28-g rifle bullet traveling 230 m/s buries itself in a 3.6-kg pendulum hanging on a 2.8-m-long string, which makes the pendulum swing upward in an arc. Determine the vertical and horizontal components of the pendulum's displacement.

33. (II) (*a*) Derive a formula for the fraction of kinetic energy lost, $\Delta\text{KE}/\text{KE}$, for the ballistic pendulum collision of Example 7–10. (*b*) Evaluate for $m = 14.0\text{ g}$ and $M = 380\text{ g}$.

34. (II) An internal explosion breaks an object, initially at rest, into two pieces, one of which has 1.5 times the mass of the other. If 7500 J were released in the explosion, how much kinetic energy did each piece acquire?

35. (II) A 920-kg sports car collides into the rear end of a 2300-kg SUV stopped at a red light. The bumpers lock, the brakes are locked, and the two cars skid forward 2.8 m before stopping. The police officer, knowing that the coefficient of kinetic friction between tires and road is 0.80, calculates the speed of the sports car at impact. What was that speed?

36. (II) A ball is dropped from a height of 1.50 m and rebounds to a height of 1.20 m. Approximately how many rebounds will the ball make before losing 90% of its energy?

37. (II) A measure of inelasticity in a head-on collision of two objects is the *coefficient of restitution*, *e*, defined as

$$e = \frac{v'_A - v'_B}{v_B - v_A},$$

where $v'_A - v'_B$ is the relative velocity of the two objects after the collision and $v_B - v_A$ is their relative velocity before it. (*a*) Show that $e = 1$ for a perfectly elastic collision, and $e = 0$ for a completely inelastic collision. (*b*) A simple method for measuring the coefficient of restitution for an object colliding with a very hard surface like steel is to drop the object onto a heavy steel plate, as shown in Fig. 7–36. Determine a formula for *e* in terms of the original height *h* and the maximum height *h'* reached after one collision.

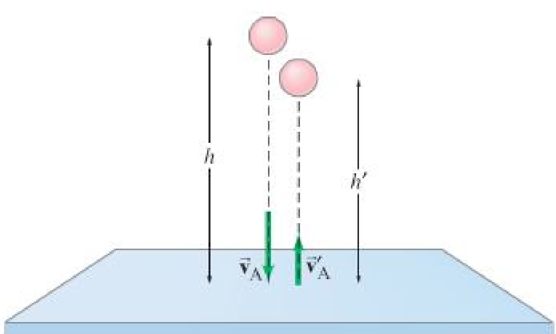


FIGURE 7–36 Problem 37. Measurement of the coefficient of restitution.

38. (II) A wooden block is cut into two pieces, one with three times the mass of the other. A depression is made in both faces of the cut, so that a firecracker can be placed in it with the block reassembled. The reassembled block is set on a rough-surfaced table, and the fuse is lit. When the firecracker explodes, the two blocks separate and slide apart. What is the ratio of distances each block travels?

39. (III) A 15.0-kg object moving in the $+x$ direction at 5.5 m/s collides head-on with a 10.0-kg object moving in the $-x$ direction at 4.0 m/s. Find the final velocity of each mass if: (*a*) the objects stick together; (*b*) the collision is elastic; (*c*) the 15.0-kg object is at rest after the collision; (*d*) the 10.0-kg object is at rest after the collision; (*e*) the 15.0-kg object has a velocity of 4.0 m/s in the $-x$ direction after the collision. Are the results in (*c*), (*d*), and (*e*) "reasonable"? Explain.

* 7–7 Collisions in Two Dimensions

*** 40.** (II) A radioactive nucleus at rest decays into a second nucleus, an electron, and a neutrino. The electron and neutrino are emitted at right angles and have momenta of $9.30 \times 10^{-23}\text{ kg}\cdot\text{m/s}$ and $5.40 \times 10^{-23}\text{ kg}\cdot\text{m/s}$, respectively. What are the magnitude and direction of the momentum of the second (recoiling) nucleus?

*** 41.** (II) An eagle $(m_A = 4.3\text{ kg})$ moving with speed $v_A = 7.8\text{ m/s}$ is on a collision course with a second eagle $(m_B = 5.6\text{ kg})$ moving at $v_B = 10.2\text{ m/s}$ in a direction perpendicular to the first. After they collide, they hold onto one another. In what direction, and with what speed, are they moving after the collision?

*** 42.** (II) Billiard ball A of mass $m_A = 0.400\text{ kg}$ moving with speed $v_A = 1.80\text{ m/s}$ strikes ball B, initially at rest, of mass $m_B = 0.500\text{ kg}$. As a result of the collision, ball A is deflected off at an angle of 30.0° with a speed $v'_A = 1.10\text{ m/s}$. (*a*) Taking the *x* axis to be the original direction of motion of ball A, write down the equations expressing the conservation of momentum for the components in the *x* and *y* directions separately. (*b*) Solve these equations for the speed v'_B and angle θ'_B of ball B. Do not assume the collision is elastic.

*** 43.** (III) After a completely inelastic collision between two objects of equal mass, each having initial speed *v*, the two move off together with speed $v/3$. What was the angle between their initial directions?

*** 44.** (III) Two billiard balls of equal mass move at right angles and meet at the origin of an *xy* coordinate system. Ball A is moving upward along the *y* axis at 2.0 m/s, and ball B is moving to the right along the *x* axis with speed 3.7 m/s. After the collision, assumed elastic, ball B is moving along the positive *y* axis (Fig. 7–37). What is the final direction of ball A and what are their two speeds?

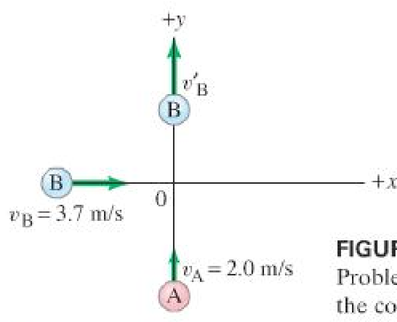


FIGURE 7–37 Problem 44. (Ball A after the collision is not shown.)

*** 45.** (III) A neon atom ($m = 20.0\text{ u}$) makes a perfectly elastic collision with another atom at rest. After the impact, the neon atom travels away at a 55.6° angle from its original direction and the unknown atom travels away at a −50.0° angle. What is the mass (in u) of the unknown atom? [*Hint*: You can use the law of sines.]

7–8 Center of Mass

46. (I) Find the center of mass of the three-mass system shown in Fig. 7–38. Specify relative to the left-hand 1.00-kg mass.

FIGURE 7–38 Problem 46.

47. (I) The distance between a carbon atom $(m_C = 12\,u)$ and an oxygen atom $(m_O = 16\,u)$ in the CO molecule is 1.13×10^{-10} m. How far from the carbon atom is the center of mass of the molecule?

48. (I) The CM of an empty 1050-kg car is 2.50 m behind the front of the car. How far from the front of the car will the CM be when two people sit in the front seat 2.80 m from the front of the car, and three people sit in the back seat 3.90 m from the front? Assume that each person has a mass of 70.0 kg.

49. (II) A square uniform raft, 18 m by 18 m, of mass 6800 kg, is used as a ferryboat. If three cars, each of mass 1200 kg, occupy its NE, SE, and SW corners, determine the CM of the loaded ferryboat.

50. (II) Three cubes, of sides l_0, $2l_0$, and $3l_0$, are placed next to one another (in contact) with their centers along a straight line and the $l = 2l_0$ cube in the center (Fig. 7–39). What is the position, along this line, of the CM of this system? Assume the cubes are made of the same uniform material.

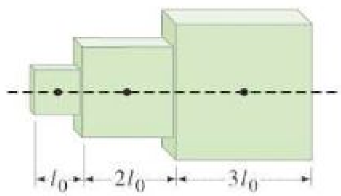


FIGURE 7–39 Problem 50.

51. (II) A (lightweight) pallet has a load of identical cases of tomato paste (see Fig. 7–40), each of which is a cube of length l. Find the center of gravity in the horizontal plane, so that the crane operator can pick up the load without tipping it.

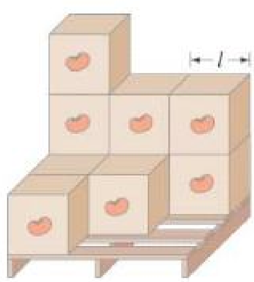


FIGURE 7–40 Problem 51.

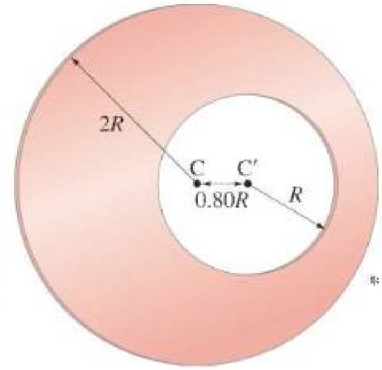


FIGURE 7–41 Problem 52.

52. (III) A uniform circular plate of radius $2R$ has a circular hole of radius R cut out of it. The center C′ of the smaller circle is a distance $0.80R$ from the center C of the larger circle, Fig. 7–41. What is the position of the center of mass of the plate? [*Hint*: Try subtraction.]

* 7–9 CM for the Human Body

* 53. (I) Assume that your proportions are the same as those in Table 7–1, and calculate the mass of one of your legs.

* 54. (I) Determine the CM of an outstretched arm using Table 7–1.

* 55. (II) Use Table 7–1 to calculate the position of the CM of an arm bent at a right angle. Assume that the person is 155 cm tall.

* 56. (II) When a high jumper is in a position such that his arms and legs are hanging vertically, and his trunk and head are horizontal, calculate how far below the torso's median line the CM will be. Will this CM be outside the body? Use Table 7–1.

* 7–10 CM and Translational Motion

* 57. (II) The masses of the Earth and Moon are 5.98×10^{24} kg and 7.35×10^{22} kg, respectively, and their centers are separated by 3.84×10^{8} m. (*a*) Where is the CM of this system located? (*b*) What can you say about the motion of the Earth–Moon system about the Sun, and of the Earth and Moon separately about the Sun?

* 58. (II) A 55-kg woman and an 80-kg man stand 10.0 m apart on frictionless ice. (*a*) How far from the woman is their CM? (*b*) If each holds one end of a rope, and the man pulls on the rope so that he moves 2.5 m, how far from the woman will he be now? (*c*) How far will the man have moved when he collides with the woman?

* 59. (II) A mallet consists of a uniform cylindrical head of mass 2.00 kg and a diameter 0.0800 m mounted on a uniform cylindrical handle of mass 0.500 kg and length 0.240 m, as shown in Fig. 7–42. If this mallet is tossed, spinning, into the air, how far above the bottom of the handle is the point that will follow a parabolic trajectory?

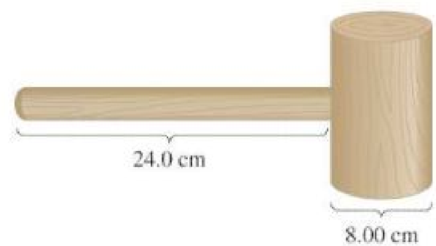


FIGURE 7–42 Problem 59.

* 60. (II) (*a*) Suppose that in Example 7–14 (Fig. 7–29), $m_{II} = 3m_I$. Where then would m_{II} land? (*b*) What if $m_I = 3m_{II}$?

* 61. (III) A helium balloon and its gondola, of mass M, are in the air and stationary with respect to the ground. A passenger, of mass m, then climbs out and slides down a rope with speed v, measured with respect to the balloon. With what speed and direction (relative to Earth) does the balloon then move? What happens if the passenger stops?

General Problems

62. A 0.145-kg baseball pitched horizontally at 35.0 m/s strikes a bat and is popped straight up to a height of 55.6 m. If the contact time is 1.4 ms, calculate the average force on the ball during the contact.

63. A rocket of mass m traveling with speed v_0 along the x axis suddenly shoots out fuel, equal to one-third of its mass, parallel to the y axis (perpendicular to the rocket as seen from the ground) with speed $2v_0$. Give the components of the final velocity of the rocket.

* 64. A novice pool player is faced with the corner pocket shot shown in Fig. 7–43. Relative dimensions are also shown. Should the player be worried about this being a "scratch shot," in which the cue ball will also fall into a pocket? Give details.

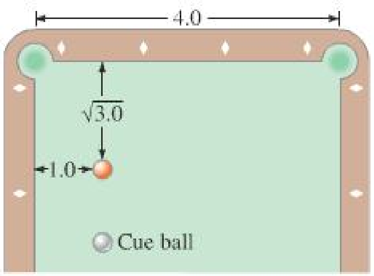


FIGURE 7–43 Problem 64.

65. A 140-kg astronaut (including space suit) acquires a speed of 2.50 m/s by pushing off with his legs from an 1800-kg space capsule. (*a*) What is the change in speed of the space capsule? (*b*) If the push lasts 0.40 s, what is the average force exerted on the astronaut by the space capsule? As the reference frame, use the position of the space capsule before the push.

66. Two astronauts, one of mass 60 kg and the other 80 kg, are initially at rest in outer space. They then push each other apart. How far apart are they when the lighter astronaut has moved 12 m?

67. A ball of mass m makes a head-on elastic collision with a second ball (at rest) and rebounds in the opposite direction with a speed equal to one-fourth its original speed. What is the mass of the second ball?

68. You have been hired as an expert witness in a court case involving an automobile accident. The accident involved car A of mass 1900 kg which crashed into stationary car B of mass 1100 kg. The driver of car A applied his brakes 15 m before he crashed into car B. After the collision, car A slid 18 m while car B slid 30 m. The coefficient of kinetic friction between the locked wheels and the road was measured to be 0.60. Show that the driver of car A was exceeding the 55-mph (90 km/h) speed limit before applying the brakes.

69. A golf ball rolls off the top of a flight of concrete steps of total vertical height 4.00 m. The ball hits four times on the way down, each time striking the horizontal part of a different step 1.0 m lower. If all collisions are perfectly elastic, what is the bounce height on the fourth bounce when the ball reaches the bottom of the stairs?

70. A bullet is fired vertically into a 1.40-kg block of wood at rest directly above it. If the bullet has a mass of 29.0 g and a speed of 510 m/s, how high will the block rise after the bullet becomes embedded in it?

71. A 25-g bullet strikes and becomes embedded in a 1.35-kg block of wood placed on a horizontal surface just in front of the gun. If the coefficient of kinetic friction between the block and the surface is 0.25, and the impact drives the block a distance of 9.5 m before it comes to rest, what was the muzzle speed of the bullet?

72. Two people, one of mass 75 kg and the other of mass 60 kg, sit in a rowboat of mass 80 kg. With the boat initially at rest, the two people, who have been sitting at opposite ends of the boat 3.2 m apart from each other, now exchange seats. How far and in what direction will the boat move?

73. A meteor whose mass was about 1.0×10^8 kg struck the Earth $(m_E = 6.0 \times 10^{24}\text{ kg})$ with a speed of about 15 km/s and came to rest in the Earth. (*a*) What was the Earth's recoil speed? (*b*) What fraction of the meteor's kinetic energy was transformed to kinetic energy of the Earth? (*c*) By how much did the Earth's kinetic energy change as a result of this collision?

74. An object at rest is suddenly broken apart into two fragments by an explosion. One fragment acquires twice the kinetic energy of the other. What is the ratio of their masses?

75. The force on a bullet is given by the formula $F = 580 - (1.8 \times 10^5)t$ over the time interval $t = 0$ to $t = 3.0 \times 10^{-3}$ s. In this formula, t is in seconds and F is in newtons. (*a*) Plot a graph of F vs. t for $t = 0$ to $t = 3.0$ ms. (*b*) Estimate, using graphical methods, the impulse given the bullet. (*c*) If the bullet achieves a speed of 220 m/s as a result of this impulse, given to it in the barrel of a gun, what must its mass be?

76. Two balls, of masses $m_A = 40$ g and $m_B = 60$ g, are suspended as shown in Fig. 7–44. The lighter ball is pulled away to a 60° angle with the vertical and released. (*a*) What is the velocity of the lighter ball before impact? (*b*) What is the velocity of each ball after the elastic collision? (*c*) What will be the maximum height of each ball after the elastic collision?

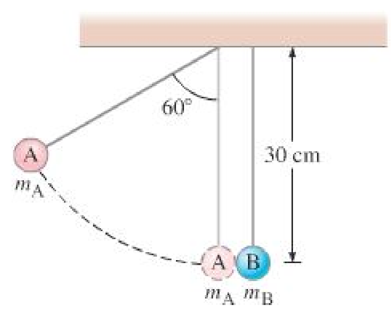


FIGURE 7–44 Problem 76.

77. An atomic nucleus at rest decays radioactively into an alpha particle and a smaller nucleus. What will be the speed of this recoiling nucleus if the speed of the alpha particle is 3.8×10^5 m/s? Assume the recoiling nucleus has a mass 57 times greater than that of the alpha particle.

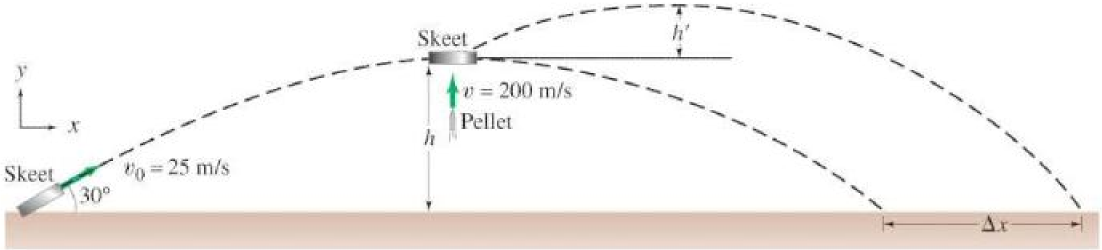


FIGURE 7–45 Problem 78.

78. A 0.25-kg skeet (clay target) is fired at an angle of 30° to the horizon with a speed of 25 m/s (Fig. 7–45). When it reaches the maximum height, it is hit from below by a 15-g pellet traveling vertically upward at a speed of 200 m/s. The pellet is embedded in the skeet. (*a*) How much higher did the skeet go up? (*b*) How much extra distance, Δx, does the skeet travel because of the collision?

79. A block of mass $m = 2.20$ kg slides down a 30.0° incline which is 3.60 m high. At the bottom, it strikes a block of mass $M = 7.00$ kg which is at rest on a horizontal surface, Fig. 7–46. (Assume a smooth transition at the bottom of the incline.) If the collision is elastic, and friction can be ignored, determine (*a*) the speeds of the two blocks after the collision, and (*b*) how far back up the incline the smaller mass will go.

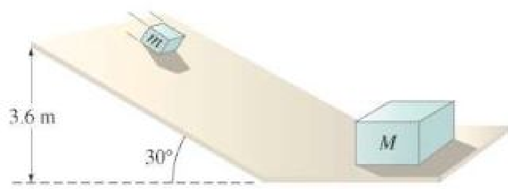


FIGURE 7–46 Problems 79 and 80.

80. In Problem 79 (Fig. 7–46), what is the upper limit on mass m if it is to rebound from M, slide up the incline, stop, slide down the incline, and collide with M again?

81. *The gravitational slingshot effect.* Figure 7–47 shows the planet Saturn moving in the negative x direction at its orbital speed (with respect to the Sun) of 9.6 km/s. The mass of Saturn is 5.69×10^{26} kg. A spacecraft with mass 825 kg approaches Saturn. When far from Saturn, it moves in the $+x$ direction at 10.4 km/s. The gravitational attraction of Saturn (a conservative force) acting on the spacecraft causes it to swing around the planet (orbit shown as dashed line) and head off in the opposite direction. Estimate the final speed of the spacecraft after it is far enough away to be considered free of Saturn's gravitational pull.

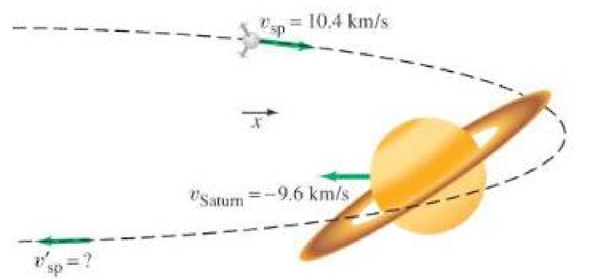


FIGURE 7–47 Problem 81.

Answers to Exercises

A: Yes, if the sports car's speed is three times greater.
B: Larger.
C: (*a*) 6.0 m/s; (*b*) almost zero; (*c*) almost 24.0 m/s.
D: The curve would be wider and less high.
E: Yes, by 300 times.
F: Yes, KE was conserved.
G: $x_{CM} = -2.0$ m; yes.
H: The boat moves in the opposite direction.