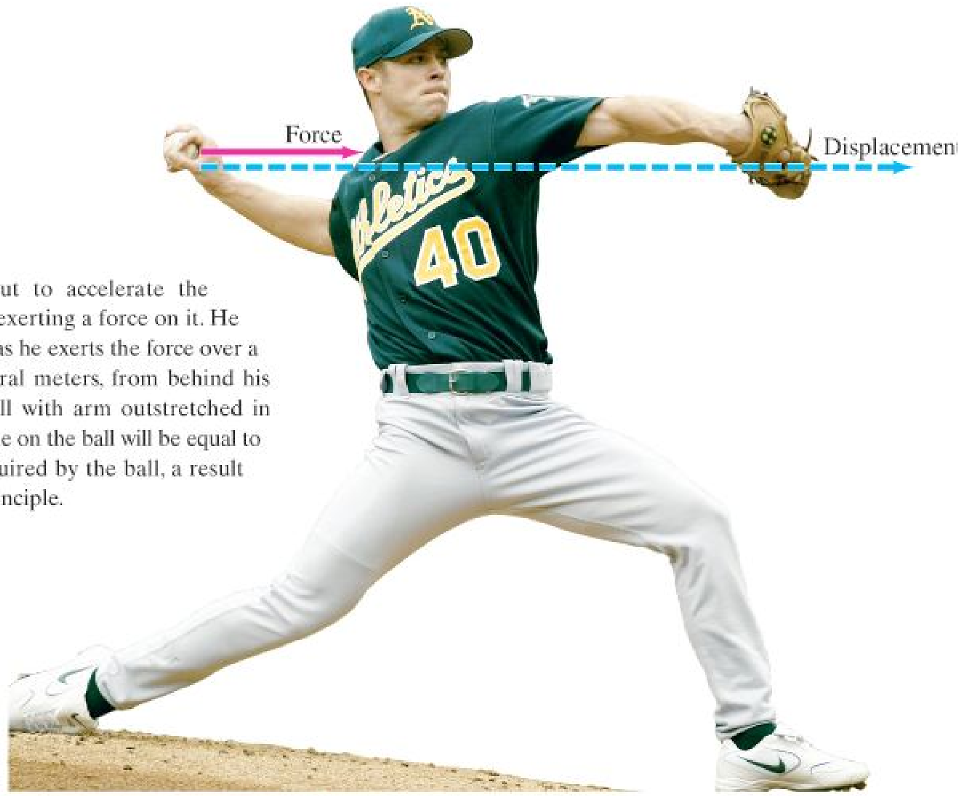


This baseball pitcher is about to accelerate the baseball to a high velocity by exerting a force on it. He will be doing work on the ball as he exerts the force over a displacement of perhaps several meters, from behind his head until he releases the ball with arm outstretched in front of him. The total work done on the ball will be equal to the kinetic energy ($\frac{1}{2}mv^2$) acquired by the ball, a result known as the work-energy principle.

CHAPTER 6

Work and Energy

Until now we have been studying the translational motion of an object in terms of Newton's three laws of motion. In this analysis, *force* has played a central role as the quantity determining the motion. In this Chapter and the next, we discuss an alternative analysis of the translational motion of objects in terms of the quantities *energy* and *momentum*. The significance of energy and momentum is that they are *conserved*. That is, in quite general circumstances they remain constant. That conserved quantities exist gives us not only a deeper insight into the nature of the world, but also gives us another way to approach solving practical problems.

The conservation laws of energy and momentum are especially valuable in dealing with systems of many objects, in which a detailed consideration of the forces involved would be difficult or impossible. These laws are applicable to a wide range of phenomena, including the atomic and subatomic worlds, where Newton's laws do not apply.

This Chapter is devoted to the very important concepts of *work* and *energy*. These two quantities are scalars and so have no direction associated with them, which often makes them easier to work with than vector quantities.

6–1 Work Done by a Constant Force

The word *work* has a variety of meanings in everyday language. But in physics, work is given a very specific meaning to describe what is accomplished when a force acts on an object, and the object moves through a distance. Specifically, the **work** done on an object by a constant force (constant in both magnitude and direction) is defined to be *the product of the magnitude of the displacement times the component of the force parallel to the displacement*. In equation form, we can write

Work

$$W = F_{\parallel} d,$$

where $F_{\parallel}$ is the component of the constant force $\vec{\mathbf{F}}$ parallel to the displacement $\vec{\mathbf{d}}$. We can also write

$$W = Fd\cos\theta, \qquad \textbf{(6–1)}$$

Work defined (for constant force)

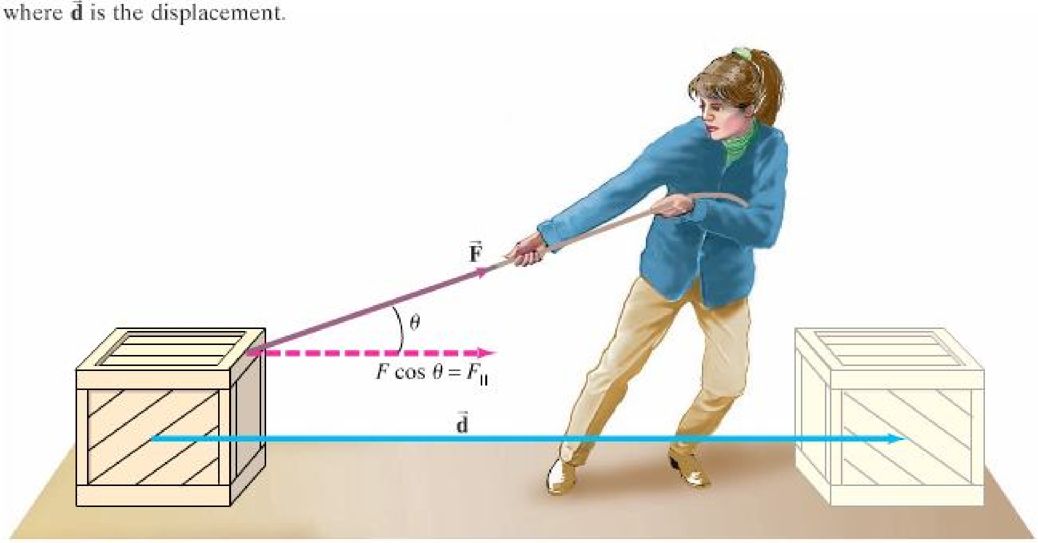
where F is the magnitude of the constant force, d is the magnitude of the displacement of the object, and θ is the angle between the directions of the force and the displacement (Fig. 6–1). The $\cos\theta$ factor appears in Eq. 6–1 because $F\cos\theta\ (=F_{\parallel})$ is the component of $\vec{\mathbf{F}}$ that is parallel to $\vec{\mathbf{d}}$. Work is a scalar quantity—it has only magnitude, which can be positive or negative.

Let us first consider the case in which the motion and the force are in the same direction, so $\theta = 0$ and $\cos\theta = 1$; in this case, $W = Fd$. For example, if you push a loaded grocery cart a distance of 50 m by exerting a horizontal force of 30 N on the cart, you do $30\,\text{N} \times 50\,\text{m} = 1500\,\text{N}\cdot\text{m}$ of work on the cart.

As this example shows, in SI units work is measured in newton-meters ($\text{N}\cdot\text{m}$). A special name is given to this unit, the **joule** (J): $1\,\text{J} = 1\,\text{N}\cdot\text{m}$. In the cgs system, the unit of work is called the *erg* and is defined as $1\,\text{erg} = 1\,\text{dyne}\cdot\text{cm}$. In British units, work is measured in foot-pounds. It is easy to show that $1\,\text{J} = 10^7\,\text{erg} = 0.7376\,\text{ft}\cdot\text{lb}$.

Units for work: the joule

FIGURE 6–1 A person pulling a crate along the floor. The work done by the force $\vec{\mathbf{F}}$ is $W = Fd\cos\theta$, where $\vec{\mathbf{d}}$ is the displacement.



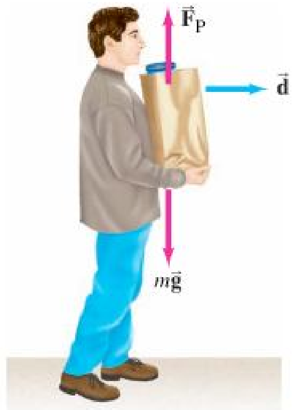


FIGURE 6–2 The person does no work on the bag of groceries since $\vec{\mathbf{F}}_P$ is perpendicular to the displacement $\vec{\mathbf{d}}$.

CAUTION
Force without work

A force can be exerted on an object and yet do no work. For example, if you hold a heavy bag of groceries in your hands at rest, you do no work on it. You do exert a force on the bag, but the displacement of the bag is zero, so the work done by you on the bag is $W = 0$. You need both a force and a displacement to do work. You also do no work on the bag of groceries if you carry it as you walk horizontally across the floor at constant velocity, as shown in Fig. 6–2. No horizontal force is required to move the bag at a constant velocity. The person shown in Fig. 6–2 does exert an upward force $\vec{\mathbf{F}}_P$ on the bag equal to its weight. But this upward force is perpendicular to the horizontal displacement of the bag and thus has nothing to do with that motion. Hence, the upward force is doing no work. This conclusion comes from our definition of work, Eq. 6–1: $W = 0$, because $\theta = 90°$ and $\cos 90° = 0$. Thus, when a particular force is perpendicular to the displacement, no work is done by that force. (When you start or stop walking, there is a horizontal acceleration and you do briefly exert a horizontal force, and thus do work on the bag.)

CAUTION
State that work is done on or by an object

When we deal with work, as with force, it is necessary to specify whether you are talking about work done *by* a specific object or done *on* a specific object. It is also important to specify whether the work done is due to one particular force (and which one), or the total (net) work done by the *net force* on the object.

EXAMPLE 6–1 Work done on a crate. A person pulls a 50-kg crate 40 m along a horizontal floor by a constant force $F_P = 100\,\text{N}$, which acts at a 37° angle as shown in Fig. 6–3. The floor is rough and exerts a friction force $F_{fr} = 50\,\text{N}$. Determine (*a*) the work done by each force acting on the crate, and (*b*) the net work done on the crate.

APPROACH We choose our coordinate system so that $\vec{\mathbf{x}}$ can be the vector that represents the 40-m displacement (that is, along the x axis). Four forces act on the crate, as shown in Fig. 6–3: the force exerted by the person $\vec{\mathbf{F}}_P$; the friction force $\vec{\mathbf{F}}_{fr}$ due to the floor; the crate's weight $m\vec{\mathbf{g}}$; and the normal force $\vec{\mathbf{F}}_N$ exerted upward by the floor. The net force on the crate is the vector sum of these four forces.

SOLUTION (*a*) The work done by the gravitational and normal forces is zero, since they are perpendicular to the displacement $\vec{\mathbf{x}}$ ($\theta = 90°$ in Eq. 6–1):

$$W_G = mgx\cos 90° = 0$$
$$W_N = F_N x\cos 90° = 0.$$

The work done by $\vec{\mathbf{F}}_P$ is

$$W_P = F_P x\cos\theta = (100\,\text{N})(40\,\text{m})\cos 37° = 3200\,\text{J}.$$

The work done by the friction force is

$$W_{fr} = F_{fr}x\cos 180° = (50\,\text{N})(40\,\text{m})(-1) = -2000\,\text{J}.$$

The angle between the displacement $\vec{\mathbf{x}}$ and the force $\vec{\mathbf{F}}_{fr}$ is 180° because they point in opposite directions. Since the force of friction is opposing the motion (and $\cos 180° = -1$), the work done by friction on the crate is *negative*.

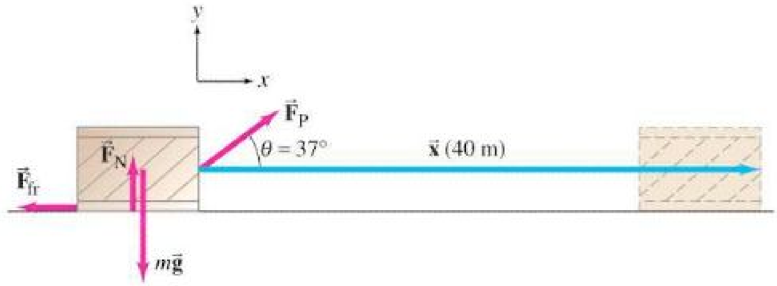


FIGURE 6–3 Example 6–1. A 50-kg crate is pulled along a floor.

(*b*) The net work can be calculated in two equivalent ways:

(1) The net work done on an object is the algebraic sum of the work done by each force, since work is a scalar:

$$\begin{aligned} W_{net} &= W_G + W_N + W_P + W_{fr} \\ &= 0 + 0 + 3200\text{ J} - 2000\text{ J} \\ &= 1200\text{ J}. \end{aligned}$$

W_{net} is the work done by all the forces acting on the object

(2) The net work can also be calculated by first determining the net force on the object and then taking its component along the displacement: $(F_{net})_x = F_P \cos\theta - F_{fr}$. Then the net work is

$$\begin{aligned} W_{net} = (F_{net})_x x &= (F_P \cos\theta - F_{fr})x \\ &= (100\text{ N} \cos 37° - 50\text{ N})(40\text{ m}) \\ &= 1200\text{ J}. \end{aligned}$$

In the vertical (y) direction, there is no displacement and no work done.

In Example 6–1 we saw that friction did negative work. In general, the work done by a force is negative whenever the force (or the component of the force, $F_{\|}$) acts in the direction opposite to the direction of motion. Also, we can see that when the work done by a force on an object is negative, that force is trying to slow the object down (and would slow it down if that were the only force acting). When the work is positive, the force involved is trying to speed up the object.

CAUTION
Negative work

EXERCISE A A box is dragged across a floor by a force $\vec{F}_P$ which makes an angle θ with the horizontal as in Fig. 6–1 or 6–3. If the magnitude of $\vec{F}_P$ is held constant but the angle θ is increased, the work done by $\vec{F}_P$ (*a*) remains the same; (*b*) increases; (*c*) decreases; (*d*) first increases, then decreases.

PROBLEM SOLVING Work

1. **Draw a free-body diagram** showing all the forces acting on the object you choose to study.
2. **Choose** an *xy* **coordinate system.** If the object is in motion, it may be convenient to choose one of the coordinate directions as the direction of one of the forces, or as the direction of motion. [Thus, for an object on an incline, you might choose one coordinate axis to be parallel to the incline.]
3. Apply **Newton's laws** to determine any unknown forces.
4. Find the **work done** ***by*** **a specific force** ***on*** the object by using $W = Fd\cos\theta$ for a constant force. Note that the work done is negative when a force tends to oppose the displacement.
5. To find the **net work** done on the object, either (*a*) find the work done by each force and add the results algebraically; or (*b*) find the net force on the object, F_{net}, and then use it to find the net work done, which for constant net force is:

$$W_{net} = F_{net}\, d \cos\theta.$$

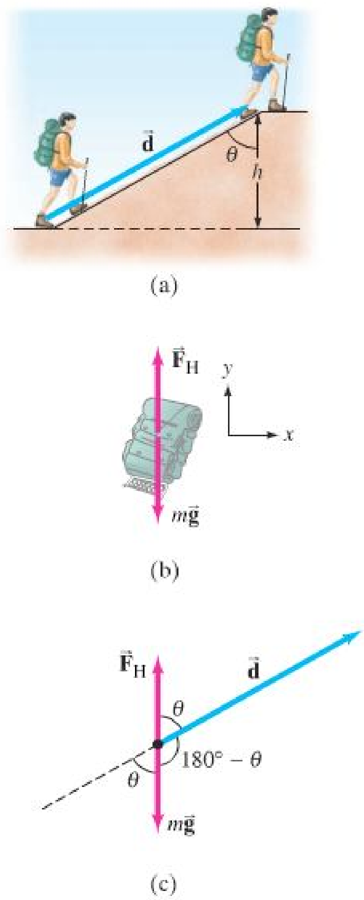


FIGURE 6–4 Example 6–2.

EXAMPLE 6–2 **Work on a backpack.** (*a*) Determine the work a hiker must do on a 15.0-kg backpack to carry it up a hill of height $h = 10.0\,\mathrm{m}$, as shown in Fig. 6–4a. Determine also (*b*) the work done by gravity on the backpack, and (*c*) the net work done on the backpack. For simplicity, assume the motion is smooth and at constant velocity (i.e., acceleration is negligible).

APPROACH We explicitly follow the Problem Solving Box step by step.

SOLUTION

1. **Draw a free-body diagram.** The forces on the backpack are shown in Fig. 6–4b: the force of gravity, $m\vec{\mathbf{g}}$, acting downward; and $\vec{\mathbf{F}}_{\mathrm{H}}$, the force the hiker must exert upward to support the backpack. Since we assume there is negligible acceleration, horizontal forces on the backpack are negligible.
2. **Choose a coordinate system.** We are interested in the vertical motion of the backpack, so we choose the y coordinate as positive vertically upward.
3. **Apply Newton's laws.** Newton's second law applied in the vertical direction to the backpack gives

$$\Sigma F_y = ma_y$$
$$F_{\mathrm{H}} - mg = 0.$$

Hence,

$$F_{\mathrm{H}} = mg = (15.0\,\mathrm{kg})(9.80\,\mathrm{m/s^2}) = 147\,\mathrm{N}.$$

4. **Find the work done by a specific force.** (*a*) To calculate the work done by the hiker on the backpack, we write Eq. 6–1 as

$$W_{\mathrm{H}} = F_{\mathrm{H}}(d\cos\theta),$$

and we note from Fig. 6–4a that $d\cos\theta = h$. So the work done by the hiker is

$$W_{\mathrm{H}} = F_{\mathrm{H}}(d\cos\theta) = F_{\mathrm{H}}h = mgh$$
$$= (147\,\mathrm{N})(10.0\,\mathrm{m}) = 1470\,\mathrm{J}.$$

Note that the work done depends only on the change in elevation and not on the angle of the hill, θ. The hiker would do the same work to lift the pack vertically the same height h.

(*b*) The work done by gravity on the backpack is (from Eq. 6–1 and Fig. 6–4c)

$$W_{\mathrm{G}} = F_{\mathrm{G}}\,d\cos(180^\circ - \theta).$$

Since $\cos(180^\circ - \theta) = -\cos\theta$, we have

$$W_{\mathrm{G}} = F_{\mathrm{G}}\,d(-\cos\theta) = mg(-d\cos\theta)$$
$$= -mgh$$
$$= -(15.0\,\mathrm{kg})(9.80\,\mathrm{m/s^2})(10.0\,\mathrm{m}) = -1470\,\mathrm{J}.$$

PROBLEM SOLVING

Work done by gravity depends on the height of the hill and not on the angle of incline

NOTE The work done by gravity (which is negative here) doesn't depend on the angle of the incline, only on the vertical height h of the hill. This is because gravity acts vertically, so only the vertical component of displacement contributes to work done.

5. **Find the net work done.** (*a*) The *net* work done on the backpack is $W_{\mathrm{net}} = 0$, since the net force on the backpack is zero (it is assumed not to accelerate significantly). We can also determine the net work done by adding the work done by each force:

$$W_{\mathrm{net}} = W_{\mathrm{G}} + W_{\mathrm{H}} = -1470\,\mathrm{J} + 1470\,\mathrm{J} = 0.$$

NOTE Even though the *net* work done by all the forces on the backpack is zero, the hiker does do work on the backpack equal to 1470 J.

CONCEPTUAL EXAMPLE 6–3 **Does the Earth do work on the Moon?** The Moon revolves around the Earth in a nearly circular orbit, kept there by the gravitational force exerted by the Earth. Does gravity do (*a*) positive work, (*b*) negative work, or (*c*) no work on the Moon?

RESPONSE The gravitational force exerted by the Earth on the Moon (Fig. 6–5) acts toward the Earth and provides its centripetal acceleration, inward along the radius of the Moon's orbit. The Moon's displacement at any moment is tangent to the circle, in the direction of its velocity, perpendicular to the radius and perpendicular to the force of gravity. Hence the angle θ between the force $\vec{\mathbf{F}}_G$ and the instantaneous displacement of the Moon is 90°, and the work done by the Earth's gravity on the Moon as it orbits is therefore zero ($\cos 90° = 0$). This is why the Moon, as well as artificial satellites, can stay in orbit without expenditure of fuel: no net work needs to be done against the force of gravity.

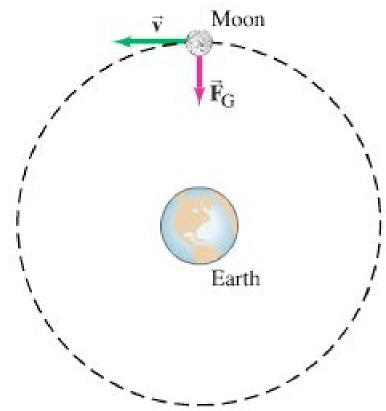


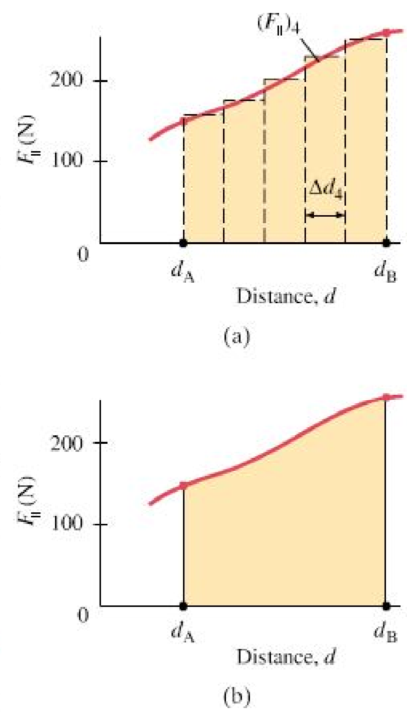
FIGURE 6–5 Example 6–3.

* 6–2 Work Done by a Varying Force

If the force acting on an object is constant, the work done by that force can be calculated using Eq. 6–1. But in many cases, the force varies in magnitude or direction during a process. For example, as a rocket moves away from Earth, work is done to overcome the force of gravity, which varies as the inverse square of the distance from the Earth's center. Other examples are the force exerted by a spring, which increases with the amount of stretch, or the work done by a varying force in pulling a box or cart up an uneven hill.

The work done by a varying force can be determined graphically. The procedure is like that for determining displacement when the velocity is known as a function of time (Section 2–8). To determine the work done by a variable force, we plot $F_{\parallel}$ ($= F\cos\theta$, the component of $\vec{\mathbf{F}}$ parallel to the direction of motion at any point) as a function of distance d, as in Fig. 6–6a. We divide the distance into small segments Δd. For each segment, we indicate the average of $F_{\parallel}$ by a horizontal dashed line. Then the work done for each segment is $\Delta W = F_{\parallel}\,\Delta d$, which is the area of a rectangle Δd wide and $F_{\parallel}$ high. The total work done to move the object a total distance $d = d_B - d_A$ is the sum of the areas of the rectangles (five in the case shown in Fig. 6–6a). Usually, the average value of $F_{\parallel}$ for each segment must be estimated, and a reasonable approximation of the work done can then be made. If we subdivide the distance into many more segments, Δd can be made smaller and our estimate of the work done would be more accurate. In the limit as Δd approaches zero, the total area of the many narrow rectangles approaches the area under the curve, Fig. 6–6b. That is, *the work done by a variable force in moving an object between two points is equal to the area under the $F_{\parallel}$ vs. d curve between those two points.*

FIGURE 6–6 The work done by a force F can be calculated by taking: (a) the sum of the areas of the rectangles; (b) the area under the curve of $F_{\parallel}$ vs. d.



6–3 Kinetic Energy, and the Work-Energy Principle

Energy is one of the most important concepts in science. Yet we cannot give a simple general definition of energy in only a few words. Nonetheless, each specific type of energy can be defined fairly simply. In this Chapter, we define translational kinetic energy and some types of potential energy. In later Chapters, we will examine other types of energy, such as that related to heat (Chapters 14 and 15). The crucial aspect of all the types of energy is that the sum of all types, the *total energy*, is the same after any process as it was before: that is, the quantity "energy" is a conserved quantity.

For the purposes of this Chapter, we can define energy in the traditional way as "the ability to do work." This simple definition is not very precise, nor is it really valid for all types of energy.[†] It is valid, however, for mechanical energy which we discuss in this Chapter, and it serves to underscore the fundamental

[†] Energy associated with heat is often not available to do work, as we will discuss in Chapter 15.

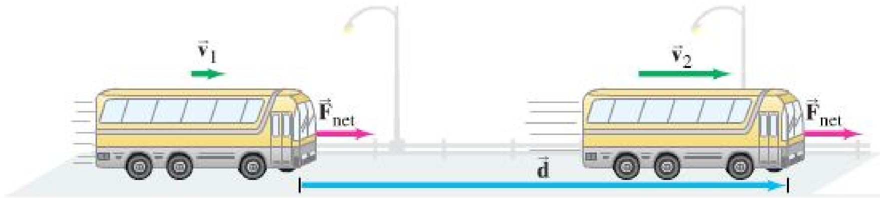


FIGURE 6–7 A constant net force F_{net} accelerates a bus from speed v_1 to speed v_2 over a displacement d. The net work done is $W_{net} = F_{net}d$.

connection between work and energy. We now define and discuss one of the basic types of energy, kinetic energy.

A moving object can do work on another object it strikes. A flying cannonball does work on a brick wall it knocks down; a moving hammer does work on a nail it drives into wood. In either case, a moving object exerts a force on a second object which undergoes a displacement. An object in motion has the ability to do work and thus can be said to have energy. The energy of motion is called **kinetic energy**, from the Greek word *kinetikos*, meaning "motion."

To obtain a quantitative definition for kinetic energy, let us consider a rigid object of mass m that is moving in a straight line with an initial speed v_1. To accelerate it uniformly to a speed v_2, a constant net force F_{net} is exerted on it parallel to its motion over a displacement d, Fig. 6–7. Then the net work done on the object is $W_{net} = F_{net}d$. We apply Newton's second law, $F_{net} = ma$, and use Eq. 2–11c, which we now write as $v_2^2 = v_1^2 + 2ad$, with v_1 as the initial speed and v_2 the final speed. We solve for a in Eq. 2–11c,

$$a = \frac{v_2^2 - v_1^2}{2d},$$

then substitute this into $F_{net} = ma$, and determine the work done:

$$W_{net} = F_{net}d = mad = m\left(\frac{v_2^2 - v_1^2}{2d}\right)d = m\left(\frac{v_2^2 - v_1^2}{2}\right)$$

or

$$W_{net} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2. \qquad \textbf{(6–2)}$$

We *define* the quantity $\frac{1}{2}mv^2$ to be the **translational kinetic energy (KE)** of the object:

Kinetic energy defined

$$\text{KE} = \tfrac{1}{2}mv^2. \qquad \textbf{(6–3)}$$

(We call this "translational" kinetic energy to distinguish it from rotational kinetic energy, which we will discuss in Chapter 8.) Equation 6–2, derived here for one-dimensional motion with a constant force, is valid in general for translational motion of an object in three dimensions and even if the force varies. We can rewrite Eq. 6–2 as:

$$W_{net} = \text{KE}_2 - \text{KE}_1$$

or

WORK-ENERGY PRINCIPLE

$$W_{net} = \Delta\text{KE}. \qquad \textbf{(6–4)}$$

Equation 6–4 (or Eq. 6–2) is an important result known as the **work-energy principle**. It can be stated in words:

WORK-ENERGY PRINCIPLE

The net work done on an object is equal to the change in the object's kinetic energy.

Notice that we made use of Newton's second law, $F_{net} = ma$, where F_{net} is the *net* force—the sum of all forces acting on the object. Thus, the work-energy principle is valid only if W is the *net work* done on the object—that is, the work done by all forces acting on the object.

CAUTION
Work-energy valid only for net work

The work-energy principle is a very useful reformulation of Newton's laws. It tells us that if (positive) net work W is done on an object, the object's kinetic energy increases by an amount W. The principle also holds true for the reverse situation: if the net work W done on an object is negative, the object's kinetic energy decreases by an amount W. That is, a net force exerted on an object opposite to the object's direction of motion decreases its speed and its kinetic energy. An example is a moving hammer (Fig. 6–8) striking a nail. The net force on the hammer ($-\vec{\mathbf{F}}$ in Fig. 6–8, where $\vec{\mathbf{F}}$ is assumed constant for simplicity) acts toward the left, whereas the displacement $\vec{\mathbf{d}}$ of the hammer is toward the right. So the net work done on the hammer, $W_h = (F)(d)(\cos 180°) = -Fd$, is negative and the hammer's kinetic energy decreases (usually to zero).

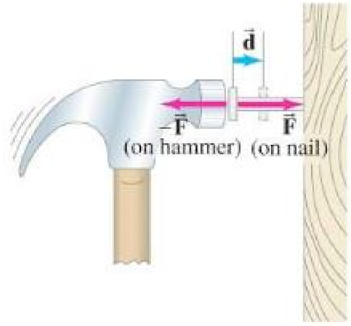


FIGURE 6–8 A moving hammer strikes a nail and comes to rest. The hammer exerts a force F on the nail; the nail exerts a force $-F$ on the hammer (Newton's third law). The work done on the nail by the hammer is positive ($W_n = Fd > 0$). The work done on the hammer by the nail is negative ($W_h = -Fd$).

Figure 6–8 also illustrates how energy can be considered the ability to do work. The hammer, as it slows down, does positive work on the nail: if the nail exerts a force $-\vec{\mathbf{F}}$ on the hammer to slow it down, the hammer exerts a force $+\vec{\mathbf{F}}$ on the nail (Newton's third law) through the distance d. Hence the work done on the nail by the hammer is $W_n = (+F)(+d) = Fd$ and is positive. We also see that $W_n = Fd = -W_h$: the work done on the nail W_n equals the negative of the work done on the hammer. That is, the decrease in kinetic energy of the hammer is equal to the work the hammer can do on another object—which is consistent with energy being the ability to do work.

Whereas the translational kinetic energy $\left(= \frac{1}{2}mv^2\right)$ is directly proportional to the mass of the object, it is proportional to the *square* of the speed. Thus, if the mass is doubled, the kinetic energy is doubled. But if the speed is doubled, the object has four times as much kinetic energy and is therefore capable of doing four times as much work.

If $W_{net} > 0$, KE increases
If $W_{net} < 0$, KE decreases

Let us summarize the relationship between work and kinetic energy (Eq. 6–4): if the net work W done on an object is positive, then the object's kinetic energy increases. If the net work W done on an object is negative, its kinetic energy decreases. If the net work done on the object is zero, its kinetic energy remains constant (which also means its speed is constant).

Energy units: the joule

Because of the direct connection between work and kinetic energy (Eq. 6–4), energy is measured in the same units as work: joules in SI units, ergs in the cgs, and foot-pounds in the British system. Like work, kinetic energy is a scalar quantity. The kinetic energy of a group of objects is the sum of the kinetic energies of the individual objects.

EXAMPLE 6–4 **KE and work done on a baseball.** A 145-g baseball is thrown so that it acquires a speed of 25 m/s. (*a*) What is its kinetic energy? (*b*) What was the net work done on the ball to make it reach this speed, if it started from rest?

APPROACH We use the definition of kinetic energy, Eq. 6–3, and then the work-energy principle, Eq. 6–4.

SOLUTION (*a*) The kinetic energy of the ball after the throw is

$$\text{KE} = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(0.145\,\text{kg})(25\,\text{m/s})^2 = 45\,\text{J}.$$

(*b*) Since the initial kinetic energy was zero, the net work done is just equal to the final kinetic energy, 45 J.

EXAMPLE 6–5 **Work on a car, to increase its KE.** How much net work is required to accelerate a 1000-kg car from 20 m/s to 30 m/s (Fig. 6–9)?

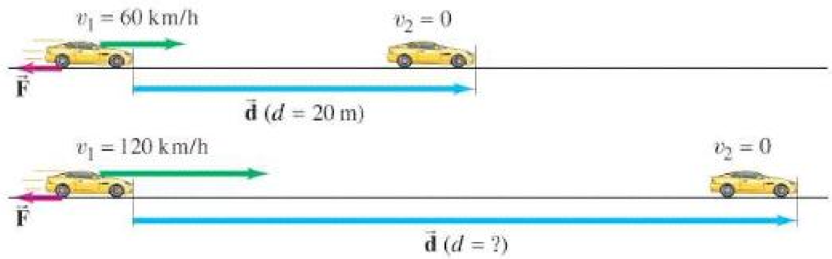
APPROACH To simplify a complex situation, let us treat the car as a particle or simple rigid object. We can then use the work-energy principle.

FIGURE 6–9 Example 6–5.

SOLUTION The net work needed is equal to the increase in kinetic energy:

$$W = \text{KE}_2 - \text{KE}_1 = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2$$
$$= \tfrac{1}{2}(1000\,\text{kg})(30\,\text{m/s})^2 - \tfrac{1}{2}(1000\,\text{kg})(20\,\text{m/s})^2 = 2.5 \times 10^5\,\text{J}.$$

NOTE You might be tempted to work this Example by finding the force and using Eq. 6–1. That won't work, however, because we don't know how far or for how long the car was accelerated. In fact, a large force could be acting for a small distance, or a small force could be acting over a long distance; both could give the same net work.



(a) (b)

FIGURE 6–10 Example 6–6.

CONCEPTUAL EXAMPLE 6–6 **Work to stop a car.** A car traveling 60 km/h can brake to a stop within a distance d of 20 m (Fig. 6–10a). If the car is going twice as fast, 120 km/h, what is its stopping distance (Fig. 6–10b)? Assume the maximum braking force is approximately independent of speed.

RESPONSE Since the stopping force F is approximately constant, the work needed to stop the car, Fd, is proportional to the distance traveled. We apply the work-energy principle, noting that $\vec{\mathbf{F}}$ and $\vec{\mathbf{d}}$ are in opposite directions and that the final speed of the car is zero:

$$W_{\text{net}} = Fd\cos 180^\circ = -Fd.$$

Then

$$-Fd = \Delta\text{KE} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2$$
$$= 0 - \tfrac{1}{2}mv_1^2.$$

Thus, since the force and mass are constant, we see that the stopping distance, d, increases with the square of the speed:

PHYSICS APPLIED
Car's stopping distance ∝ initial speed squared

$$d \propto v^2.$$

If the car's initial speed is doubled, the stopping distance is $(2)^2 = 4$ times as great, or 80 m.

EXERCISE B Can kinetic energy ever be negative?

6–4 Potential Energy

Potential energy

We have just discussed how an object is said to have energy by virtue of its motion, which we call kinetic energy. But it is also possible to have **potential energy**, which is the energy associated with forces that depend on the position or configuration of an object (or objects) relative to the surroundings. Various types of potential energy (PE) can be defined, and each type is associated with a particular force.

The spring of a wind-up toy is an example of an object with potential energy. The spring acquired its potential energy because work was done *on* it by the person winding the toy. As the spring unwinds, it exerts a force and does work to make the toy move.

Gravitational PE

Perhaps the most common example of potential energy is *gravitational potential energy*. A heavy brick held high in the air has potential energy because of its position relative to the Earth. The raised brick has the ability to do work, for if it is released, it will fall to the ground due to the gravitational force, and can do work on, say, a stake, driving it into the ground. Let us seek the form for the gravitational potential energy of an object near the surface of the Earth. For an object of mass m to be lifted vertically, an upward force at least equal to its weight, mg, must be exerted on it, say by a person's hand.

To lift it without acceleration a vertical displacement of height h, from position y_1 to y_2 in Fig. 6–11 (upward direction chosen positive), a person must do work equal to the product of the needed external force, $F_{ext} = mg$ upward, and the vertical displacement h. That is,

$$\begin{aligned} W_{ext} &= F_{ext} d \cos 0^\circ = mgh \\ &= mg(y_2 - y_1). \end{aligned} \quad \textbf{(6–5a)}$$

Gravity is also acting on the object as it moves from y_1 to y_2, and does work on it equal to

$$W_G = F_G d \cos\theta = mgh \cos 180^\circ,$$

where $\theta = 180^\circ$ because $\vec{\mathbf{F}}_G$ and $\vec{\mathbf{d}}$ point in opposite directions. So

$$\begin{aligned} W_G &= -mgh \\ &= -mg(y_2 - y_1). \end{aligned} \quad \textbf{(6–5b)}$$

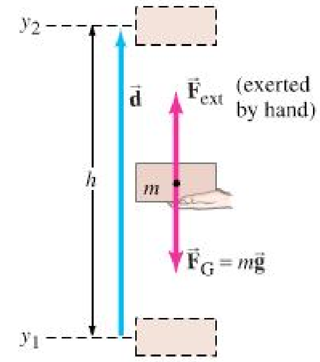


FIGURE 6–11 A person exerts an upward force $F_{ext} = mg$ to lift a brick from y_1 to y_2.

If we now allow the object to start from rest and fall freely under the action of gravity, it acquires a velocity given by $v^2 = 2gh$ (Eq. 2–11c) after falling a height h. It then has kinetic energy $\frac{1}{2}mv^2 = \frac{1}{2}m(2gh) = mgh$, and if it strikes a stake it can do work on the stake equal to mgh (work-energy principle). Thus, to raise an object of mass m to a height h *requires* an amount of work equal to mgh (Eq. 6–5a). And once at height h, the object has the *ability* to do an amount of work equal to mgh.

We therefore define the **gravitational potential energy** of an object, due to Earth's gravity, as the product of the object's weight mg and its height y above some reference level (such as the ground):

Gravitational PE

$$\text{PE}_{grav} = mgy. \quad \textbf{(6–6)}$$

The higher an object is above the ground, the more gravitational potential energy it has. We combine Eq. 6–5a with Eq. 6–6:

$$\begin{aligned} W_{ext} &= mg(y_2 - y_1) \\ W_{ext} &= \text{PE}_2 - \text{PE}_1 = \Delta\text{PE}. \end{aligned} \quad \textbf{(6–7a)}$$

That is, the work done by an external force to move the object of mass m from point 1 to point 2 (without acceleration) is equal to the change in potential energy between positions 1 and 2.

Alternatively, we can write the change in potential energy, ΔPE, in terms of the work done by gravity itself: starting from Eq. 6–5b, we obtain

$$\begin{aligned} W_G &= -mg(y_2 - y_1) \\ W_G &= -(\text{PE}_2 - \text{PE}_1) = -\Delta\text{PE}. \end{aligned} \quad \textbf{(6–7b)}$$

That is, the work done by gravity as the object of mass m moves from point 1 to point 2 is equal to the negative of the difference in potential energy between positions 1 and 2.

CAUTION
Potential energy belongs to a system, not to a single object

Potential energy belongs to a system, and not to a single object alone. Potential energy is associated with a force, and a force on one object is always exerted by some other object. Thus potential energy is a property of the system as a whole. For an object raised to a height y above the Earth's surface, the change in gravitational potential energy is mgy. The system here is the object plus the Earth, and properties of both are involved: object (m) and Earth (g).

Gravitational potential energy depends on the *vertical height* of the object *above some reference level* (Eq. 6–6). In some situations, you may wonder from what point to measure the height y. The gravitational potential energy of a book held high above a table, for example, depends on whether we measure y from the top of the table, from the floor, or from some other reference point.

CAUTION
Change in PE is what is physically meaningful

What is physically important in any situation is the *change* in potential energy, ΔPE, because that is what is related to the work done, Eqs. 6–7; and it is ΔPE that can be measured. We can thus choose to measure y from any reference point that is convenient, but we must choose the reference point at the start and be consistent throughout a calculation. The *change* in potential energy between any two points does not depend on this choice.

Grav. PE depends on vertical *height*

An important result we discussed earlier (see Example 6–2 and Fig. 6–4) concerns the gravity force, which does work only in the vertical direction: the work done by gravity depends only on the vertical height h, and not on the path taken, whether it be purely vertical motion or, say, motion along an incline. Thus, from Eqs. 6–7 we see that changes in gravitational potential energy depend only on the change in vertical height and not on the path taken.

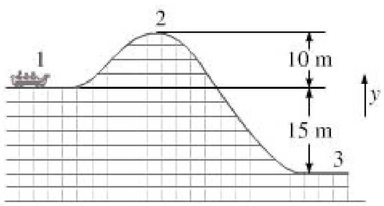


FIGURE 6–12 Example 6–7.

EXAMPLE 6–7 Potential energy changes for a roller coaster. A 1000-kg roller-coaster car moves from point 1, Fig. 6–12, to point 2 and then to point 3. (*a*) What is the gravitational potential energy at 2 and 3 relative to point 1? That is, take $y = 0$ at point 1. (*b*) What is the change in potential energy when the car goes from point 2 to point 3? (*c*) Repeat parts (*a*) and (*b*), but take the reference point ($y = 0$) to be at point 3.

APPROACH We are interested in the potential energy of the car–Earth system. We take upward as the positive y direction, and use the definition of gravitational potential energy to calculate PE.

SOLUTION (*a*) We measure heights from point 1, which means initially that the gravitational potential energy is zero. At point 2, where $y_2 = 10\,\text{m}$,

$$\text{PE}_2 = mgy_2 = (1000\,\text{kg})(9.8\,\text{m/s}^2)(10\,\text{m}) = 9.8 \times 10^4\,\text{J}.$$

At point 3, $y_3 = -15\,\text{m}$, since point 3 is below point 1. Therefore,

$$\text{PE}_3 = mgy_3 = (1000\,\text{kg})(9.8\,\text{m/s}^2)(-15\,\text{m}) = -1.5 \times 10^5\,\text{J}.$$

(*b*) In going from point 2 to point 3, the potential energy change $(\text{PE}_{\text{final}} - \text{PE}_{\text{initial}})$ is

$$\begin{aligned}\text{PE}_3 - \text{PE}_2 &= (-1.5 \times 10^5\,\text{J}) - (9.8 \times 10^4\,\text{J}) \\ &= -2.5 \times 10^5\,\text{J}.\end{aligned}$$

The gravitational potential energy decreases by 2.5×10^5 J.

(*c*) In this instance, $y_1 = +15\,\text{m}$ at point 1, so the potential energy initially (at point 1) is

$$\text{PE}_1 = (1000\,\text{kg})(9.8\,\text{m/s}^2)(15\,\text{m}) = 1.5 \times 10^5\,\text{J}.$$

At point 2, $y_2 = 25\,\text{m}$, so the potential energy is

$$\text{PE}_2 = 2.5 \times 10^5\,\text{J}.$$

At point 3, $y_3 = 0$, so the potential energy is zero. The change in potential energy going from point 2 to point 3 is

$$\text{PE}_3 - \text{PE}_2 = 0 - 2.5 \times 10^5\,\text{J} = -2.5 \times 10^5\,\text{J},$$

which is the same as in part (*b*).

PE defined in general

There are other kinds of potential energy besides gravitational. Each form of potential energy is associated with a particular force, and can be defined analogously to gravitational potential energy. In general, the *change in potential energy associated with a particular force is equal to the negative of the work done by that force if the object is moved from one point to a second point* (as in Eq. 6–7b for gravity). Alternatively, because of Newton's third law, we can define the *change in potential energy as the work required of an external force to move the object without acceleration between the two points*, as in Eq. 6–7a.

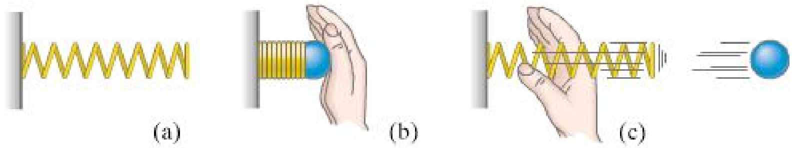


FIGURE 6–13 (a) A spring can store energy (elastic PE) when compressed as in (b) and can do work when released (c).

PE of elastic spring

We now consider another type of potential energy, that associated with elastic materials. This includes a great variety of practical applications. Consider the simple coil spring shown in Fig. 6–13. The spring has potential energy when compressed (or stretched), for when it is released, it can do work on a ball as shown. To hold a spring either stretched or compressed an amount x from its natural (unstretched) length requires the hand to exert a force on the spring, F_P, that is directly proportional to x. That is,

$$F_P = kx,$$

where k is a constant, called the *spring stiffness constant*, and is a measure of the stiffness of the particular spring. The stretched or compressed spring exerts a force F_S in the opposite direction on the hand, as shown in Fig. 6–14:

$$F_S = -kx. \quad \textbf{(6–8)}$$

This force is sometimes called a "restoring force" because the spring exerts its force in the direction opposite the displacement (hence the minus sign), acting to return it to its natural length. Equation 6–8 is known as the **spring equation** and also as **Hooke's law**, and is accurate for springs as long as x is not too great.

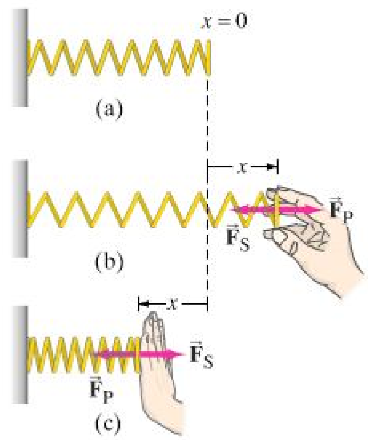


FIGURE 6–14 (a) Spring in natural (unstretched) position. (b) Spring is stretched by a person exerting a force $\vec{\mathbf{F}}_P$ to the right (positive direction). The spring pulls back with a force $\vec{\mathbf{F}}_S$, where $F_S = -kx$. (c) Person compresses the spring ($x < 0$) by exerting a force $\vec{\mathbf{F}}_P$ to the left; the spring pushes back with a force $F_S = -kx$, where $F_S > 0$ because $x < 0$.

To calculate the potential energy of a stretched spring, let us calculate the work required to stretch it (Fig. 6–14b). We might expect to use Eq. 6–1 for the work done on it, $W = Fx$, where x is the amount it is stretched from its natural length. But this would be incorrect since the force $F_P\,(= kx)$ is not constant but varies over this distance, becoming greater the more the spring is stretched, as shown graphically in Fig. 6–15. So let us use the average force, $\overline{F}$. Since F_P varies linearly—from zero at the unstretched position to kx when stretched to x—the average force is $\overline{F} = \frac{1}{2}[0 + kx] = \frac{1}{2}kx$, where x here is the final amount stretched (shown as x_f in Fig. 6–15 for clarity). The work done is then

$$W = \overline{F}x = \left(\tfrac{1}{2}kx\right)(x) = \tfrac{1}{2}kx^2.$$

Hence the **elastic potential energy** is proportional to the square of the amount stretched:

$$\text{elastic PE} = \tfrac{1}{2}kx^2. \quad \textbf{(6–9)}$$

Elastic PE

If a spring is *compressed* a distance x from its natural length, the average force is again $\overline{F} = \frac{1}{2}kx$, and again the potential energy is given by Eq. 6–9. Thus x can be either the amount compressed or amount stretched from the spring's natural length.[†] Note that for a spring, we choose the reference point for zero PE at the spring's natural position.

[†] We can also obtain Eq. 6–9 using Section 6–2. The work done, and hence ΔPE, equals the area under the F vs. x graph of Fig. 6–15. This area is a triangle (colored in Fig. 6–15) of altitude kx and base x, and hence of area (for a triangle) equal to $\frac{1}{2}(kx)(x) = \frac{1}{2}kx^2$.

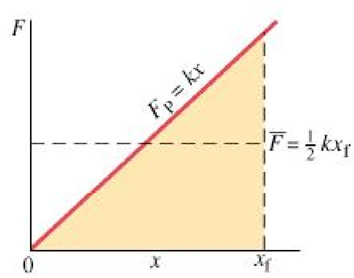


FIGURE 6–15 As a spring is stretched (or compressed), the force needed increases linearly as x increases: graph of $F = kx$ vs. x from $x = 0$ to $x = x_f$.

In each of the above examples of potential energy—from a brick held at a height y, to a stretched or compressed spring—an object has the capacity or *potential* to do work even though it is not yet actually doing it. These examples show that energy can be *stored*, for later use, in the form of potential energy (Fig. 6–13, for instance, for a spring).

Note that there is a single universal formula for the translational kinetic energy of an object, $\frac{1}{2}mv^2$, but there is no single formula for potential energy. Instead, the mathematical form of the potential energy depends on the force involved.

6–5 Conservative and Nonconservative Forces

The work done against gravity in moving an object from one point to another does not depend on the path taken. For example, it takes the same work ($= mgy$) to lift an object of mass m vertically a certain height as to carry it up an incline of the same vertical height, as in Fig. 6–4 (see Example 6–2). Forces such as gravity, for which the work done does not depend on the path taken but only on the initial and final positions, are called **conservative forces**. The elastic force of a spring (or other elastic material) in which $F = -kx$, is also a conservative force. An object that starts at a given point and returns to that same point under the action of a conservative force has no net work done on it because the potential energy is the same at the start and the finish of such a round trip.

Friction, on the other hand, is a **nonconservative force** since the work it does depends on the path. For example, when a crate is moved across a floor from one point to another, the work done depends on whether the path taken is straight, or is curved or zigzag. As shown in Fig. 6–16, if a crate is pushed from point 1 to point 2 along the longer semicircular path rather than along the straight path, more work is done against friction. That is because the distance is greater and, unlike the gravitational force, the friction force is always directed opposite to the direction of motion. (The $\cos\theta$ term in Eq. 6–1 is always $\cos 180° = -1$ at all points on the path for the friction force.) Thus the work done by friction in Fig. 6–16 does not depend *only* on points 1 and 2. Other forces that are nonconservative include the force exerted by a person and tension in a rope (see Table 6–1).

TABLE 6–1 Conservative and Nonconservative Forces

Conservative Forces	Nonconservative Forces
Gravitational	Friction
Elastic	Air resistance
Electric	Tension in cord
	Motor or rocket propulsion
	Push or pull by a person

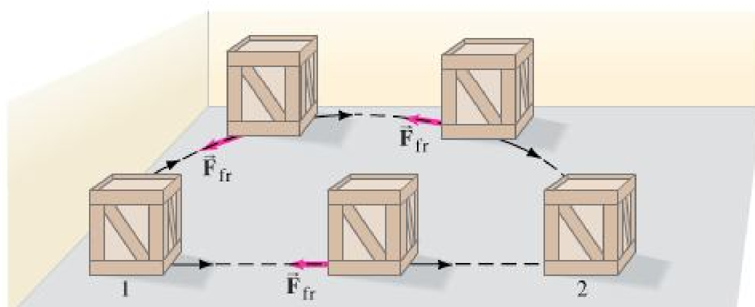


FIGURE 6–16 A crate is pushed across the floor from position 1 to position 2 via two paths, one straight and one curved. The friction force is always in the direction exactly opposed to the direction of motion. Hence, for a constant magnitude friction force, $W_{fr} = -F_{fr}d$, so if d is greater (as for the curved path), then W is greater. The work done does not depend only on points 1 and 2.

Because potential energy is energy associated with the position or configuration of objects, potential energy can only make sense if it can be stated uniquely for a given point. This cannot be done with nonconservative forces since the work done depends on the path taken (as in Fig. 6–16). Hence, *potential energy can be defined only for a conservative force.* Thus, although potential energy is always associated with a force, not all forces have a potential energy. For example, there is no potential energy for friction.

PE is defined only for a conservative force

There is no PE for friction

EXERCISE C An object acted on by a constant force F moves from point 1 to point 2 and back again. The work done by the force F in this round trip is 60 J. Can you determine from this information if F is a conservative or nonconservative force?

We can now extend the **work-energy principle** (discussed in Section 6–3) to include potential energy. Suppose several forces act on an object which can undergo translational motion. And suppose only some of these forces are conservative. We write the total (net) work W_{net} as a sum of the work done by conservative forces, W_C, and the work done by nonconservative forces, W_{NC}:

$$W_{net} = W_C + W_{NC}.$$

Then, from the work-energy principle, Eq. 6–4, we have

$$W_{net} = \Delta KE$$

$$W_C + W_{NC} = \Delta KE$$

where $\Delta KE = KE_2 - KE_1$. Then

$$W_{NC} = \Delta KE - W_C.$$

Work done by a conservative force can be written in terms of potential energy, as we saw in Eq. 6–7b for gravitational potential energy:

$$W_C = -\Delta PE.$$

We combine these last two equations:

$$W_{NC} = \Delta KE + \Delta PE. \quad \textbf{(6–10)}$$

WORK-ENERGY PRINCIPLE (general form)

Thus, *the work W_{NC} done by the nonconservative forces acting on an object is equal to the total change in kinetic and potential energies.*

It must be emphasized that *all* the forces acting on an object must be included in Eq. 6–10, either in the potential energy term on the right (if it is a conservative force), or in the work term on the left (but not in both!).

6–6 Mechanical Energy and Its Conservation

If only conservative forces are acting in a system, we arrive at a particularly simple and beautiful relation involving energy.

When no nonconservative forces are present, then $W_{NC} = 0$ in Eq. 6–10, the general form of the work-energy principle. Then we have

$$\Delta KE + \Delta PE = 0 \qquad \begin{bmatrix}\text{conservative}\\ \text{forces only}\end{bmatrix} \quad \textbf{(6–11a)}$$

or

$$(KE_2 - KE_1) + (PE_2 - PE_1) = 0. \qquad \begin{bmatrix}\text{conservative}\\ \text{forces only}\end{bmatrix} \quad \textbf{(6–11b)}$$

We now define a quantity E, called the **total mechanical energy** of our system, as the sum of the kinetic and potential energies at any moment:

$$E = KE + PE.$$

Total mechanical energy defined

Now we can rewrite Eq. 6–11b as

$$KE_2 + PE_2 = KE_1 + PE_1 \qquad \begin{bmatrix}\text{conservative}\\ \text{forces only}\end{bmatrix} \quad \textbf{(6–12a)}$$

CONSERVATION OF MECHANICAL ENERGY

or

$$E_2 = E_1 = \text{constant.} \qquad \begin{bmatrix}\text{conservative}\\ \text{forces only}\end{bmatrix} \quad \textbf{(6–12b)}$$

Equations 6–12 express a useful and profound principle regarding the total mechanical energy of a system—namely, that it is a **conserved quantity**. The total mechanical energy E remains constant as long as no nonconservative forces act: $(KE + PE)$ at some initial time 1 is equal to the $(KE + PE)$ at any later time 2.

To say it another way, consider Eq. 6–11a which tells us $\Delta PE = -\Delta KE$; that is, if the kinetic energy KE of a system increases, then the potential energy PE must decrease by an equivalent amount to compensate. Thus, the total, $KE + PE$, remains constant:

If only conservative forces are acting, the total mechanical energy of a system neither increases nor decreases in any process. It stays constant—it is conserved.

CONSERVATION OF MECHANICAL ENERGY

This is the **principle of conservation of mechanical energy** for conservative forces.

In the next Section we shall see the great usefulness of the conservation of mechanical energy principle in a variety of situations, and how it is often easier to use than the kinematic equations or Newton's laws. After that we will discuss how other forms of energy can be included in the general conservation of energy law that includes energy associated with nonconservative forces.

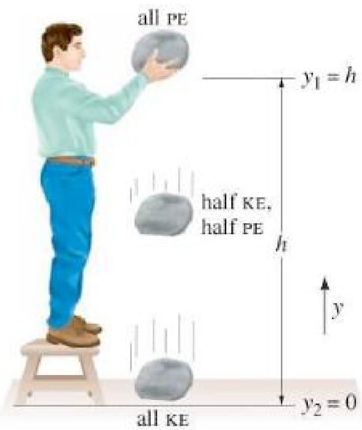


FIGURE 6–17 As it falls, the rock's potential energy changes to kinetic energy.

6–7 Problem Solving Using Conservation of Mechanical Energy

A simple example of the conservation of mechanical energy (neglecting air resistance) is a rock allowed to fall under gravity from a height h above the ground, as shown in Fig. 6–17. If the rock starts from rest, all of the initial energy is potential energy. As the rock falls, the potential energy decreases (because y decreases), but the rock's kinetic energy increases to compensate, so that the sum of the two remains constant. At any point along the path, the total mechanical energy is given by

$$E = \text{KE} + \text{PE} = \tfrac{1}{2}mv^2 + mgy$$

where y is the rock's height above the ground at a given instant and v is its speed at that point. If we let the subscript 1 represent the rock at one point along its path (for example, the initial point), and the subscript 2 represent it at some other point, then we can write

total mechanical energy at point 1 = total mechanical energy at point 2

or (see also Eq. 6–12a)

Conservation of mechanical energy when only gravity acts

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2. \qquad \text{[grav. PE only]} \quad \textbf{(6–13)}$$

Just before the rock hits the ground, where we chose $y = 0$, all of the initial potential energy will have been transformed into kinetic energy.

EXAMPLE 6–8 **Falling rock.** If the original height of the rock in Fig. 6–17 is $y_1 = h = 3.0\text{ m}$, calculate the rock's speed when it has fallen to 1.0 m above the ground.

APPROACH One approach is to use the kinematic equations of Chapter 2. Let us instead apply the principle of conservation of mechanical energy, Eq. 6–13, assuming that only gravity acts on the rock. We choose the ground as our reference level ($y = 0$).

SOLUTION At the moment of release (point 1) the rock's position is $y_1 = 3.0\text{ m}$ and it is at rest: $v_1 = 0$. We want to find v_2 when the rock is at position $y_2 = 1.0\text{ m}$. Equation 6–13 gives

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2.$$

The m's cancel out; setting $v_1 = 0$ and solving for v_2^2 we find

$$v_2^2 = 2g(y_1 - y_2)$$
$$= 2(9.8\text{ m/s}^2)[(3.0\text{ m}) - (1.0\text{ m})] = 39.2\text{ m}^2/\text{s}^2,$$

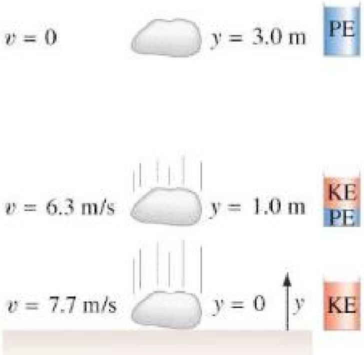
and

$$v_2 = \sqrt{39.2}\text{ m/s} = 6.3\text{ m/s}.$$

The rock's speed 1.0 m above the ground is 6.3 m/s downward.

NOTE The velocity at point 2 is independent of the rock's mass.

FIGURE 6–18 Energy buckets (for Example 6–8). Kinetic energy is red and potential energy is blue. The total (KE + PE) is the same for the three points shown. The speed at $y = 0$, just before the rock hits the ground, is $\sqrt{2(9.8\text{ m/s}^2)(3.0\text{ m})} = 7.7\text{ m/s}$.



EXERCISE D Solve Example 6–8 by using the work-energy principle applied to the rock, without the concept of potential energy. Show all equations you use, starting with Eq. 6–4.

A simple way to visualize energy conservation is with an "energy bucket" as shown in Fig. 6–18. At each point in the fall of the rock, for example, the amount of kinetic energy and potential energy are shown as if they were two differently colored materials in the bucket. The total amount of material in the bucket (= total mechanical energy) remains constant.

Equation 6–13 can be applied to any object moving without friction under the action of gravity. For example, Fig. 6–19 shows a roller-coaster car starting from rest at the top of a hill, and coasting without friction to the bottom and up the hill on the other side.[†] Initially, the car has only potential energy. As it coasts down the hill, it loses potential energy and gains in kinetic energy, but the sum of the two remains constant. At the bottom of the hill it has its maximum kinetic energy; as it climbs up the other side, the kinetic energy changes back to potential energy. When the car comes to rest again, all of its energy will be potential energy. Given that the potential energy is proportional to the vertical height, energy conservation tells us that (in the absence of friction) the car comes to rest at a height equal to its original height. If the two hills are the same height, the car will just barely reach the top of the second hill when it stops. If the second hill is lower than the first, not all of the car's kinetic energy will be transformed to potential energy and the car can continue over the top and down the other side. If instead the second hill is higher, the car will only reach a height on it equal to its original height on the first hill. This is true (in the absence of friction) no matter how steep the hill is, since potential energy depends only on the vertical height (Eq. 6–6).

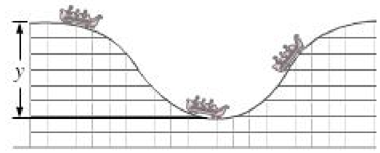


FIGURE 6–19 A roller-coaster car moving without friction illustrates the conservation of mechanical energy.

Grav. PE depends on vertical height, not path length (Eq. 6–6)

EXAMPLE 6–9 Roller-coaster speed using energy conservation. Assuming the height of the hill in Fig. 6–19 is 40 m, and the roller-coaster car starts from rest at the top, calculate (*a*) the speed of the roller-coaster car at the bottom of the hill, and (*b*) at what height it will have half this speed. Take $y = 0$ at the bottom of the hill.

APPROACH We choose point 1 to be where the car starts from rest $(v_1 = 0)$ at the top of the hill $(y_1 = 40\text{ m})$. Point 2 is the bottom of the hill, which we choose as our reference level, so $y_2 = 0$. We use conservation of mechanical energy.

SOLUTION (*a*) We use Eq. 6–13 with $v_1 = 0$ and $y_2 = 0$. Then

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2$$
$$mgy_1 = \tfrac{1}{2}mv_2^2.$$

The m's cancel out and, setting $y_1 = 40\text{ m}$, we find

$$v_2 = \sqrt{2gy_1} = \sqrt{2(9.8\text{ m/s}^2)(40\text{ m})} = 28\text{ m/s}.$$

(*b*) We again use conservation of energy,

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2,$$

but now $v_2 = 14\text{ m/s}$ (half of 28 m/s) and y_2 is unknown. We cancel the m's, set $v_1 = 0$, and solve for y_2:

$$y_2 = y_1 - \frac{v_2^2}{2g} = 30\text{ m}.$$

That is, the car has a speed of 14 m/s when it is 30 *vertical* meters above the lowest point, both when descending the left-hand hill and when ascending the right-hand hill.

NOTE The mathematics of this Example is almost the same as that in Example 6–8. But there is an important difference between them. Example 6–8 could have been solved using force, acceleration, and the kinematic equations (Eqs. 2–11). But here, where the motion is not vertical, that approach would have been too complicated, whereas energy conservation readily gives us the answer.

[†] The forces on the car are gravity, the normal force exerted by the track, and friction (here, assumed zero). The normal force acts perpendicular to the track, and so is always perpendicular to the motion and does no work. Thus $W_{NC} = 0$ in Eq. 6–10 (so mechanical energy is conserved) and we can use Eq. 6–13 with the potential energy being only gravitational potential energy. We will see how to deal with friction, for which $W_{NC} \neq 0$, in Section 6–9.

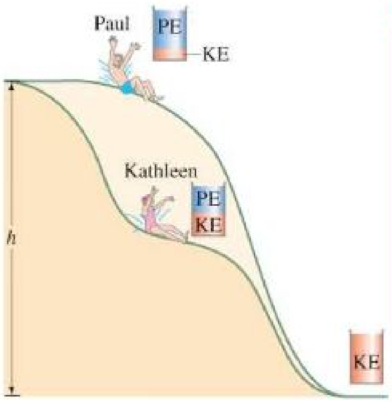


FIGURE 6–20 Example 6–10.

CONCEPTUAL EXAMPLE 6–10 **Speeds on two water slides.** Two water slides at a pool are shaped differently, but have the same length and start at the same height h (Fig. 6–20). Two riders, Paul and Kathleen, start from rest at the same time on different slides. (*a*) Which rider, Paul or Kathleen, is traveling faster at the bottom? (*b*) Which rider makes it to the bottom first? Ignore friction.

RESPONSE (*a*) Each rider's initial potential energy mgh gets transformed to kinetic energy, so the speed v at the bottom is obtained from $\frac{1}{2}mv^2 = mgh$. The mass cancels and so the speed will be the same, regardless of the mass of the rider. Since they descend the same vertical height, they will finish with the same speed.

(*b*) Note that Kathleen is consistently at a lower elevation than Paul at any instant, until the end. This means she has converted her potential energy to kinetic energy earlier. Consequently, she is traveling faster than Paul for the whole trip, except toward the end where Paul finally gets up to the same speed. Since she was going faster for the whole trip, and the distance is the same, Kathleen gets to the bottom first.

EXERCISE E Two balls are released from the same height above the floor. Ball A falls freely through the air, whereas ball B slides on a curved frictionless track to the floor. How do the speeds of the balls compare when they reach the floor?

PROBLEM SOLVING
Whether to use energy, or Newton's laws?

You may wonder sometimes whether to approach a problem using work and energy, or instead to use Newton's laws. As a rough guideline, if the force(s) involved are constant, either approach may succeed. If the forces are not constant, and/or the path is not simple, energy may be the surest approach.

PHYSICS APPLIED
Sports

There are many interesting examples of the conservation of energy in sports, such as the pole vault illustrated in Fig. 6–21. We often have to make approximations, but the sequence of events in broad outline for the pole vault is as follows. The initial kinetic energy of the running athlete is transformed into elastic potential energy of the bending pole and, as the athlete leaves the ground, into gravitational potential energy. When the vaulter reaches the top and the pole has straightened out again, the energy has all been transformed into gravitational potential energy (if we ignore the vaulter's low horizontal speed over the bar). The pole does not supply any energy, but it acts as a device to *store* energy and thus aid in the transformation of kinetic energy into gravitational potential energy, which is the net result. The energy required to pass over the bar depends on how high the center of mass (CM) of the vaulter must be raised. By bending their bodies, pole vaulters keep their CM so low that it can actually pass slightly beneath the bar (Fig. 6–22), thus enabling them to cross over a higher bar than would otherwise be possible. (Center of mass is covered in Chapter 7.)

FIGURE 6–21 Transformation of energy during a pole vault.

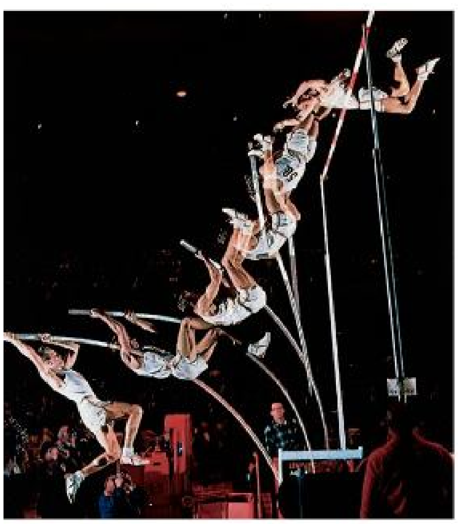


FIGURE 6–22 By bending their bodies, pole vaulters can keep their center of mass so low that it may even pass below the bar. By changing their kinetic energy (of running) into gravitational potential energy ($= mgy$) in this way, vaulters can cross over a higher bar than if the change in potential energy were accomplished without carefully bending the body.

As another example of the conservation of mechanical energy, let us consider an object of mass m connected to a horizontal spring whose own mass can be neglected and whose spring stiffness constant is k. The mass m has speed v at any moment. The potential energy of the system (object plus spring) is given by Eq. 6–9, PE $= \frac{1}{2}kx^2$, where x is the displacement of the spring from its unstretched length. If neither friction nor any other force is acting, conservation of mechanical energy tells us that

$$\tfrac{1}{2}mv_1^2 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2 + \tfrac{1}{2}kx_2^2, \qquad \text{[elastic PE only]} \quad \textbf{(6–14)}$$

Conservation of mechanical energy when PE is elastic

where the subscripts 1 and 2 refer to the velocity and displacement at two different moments.

EXAMPLE 6–11 **Toy dart gun.** A dart of mass 0.100 kg is pressed against the spring of a toy dart gun as shown in Fig. 6–23a. The spring (with spring stiffness constant $k = 250\ \mathrm{N/m}$) is compressed 6.0 cm and released. If the dart detaches from the spring when the spring reaches its natural length $(x = 0)$, what speed does the dart acquire?

APPROACH The dart is initially at rest (point 1), so $\mathrm{KE}_1 = 0$. We ignore friction and use conservation of mechanical energy; the only potential energy is elastic.

SOLUTION We use Eq. 6–14 with point 1 being at the maximum compression of the spring, so $v_1 = 0$ (dart not yet released) and $x_1 = -0.060\ \mathrm{m}$. Point 2 we choose to be the instant the dart flies off the end of the spring (Fig. 6–23b), so $x_2 = 0$ and we want to find v_2. Thus Eq. 6–14 can be written

$$0 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2 + 0.$$

Then

$$v_2^2 = \frac{kx_1^2}{m}$$

$$= \frac{(250\ \mathrm{N/m})(-0.060\ \mathrm{m})^2}{(0.100\ \mathrm{kg})} = 9.0\ \mathrm{m^2/s^2}$$

so $v_2 = \sqrt{v_2^2} = 3.0\ \mathrm{m/s}$.

NOTE In the horizontal direction, the only force on the dart (neglecting friction) was the force exerted by the spring. Vertically, gravity was counterbalanced by the normal force exerted on the dart by the gun barrel. After it leaves the barrel, the dart will follow a projectile's path under gravity.

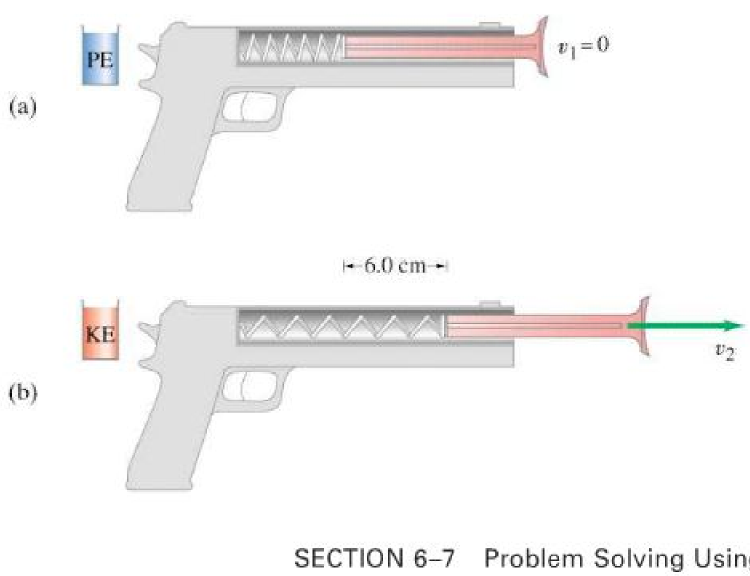


FIGURE 6–23 Example 6–11. (a) A dart is pushed against a spring, compressing it 6.0 cm. The dart is then released, and in (b) it leaves the spring at velocity v_2.

Additional Example

The next Example shows how to solve a problem involving two types of potential energy.

EXAMPLE 6–12 **Two kinds of PE.** A ball of mass $m = 2.60\,\mathrm{kg}$, starting from rest, falls a vertical distance $h = 55.0\,\mathrm{cm}$ before striking a vertical coiled spring, which it compresses an amount $Y = 15.0\,\mathrm{cm}$ (Fig. 6–24). Determine the spring stiffness constant of the spring. Assume the spring has negligible mass, and ignore air resistance. Measure all distances from the point where the ball first touches the uncompressed spring ($y = 0$ at this point).

APPROACH The forces acting on the ball are the gravitational pull of the Earth and the elastic force exerted by the spring. Both forces are conservative, so we can use conservation of mechanical energy, including both types of potential energy. We must be careful, however: gravity acts throughout the fall (Fig. 6–24), whereas the elastic force does not act until the ball touches the spring (Fig. 6–24b). We choose y positive upward, and $y = 0$ at the end of the spring in its natural (uncompressed) state.

SOLUTION We divide this solution into two parts. (An alternate solution follows.) *Part 1*: Let us first consider the energy changes as the ball falls from a height $y_1 = h = 0.55\,\mathrm{m}$, Fig. 6–24a, to $y_2 = 0$, just as it touches the spring, Fig. 6–24b. Our system is the ball acted on by gravity plus the spring, which up to this point doesn't do anything. Thus

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2$$
$$0 + mgh = \tfrac{1}{2}mv_2^2 + 0.$$

We solve for $v_2 = \sqrt{2gh} = \sqrt{2(9.80\,\mathrm{m/s^2})(0.550\,\mathrm{m})} = 3.283\,\mathrm{m/s} \approx 3.28\,\mathrm{m/s}$. This is the speed of the ball just as it touches the top of the spring, Fig. 6–24b. *Part 2*: Let's see what happens as the ball compresses the spring, Figs. 6–24b to c. Now there are two conservative forces on the ball—gravity and the spring force. So our conservation of energy equation becomes

Conservation of energy: gravity and elastic PE

$$E(\text{ball touches spring}) = E(\text{spring compresses})$$
$$\tfrac{1}{2}mv_2^2 + mgy_2 + \tfrac{1}{2}ky_2^2 = \tfrac{1}{2}mv_3^2 + mgy_3 + \tfrac{1}{2}ky_3^2.$$

We take point 2 to be the instant when the ball just touches the spring, so $y_2 = 0$ and $v_2 = 3.283\,\mathrm{m/s}$ (keeping an extra digit for now). We take point 3 to be when the ball comes to rest (for an instant) and the spring is fully compressed, so $v_3 = 0$ and $y_3 = -Y = -0.150\,\mathrm{m}$ (given). Substituting into the above energy equation, we get

$$\tfrac{1}{2}mv_2^2 + 0 + 0 = 0 - mgY + \tfrac{1}{2}kY^2.$$

We know m, v_2, and Y, so we can solve for k:

$$k = \frac{2}{Y^2}\left[\tfrac{1}{2}mv_2^2 + mgY\right] = \frac{m}{Y^2}\left[v_2^2 + 2gY\right]$$
$$= \frac{(2.60\,\mathrm{kg})}{(0.150\,\mathrm{m})^2}\left[(3.283\,\mathrm{m/s})^2 + 2(9.80\,\mathrm{m/s^2})(0.150\,\mathrm{m})\right] = 1590\,\mathrm{N/m},$$

which is the result we sought.

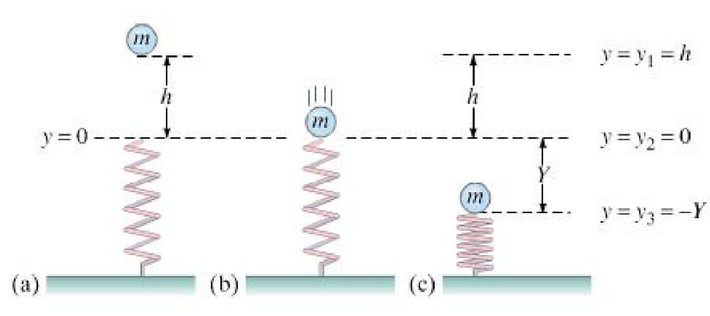


FIGURE 6–24 Example 6–12.

Alternate Solution Instead of dividing the solution into two parts, we can do it all at once. After all, we get to choose what two points are used on the left and right of the energy equation. Let us write the energy equation for points 1 and 3 (Fig. 6–24). Point 1 is the initial point just before the ball starts to fall (Fig. 6–24a), so $v_1 = 0$, $y_1 = h = 0.550\text{ m}$; and point 3 is when the spring is fully compressed (Fig. 6–24c), so $v_3 = 0$, $y_3 = -Y = -0.150\text{ m}$. The forces on the ball in this process are gravity and (at least part of the time) the spring. So conservation of energy tells us

$$\tfrac{1}{2}mv_1^2 + mgy_1 + \tfrac{1}{2}k(0)^2 = \tfrac{1}{2}mv_3^2 + mgy_3 + \tfrac{1}{2}ky_3^2$$
$$0 \quad + mgh + \quad 0 \quad = \quad 0 \quad - mgY + \tfrac{1}{2}kY^2$$

where we have set $y = 0$ for the spring at point 1 because it is not acting and is not compressed or stretched at point 1. We solve for k:

$$k = \frac{2\,mg(h + Y)}{Y^2} = \frac{2(2.60\text{ kg})(9.80\text{ m/s}^2)(0.550\text{ m} + 0.150\text{ m})}{(0.150\text{ m})^2} = 1590\text{ N/m}$$

just as in our first method of solution.

PROBLEM SOLVING
Alternate Solution

6–8 Other Forms of Energy; Energy Transformations and the Law of Conservation of Energy

Besides the kinetic energy and potential energy of ordinary objects, other forms of energy can be defined as well. These include electric energy, nuclear energy, thermal energy, and the chemical energy stored in food and fuels. With the advent of the atomic theory, these other forms of energy have come to be considered as kinetic or potential energy at the atomic or molecular level. For example, according to the atomic theory, thermal energy is the kinetic energy of rapidly moving molecules—when an object is heated, the molecules that make up the object move faster. On the other hand, the energy stored in food and fuel such as gasoline is potential energy stored by virtue of the relative positions of the atoms within a molecule due to electric forces between the atoms (referred to as chemical bonds). For the energy in chemical bonds to be used to do work, it must be released, usually through chemical reactions. This is analogous to a compressed spring which, when released, can do work. Electric, magnetic, and nuclear energies also can be considered examples of kinetic and potential (or stored) energies. We will deal with these other forms of energy in detail in later Chapters.

Energy can be transformed from one form to another, and we have already encountered several examples of this. A rock held high in the air has potential energy; as it falls, it loses potential energy, since its height above the ground decreases. At the same time, it gains in kinetic energy, since its velocity is increasing. Potential energy is being transformed into kinetic energy.

Often the transformation of energy involves a transfer of energy from one object to another. The potential energy stored in the spring of Fig. 6–13b is transformed into the kinetic energy of the ball, Fig. 6–13c. Water at the top of a dam has potential energy, which is transformed into kinetic energy as the water falls. At the base of the dam, the kinetic energy of the water can be transferred to turbine blades and further transformed into electric energy, as we shall see in a later Chapter. The potential energy stored in a bent bow can be transformed into kinetic energy of the arrow (Fig. 6–25).

FIGURE 6–25 Potential energy of a bent bow about to be transformed into kinetic energy of an arrow.

In each of these examples, the transfer of energy is accompanied by the performance of work. The spring of Fig. 6–13 does work on the ball. Water does work on turbine blades. A bow does work on an arrow. This observation gives us a further insight into the relation between work and energy: *work is done when energy is transferred from one object to another.*[†] A person throwing a ball or pushing a grocery cart provides another example. The work done is a manifestation of energy being transferred from the person (ultimately derived from the chemical energy of food) to the ball or cart.

Work is done when energy is transferred from one object to another

[†] If the objects are at different temperatures, heat can flow between them instead, or in addition. See Chapters 14 and 15.

One of the great results of physics is that whenever energy is transferred or transformed, it is found that no energy is gained or lost in the process.

This is the **law of conservation of energy**, one of the most important principles in physics; it can be stated as:

LAW OF CONSERVATION OF ENERGY

The total energy is neither increased nor decreased in any process. Energy can be transformed from one form to another, and transferred from one object to another, but the total amount remains constant.

We have already discussed the conservation of energy for mechanical systems involving conservative forces, and we saw how it could be derived from Newton's laws and thus is equivalent to them. But in its full generality, the validity of the law of conservation of energy, encompassing all forms of energy including those associated with nonconservative forces like friction, rests on experimental observation. Even though Newton's laws are found to fail in the submicroscopic world of the atom, the law of conservation of energy has been found to hold in every experimental situation so far tested.

6–9 Energy Conservation with Dissipative Forces: Solving Problems

In our applications of energy conservation in Section 6–7, we neglected friction, a nonconservative force. But in many situations it cannot be ignored. In a real situation, the roller-coaster car in Fig. 6–19, for example, will not in fact reach the same height on the second hill as it had on the first hill because of friction. In this, and in other natural processes, the mechanical energy (sum of the kinetic and potential energies) does not remain constant but decreases. Because frictional forces reduce the mechanical energy (but *not* the total energy), they are called **dissipative forces**. Historically, the presence of dissipative forces hindered the formulation of a comprehensive conservation of energy law until well into the nineteenth century. It was only then that heat, which is always produced when there is friction (try rubbing your hands together), was interpreted in terms of energy. Quantitative studies by nineteenth-century scientists (discussed in Chapters 14 and 15) demonstrated that if heat is considered as a transfer of energy (thermal energy), then the total energy is conserved in any process. For example, if the roller-coaster car in Fig. 6–19 is subject to frictional forces, then the initial total energy of the car will be equal to the kinetic plus potential energy of the car at any subsequent point along its path plus the amount of thermal energy produced in the process. The thermal energy produced by a constant friction force F_{fr} is equal to the work done by friction. We now apply the general form of the work-energy principle, Eq. 6–10:

Dissipative forces

$$W_{\text{NC}} = \Delta\text{KE} + \Delta\text{PE}.$$

We can write $W_{\text{NC}} = -F_{\text{fr}}\,d$, where d is the distance over which the friction force acts. ($\vec{\mathbf{F}}$ and $\vec{\mathbf{d}}$ are in opposite directions, hence the minus sign.) Thus, with $\text{KE} = \frac{1}{2}mv^2$ and $\text{PE} = mgy$, we have

$$-F_{\text{fr}}\,d = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 + mgy_2 - mgy_1$$

or

Conservation of energy with gravity and friction

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 + F_{\text{fr}}\,d, \quad \begin{bmatrix}\text{gravity and}\\ \text{friction acting}\end{bmatrix} \quad \textbf{(6–15)}$$

where d is the distance along the path traveled by the object in going from point 1 to point 2. Equation 6–15 can be seen to be Eq. 6–13 modified to include friction. It can be interpreted in a simple way: the initial mechanical energy of the car (point 1) equals the (reduced) final mechanical energy of the car plus the energy transformed by friction into thermal energy.

When other forms of energy are involved, such as chemical or electrical energy, the total amount of energy is always found to be conserved. Hence the law of conservation of energy is believed to be universally valid.

Work-Energy versus Energy Conservation

The work-energy principle and the law of conservation of energy are basically equivalent. The difference between them is in how you use them, and in particular on your *choice of the system* under study. If you choose as your system one or more objects on which external forces do work, then you must use the work-energy principle: the work done by the external forces on your system equals the total change in energy of your chosen system.

On the other hand, if you choose a system on which no external forces do work, then you can apply conservation of energy to that system.

Consider, for example, a spring connected to a block on a frictionless table (Fig. 6–26). If you choose the block as your system, then the work done on the block by the spring equals the change in kinetic energy of the block: the work-energy principle. (Energy conservation does not apply to this system—the block's energy changes.) If instead you choose the block plus the spring as your system, no external forces do work (since the spring is part of the chosen system). To this system you can apply conservation of energy: if you compress the spring and then release it, the spring still exerts a force on the block, but the subsequent motion can be discussed in terms of kinetic energy $\left(\frac{1}{2}mv^2\right)$ plus potential energy $\left(\frac{1}{2}kx^2\right)$, whose total remains constant.

Conservation of energy applies to any system on which no work is done by external forces.

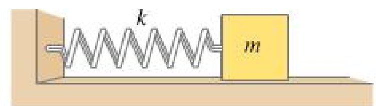


FIGURE 6–26 A spring connected to a block on a frictionless table. If you choose your system to be the block plus spring, then

$$E = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2$$

is conserved.

PROBLEM SOLVING Conservation of Energy

1. **Draw a picture** of the physical situation.
2. Determine **the system** for which energy will be conserved: the object or objects and the forces acting.
3. Ask yourself what quantity you are looking for, and decide what are **the initial** (point 1) **and final** (point 2) **positions**.
4. If the object under investigation changes its height during the problem, then **choose a reference frame** with a convenient $y = 0$ level for gravitational potential energy; the lowest point in the problem is often a good choice.

   If springs are involved, choose the unstretched spring position to be x (or y) $= 0$.
5. **Apply conservation of energy.** If no friction or other nonconservative forces act, then conservation of mechanical energy holds:
   $$\text{KE}_1 + \text{PE}_1 = \text{KE}_2 + \text{PE}_2.$$
   If friction or other nonconservative forces are present, then an additional term (W_{NC}) will be needed:
   $$W_{\text{NC}} = \Delta\text{KE} + \Delta\text{PE}.$$
   To be sure which sign to give W_{NC}, you can use your intuition: is the total mechanical energy increased or decreased in the process?
6. Use the equation(s) you develop to **solve** for the unknown quantity.

EXAMPLE 6–13 **Friction on the roller coaster.** The roller-coaster car in Example 6–9 reaches a vertical height of only 25 m on the second hill before coming to a momentary stop (Fig. 6–27). It traveled a total distance of 400 m. Estimate the average friction force (assume constant) on the car, whose mass is 1000 kg.

APPROACH We explicitly follow the Problem Solving Box step by step.

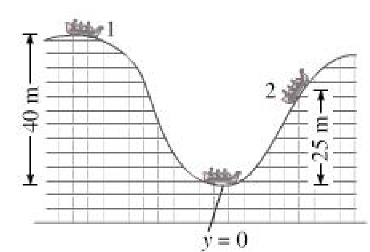
SOLUTION **1. Draw a Picture.** See Fig. 6–27.

2. **The system.** The system is the roller-coaster car (and the Earth since it exerts the gravitational force). The forces acting on the car are gravity and friction. (The normal force also acts on the car, but does no work, so it does not affect the energy.)
3. **Choose initial and final positions.** We take point 1 to be the instant when the car started coasting (at the top of the first hill), and point 2 to be the instant it stopped 25 m up the second hill.
4. **Choose a reference frame.** We choose the lowest point in the motion to be $y = 0$ for the gravitational potential energy.
5. **Apply conservation of energy.** There is friction acting on the car, so we use conservation of energy in the form of Eq. 6–15, with $v_1 = 0$, $y_1 = 40$ m, $v_2 = 0$, $y_2 = 25$ m, and $d = 400$ m. Thus

$$0 + (1000\ \text{kg})(9.8\ \text{m/s}^2)(40\ \text{m}) = 0 + (1000\ \text{kg})(9.8\ \text{m/s}^2)(25\ \text{m}) + F_{\text{fr}}(400\ \text{m}).$$

6. **Solve.** We can solve this equation for F_{fr}: $F_{\text{fr}} = 370$ N.

FIGURE 6–27 Example 6–13. Because of friction, a roller coaster car does not reach the original height on the second hill.



Problem solving is not a process that can be done by following a set of rules. The Problem Solving Box on page 157 is thus not a prescription, but is a *summary* of steps to help you get started in solving problems involving energy.

6–10 Power

Power defined

Power is defined as the *rate at which work is done*. Average power equals the work done divided by the time to do it. Power can also be defined as the *rate at which energy is transformed*. Thus

Average power

$$\overline{P} = \text{average power} = \frac{\text{work}}{\text{time}} = \frac{\text{energy transformed}}{\text{time}}. \qquad \textbf{(6–16)}$$

Power units: the watt

The horsepower

The power of a horse refers to how much work it can do per unit time. The power rating of an engine refers to how much chemical or electrical energy can be transformed into mechanical energy per unit time. In SI units, power is measured in joules per second, and this unit is given a special name, the **watt** (W): $1\text{ W} = 1\text{ J/s}$. We are most familiar with the watt for electrical devices: the rate at which an electric lightbulb or heater changes electric energy into light or thermal energy; but the watt is used for other types of energy transformations as well. In the British system, the unit of power is the foot-pound per second $(\text{ft}\cdot\text{lb/s})$. For practical purposes, a larger unit is often used, the **horsepower**. One horsepower† (hp) is defined as $550\text{ ft}\cdot\text{lb/s}$, which equals 746 W.

CAUTION

Distinguish between power and energy

To see the distinction between energy and power, consider the following example. A person is limited in the work he or she can do, not only by the total energy required, but also by how fast this energy is transformed: that is, by power. For example, a person may be able to walk a long distance or climb many flights of stairs before having to stop because so much energy has been expended. On the other hand, a person who runs very quickly upstairs may fall exhausted after only a flight or two. He or she is limited in this case by power, the rate at which his or her body can transform chemical energy into mechanical energy.

FIGURE 6–28 Example 6–14.

EXAMPLE 6–14 **Stair-climbing power.** A 60-kg jogger runs up a long flight of stairs in 4.0 s (Fig. 6–28). The vertical height of the stairs is 4.5 m. (*a*) Estimate the jogger's power output in watts and horsepower. (*b*) How much energy did this require?

APPROACH The work done by the jogger is against gravity, and equals $W = mgy$. To get her power output we divide W by the time it took.

SOLUTION (*a*) The average power output was

$$\overline{P} = \frac{W}{t} = \frac{mgy}{t} = \frac{(60\text{ kg})(9.8\text{ m/s}^2)(4.5\text{ m})}{4.0\text{ s}} = 660\text{ W}.$$

Since there are 746 W in 1 hp, the jogger is doing work at a rate of just under 1 hp. A human cannot do work at this rate for very long.

(*b*) The energy required is $E = \overline{P}t$ (Eq. 6–16). Since $\overline{P} = 660\text{ W} = 660\text{ J/s}$, then $E = (660\text{ J/s})(4.0\text{ s}) = 2600\text{ J}$. This result equals $W = mgy$.

NOTE The person had to transform more energy than this 2600 J. The total energy transformed by a person or an engine always includes some thermal energy (recall how hot you get running up stairs).

†The unit was chosen by James Watt (1736–1819), who needed a way to specify the power of his newly developed steam engines. He found by experiment that a good horse can work all day at an average rate of about $360\text{ ft}\cdot\text{lb/s}$. So as not to be accused of exaggeration in the sale of his steam engines, he multiplied this by $1\frac{1}{2}$ when he defined the hp.

Automobile engines do work to overcome the force of friction (including air resistance), to climb hills, and to accelerate. A car is limited by the rate at which it can do work, which is why automobile engines are rated in horsepower. A car needs power most when climbing hills and when accelerating. In the next Example, we will calculate how much power is needed in these situations for a car of reasonable size. Even when a car travels on a level road at constant speed, it needs some power just to do work to overcome the retarding forces of internal friction and air resistance. These forces depend on the conditions and speed of the car, but are typically in the range 400 N to 1000 N.

It is often convenient to write power in terms of the net force F applied to an object and its speed v. This is readily done since $\overline{P} = W/t$ and $W = Fd$, where d is the distance traveled. Then

$$\overline{P} = \frac{W}{t} = \frac{Fd}{t} = F\overline{v}, \tag{6–17}$$

where $\overline{v} = d/t$ is the average speed of the object.

EXAMPLE 6–15 **Power needs of a car.** Calculate the power required of a 1400-kg car under the following circumstances: (*a*) the car climbs a 10° hill (a fairly steep hill) at a steady 80 km/h; and (*b*) the car accelerates along a level road from 90 to 110 km/h in 6.0 s to pass another car. Assume the retarding force on the car is $F_R = 700\text{ N}$ throughout. See Fig. 6–29.

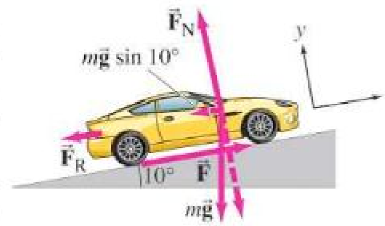


FIGURE 6–29 Example 6–15a. Calculation of power needed for a car to climb a hill.

APPROACH First we must be careful not to confuse $\vec{F}_R$, which is due to air resistance and friction that retards the motion, with the force $\vec{F}$ needed to accelerate the car, which is the frictional force exerted by the road on the tires—the reaction to the motor-driven tires pushing against the road. We must determine the latter force F before calculating the power.

SOLUTION (*a*) To move at a steady speed up the hill, the car must, by Newton's second law, exert a force F equal to the sum of the retarding force, 700 N, and the component of gravity parallel to the hill, $mg\sin 10°$. Thus

$$\begin{aligned} F &= 700\text{ N} + mg\sin 10° \\ &= 700\text{ N} + (1400\text{ kg})(9.80\text{ m/s}^2)(0.174) = 3100\text{ N}. \end{aligned}$$

Since $\overline{v} = 80\text{ km/h} = 22\text{ m/s}$ and is parallel to $\vec{F}$, then (Eq. 6–17) the power is

$$\overline{P} = F\overline{v} = (3100\text{ N})(22\text{ m/s}) = 6.80 \times 10^4\text{ W} = 91\text{ hp}.$$

(*b*) The car accelerates from 25.0 m/s to 30.6 m/s (90 to 110 km/h). Thus the car must exert a force that overcomes the 700-N retarding force plus that required to give it the acceleration

$$\overline{a}_x = \frac{(30.6\text{ m/s} - 25.0\text{ m/s})}{6.0\text{ s}} = 0.93\text{ m/s}^2.$$

We apply Newton's second law with x being the direction of motion:

$$ma_x = \Sigma F_x = F - F_R.$$

Then the force required, F, is

$$\begin{aligned} F &= ma_x + F_R \\ &= (1400\text{ kg})(0.93\text{ m/s}^2) + 700\text{ N} \\ &= 1300\text{ N} + 700\text{ N} = 2000\text{ N}. \end{aligned}$$

Since $\overline{P} = F\overline{v}$, the required power increases with speed and the motor must be able to provide a maximum power output of

$$\overline{P} = (2000\text{ N})(30.6\text{ m/s}) = 6.12 \times 10^4\text{ W} = 82\text{ hp}.$$

NOTE Even taking into account the fact that only 60 to 80% of the engine's power output reaches the wheels, it is clear from these calculations that an engine of 100 to 150 hp is quite adequate from a practical point of view.

We mentioned in Example 6–15 that only part of the energy output of a car engine reaches the wheels. Not only is some energy wasted in getting from the engine to the wheels, in the engine itself much of the input energy (from the gasoline) does not do useful work. An important characteristic of all engines is their overall *efficiency* e, defined as the ratio of the useful power output of the engine, P_{out}, to the power input, P_{in}:

Efficiency

$$e = \frac{P_{out}}{P_{in}}.$$

The efficiency is always less than 1.0 because no engine can create energy, and no engine can even transform energy from one form to another without some energy going to friction, thermal energy, and other nonuseful forms of energy. For example, an automobile engine converts chemical energy released in the burning of gasoline into mechanical energy that moves the pistons and eventually the wheels. But nearly 85% of the input energy is "wasted" as thermal energy that goes into the cooling system or out the exhaust pipe, plus friction in the moving parts. Thus car engines are roughly only about 15% efficient. We will discuss efficiency in detail in Chapter 15.

Summary

Work is done on an object by a force when the object moves through a distance d. If the direction of a constant force F makes an angle θ with the direction of motion, the work done by this force is

$$W = Fd\cos\theta. \qquad \textbf{(6–1)}$$

Energy can be defined as the ability to do work. In SI units, work and energy are measured in **joules** ($1\ \text{J} = 1\ \text{N}\cdot\text{m}$).

Kinetic energy (KE) is energy of motion. An object of mass m and speed v has translational kinetic energy

$$\text{KE} = \tfrac{1}{2}mv^2. \qquad \textbf{(6–3)}$$

Potential energy (PE) is energy associated with forces that depend on the position or configuration of objects. Gravitational potential energy is

$$\text{PE}_{\text{grav}} = mgy, \qquad \textbf{(6–6)}$$

where y is the height of the object of mass m above an arbitrary reference point. Elastic potential energy is given by

$$\text{elastic PE} = \tfrac{1}{2}kx^2 \qquad \textbf{(6–9)}$$

for a stretched or compressed spring, where x is the displacement from the unstretched position and k is the spring stiffness constant. Other potential energies include chemical, electrical, and nuclear energy. The change in potential energy when an object changes position is equal to the external work needed to take the object from one position to the other.

The **work-energy principle** states that the *net* work done on an object (by the *net* force) equals the change in kinetic energy of that object:

$$W_{\text{net}} = \Delta\text{KE} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2. \qquad \textbf{(6–2, 6–4)}$$

The **law of conservation of energy** states that energy can be transformed from one type to another, but the total energy remains constant. It is valid even when friction is present, since the heat generated can be considered a form of energy transfer. When only *conservative forces* act, the total mechanical energy is conserved:

$$\text{KE} + \text{PE} = \text{constant}.$$

When nonconservative forces such as friction act, then

$$W_{\text{NC}} = \Delta\text{KE} + \Delta\text{PE}, \qquad \textbf{(6–10)}$$

where W_{NC} is the work done by nonconservative forces.

Power is defined as the rate at which work is done, or the rate at which energy is transformed. The SI unit of power is the **watt** ($1\ \text{W} = 1\ \text{J/s}$).

Questions

1. In what ways is the word "work" as used in everyday language the same as that defined in physics? In what ways is it different? Give examples of both.
2. Can a centripetal force ever do work on an object? Explain.
3. Can the normal force on an object ever do work? Explain.
4. A woman swimming upstream is not moving with respect to the shore. Is she doing any work? If she stops swimming and merely floats, is work done on her?
5. Is the work done by kinetic friction forces always negative? [*Hint*: Consider what happens to the dishes when you pull a tablecloth out from under them.]
6. Why is it tiring to push hard against a solid wall even though you are doing no work?
7. You have two springs that are identical except that spring 1 is stiffer than spring 2 ($k_1 > k_2$). On which spring is more work done (a) if they are stretched using the same force, (b) if they are stretched the same distance?

8. A hand exerts a constant horizontal force on a block that is free to slide on a frictionless surface (Fig. 6–30). The block starts from rest at point A, and by the time it has traveled a distance d to point B it is traveling with speed v_B. When the block has traveled another distance d to point C, will its speed be greater than, less than, or equal to $2v_B$? Explain your reasoning.

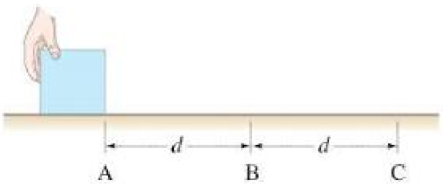


FIGURE 6–30 Question 8.

9. By approximately how much does your gravitational potential energy change when you jump as high as you can?

10. In Fig. 6–31, water balloons are tossed from the roof of a building, all with the same speed but with different launch angles. Which one has the highest speed on impact? Ignore air resistance.

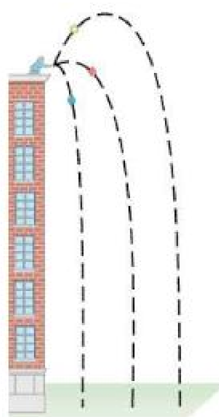


FIGURE 6–31 Question 10.

11. A pendulum is launched from a point that is a height h above its lowest point in two different ways (Fig. 6–32). During both launches, the pendulum is given an initial speed of 3.0 m/s. On the first launch, the initial velocity of the pendulum is directed upward along the trajectory, and on the second launch it is directed downward along the trajectory. Which launch will cause it to swing the largest angle from the equilibrium position? Explain.

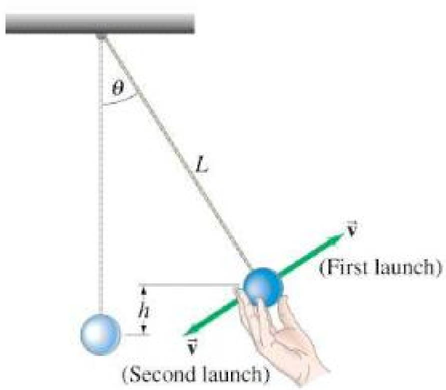


FIGURE 6–32 Question 11.

12. A coil spring of mass m rests upright on a table. If you compress the spring by pressing down with your hand and then release it, can the spring leave the table? Explain, using the law of conservation of energy.

13. A bowling ball is hung from the ceiling by a steel wire (Fig. 6–33). The instructor pulls the ball back and stands against the wall with the ball against his nose. To avoid injury the instructor is supposed to release the ball without pushing it. Why?

FIGURE 6–33 Question 13.

14. What happens to the gravitational potential energy when water at the top of a waterfall falls to the pool below?

15. Describe the energy transformations when a child hops around on a pogo stick.

16. Describe the energy transformations that take place when a skier starts skiing down a hill, but after a time is brought to rest by striking a snowdrift.

17. A child on a sled (total mass m) starts from rest at the top of a hill of height h and slides down. Does the velocity at the bottom depend on the angle of the hill if (a) it is icy and there is no friction, and (b) there is friction (deep snow)?

18. Seasoned hikers prefer to step over a fallen log in their path rather than stepping on top and jumping down on the other side. Explain.

19. Two identical arrows, one with twice the speed of the other, are fired into a bale of hay. Assuming the hay exerts a constant frictional force on the arrows, the faster arrow will penetrate how much farther than the slower arrow? Explain.

20. Analyze the motion of a simple swinging pendulum in terms of energy, (a) ignoring friction, and (b) taking friction into account. Explain why a grandfather clock has to be wound up.

21. When a "superball" is dropped, can it rebound to a height greater than its original height? Explain.

22. Suppose you lift a suitcase from the floor to a table. The work you do on the suitcase depends on which of the following: (a) whether you lift it straight up or along a more complicated path, (b) the time it takes, (c) the height of the table, and (d) the weight of the suitcase?

23. Repeat Question 22 for the *power* needed rather than the work.

24. Why is it easier to climb a mountain via a zigzag trail than to climb straight up?

25. Recall from Chapter 4, Example 4–14, that you can use a pulley and ropes to decrease the force needed to raise a heavy load (see Fig. 6–34). But for every meter the load is raised, how much rope must be pulled up? Account for this, using energy concepts.

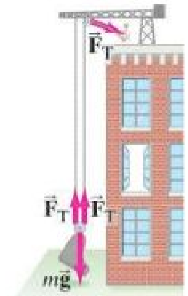


FIGURE 6–34 Question 25.

Problems

6–1 Work, Constant Force

1. (I) How much work is done by the gravitational force when a 265-kg pile driver falls 2.80 m?
2. (I) A 65.0-kg firefighter climbs a flight of stairs 20.0 m high. How much work is required?
3. (I) A 1300-N crate rests on the floor. How much work is required to move it at constant speed (*a*) 4.0 m along the floor against a friction force of 230 N, and (*b*) 4.0 m vertically?
4. (I) How much work did the movers do (horizontally) pushing a 160-kg crate 10.3 m across a rough floor without acceleration, if the effective coefficient of friction was 0.50?
5. (II) A box of mass 5.0 kg is accelerated from rest across a floor at a rate of $2.0\ \text{m/s}^2$ for 7.0 s. Find the net work done on the box.
6. (II) Eight books, each 4.3 cm thick with mass 1.7 kg, lie flat on a table. How much work is required to stack them one on top of another?
7. (II) A lever such as that shown in Fig. 6–35 can be used to lift objects we might not otherwise be able to lift. Show that the ratio of output force, F_O, to input force, F_I, is related to the lengths l_I and l_O from the pivot point by $F_O/F_I = l_I/l_O$ (ignoring friction and the mass of the lever), given that the work output equals work input.

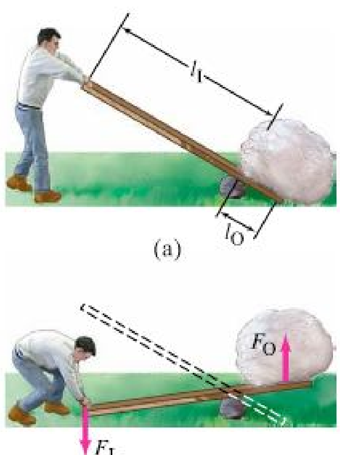


FIGURE 6–35 Problem 7. A simple lever.

8. (II) A 330-kg piano slides 3.6 m down a 28° incline and is kept from accelerating by a man who is pushing back on it *parallel to the incline* (Fig. 6–36). The effective coefficient of kinetic friction is 0.40. Calculate: (*a*) the force exerted by the man, (*b*) the work done by the man on the piano, (*c*) the work done by the friction force, (*d*) the work done by the force of gravity, and (*e*) the net work done on the piano.

FIGURE 6–36 Problem 8.

9. (II) (*a*) Find the force required to give a helicopter of mass M an acceleration of $0.10\,g$ upward. (*b*) Find the work done by this force as the helicopter moves a distance h upward.
10. (II) What is the minimum work needed to push a 950-kg car 810 m up along a 9.0° incline? (*a*) Ignore friction. (*b*) Assume the effective coefficient of friction retarding the car is 0.25.

* 6–2 Work, Varying Force

* 11. (II) In Fig. 6–6a, assume the distance axis is linear and that $d_A = 10.0$ m and $d_B = 35.0$ m. Estimate the work done by force F in moving a 2.80-kg object from d_A to d_B.

* 12. (II) The force on an object, acting along the x axis, varies as shown in Fig. 6–37. Determine the work done by this force to move the object (*a*) from $x = 0.0$ to $x = 10.0$ m, and (*b*) from $x = 0.0$ to $x = 15.0$ m.

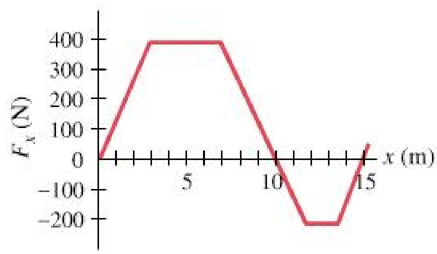


FIGURE 6–37 Problem 12.

* 13. (II) A spring has $k = 88$ N/m. Use a graph to determine the work needed to stretch it from $x = 3.8$ cm to $x = 5.8$ cm, where x is the displacement from its unstretched length.

* 14. (II) The net force exerted on a particle acts in the $+x$ direction. Its magnitude increases linearly from zero at $x = 0$, to 24.0 N at $x = 3.0$ m. It remains constant at 24.0 N from $x = 3.0$ m to $x = 8.0$ m, and then decreases linearly to zero at $x = 13.0$ m. Determine the work done to move the particle from $x = 0$ to $x = 13.0$ m graphically by determining the area under the F_x vs. x graph.

6–3 Kinetic Energy; Work-Energy Principle

15. (I) At room temperature, an oxygen molecule, with mass of 5.31×10^{-26} kg, typically has a KE of about 6.21×10^{-21} J. How fast is the molecule moving?
16. (I) (*a*) If the KE of an arrow is doubled, by what factor has its speed increased? (*b*) If its speed is doubled, by what factor does its KE increase?
17. (I) How much work is required to stop an electron $(m = 9.11 \times 10^{-31}\ \text{kg})$ which is moving with a speed of 1.90×10^6 m/s?
18. (I) How much work must be done to stop a 1250-kg car traveling at 105 km/h?
19. (II) An 88-g arrow is fired from a bow whose string exerts an average force of 110 N on the arrow over a distance of 78 cm. What is the speed of the arrow as it leaves the bow?
20. (II) A baseball $(m = 140\ \text{g})$ traveling 32 m/s moves a fielder's glove backward 25 cm when the ball is caught. What was the average force exerted by the ball on the glove?
21. (II) If the speed of a car is increased by 50%, by what factor will its minimum braking distance be increased, assuming all else is the same? Ignore the driver's reaction time.

22. (II) At an accident scene on a level road, investigators measure a car's skid mark to be 88 m long. The accident occurred on a rainy day, and the coefficient of kinetic friction was estimated to be 0.42. Use these data to determine the speed of the car when the driver slammed on (and locked) the brakes. (Why does the car's mass not matter?)

23. (II) A softball having a mass of 0.25 kg is pitched at 95 km/h. By the time it reaches the plate, it may have slowed by 10%. Neglecting gravity, estimate the average force of air resistance during a pitch, if the distance between the plate and the pitcher is about 15 m.

24. (II) How high will a 1.85-kg rock go if thrown straight up by someone who does 80.0 J of work on it? Neglect air resistance.

25. (III) A 285-kg load is lifted 22.0 m vertically with an acceleration $a = 0.160\,g$ by a single cable. Determine (*a*) the tension in the cable, (*b*) the net work done on the load, (*c*) the work done by the cable on the load, (*d*) the work done by gravity on the load, and (*e*) the final speed of the load assuming it started from rest.

6–4 and 6–5 Potential Energy

26. (I) A spring has a spring stiffness constant, k, of 440 N/m. How much must this spring be stretched to store 25 J of potential energy?

27. (I) A 7.0-kg monkey swings from one branch to another 1.2 m higher. What is the change in potential energy?

28. (I) By how much does the gravitational potential energy of a 64-kg pole vaulter change if his center of mass rises about 4.0 m during the jump?

29. (II) A 1200-kg car rolling on a horizontal surface has speed $v = 65$ km/h when it strikes a horizontal coiled spring and is brought to rest in a distance of 2.2 m. What is the spring stiffness constant of the spring?

30. (II) A 1.60-m tall person lifts a 2.10-kg book from the ground so it is 2.20 m above the ground. What is the potential energy of the book relative to (*a*) the ground, and (*b*) the top of the person's head? (*c*) How is the work done by the person related to the answers in parts (*a*) and (*b*)?

31. (II) A 55-kg hiker starts at an elevation of 1600 m and climbs to the top of a 3300-m peak. (*a*) What is the hiker's change in potential energy? (*b*) What is the minimum work required of the hiker? (*c*) Can the actual work done be more than this? Explain why.

32. (II) A spring with $k = 53$ N/m hangs vertically next to a ruler. The end of the spring is next to the 15-cm mark on the ruler. If a 2.5-kg mass is now attached to the end of the spring, where will the end of the spring line up with the ruler marks?

6–6 and 6–7 Conservation of Mechanical Energy

33. (I) Jane, looking for Tarzan, is running at top speed (5.3 m/s) and grabs a vine hanging vertically from a tall tree in the jungle. How high can she swing upward? Does the length of the vine affect your answer?

34. (I) A novice skier, starting from rest, slides down a frictionless 35.0° incline whose vertical height is 185 m. How fast is she going when she reaches the bottom?

35. (I) A sled is initially given a shove up a frictionless 28.0° incline. It reaches a maximum vertical height 1.35 m higher than where it started. What was its initial speed?

36. (II) In the high jump, Fran's kinetic energy is transformed into gravitational potential energy without the aid of a pole. With what minimum speed must Fran leave the ground in order to lift her center of mass 2.10 m and cross the bar with a speed of 0.70 m/s?

37. (II) A 65-kg trampoline artist jumps vertically upward from the top of a platform with a speed of 5.0 m/s. (*a*) How fast is he going as he lands on the trampoline, 3.0 m below (Fig. 6–38)? (*b*) If the trampoline behaves like a spring with spring stiffness constant 6.2×10^4 N/m, how far does he depress it?

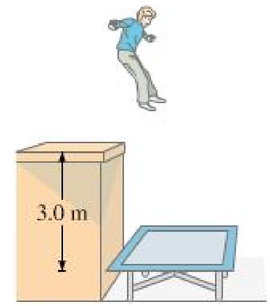


FIGURE 6–38 Problem 37.

38. (II) A projectile is fired at an upward angle of 45.0° from the top of a 265-m cliff with a speed of 185 m/s. What will be its speed when it strikes the ground below? (Use conservation of energy.)

39. (II) A vertical spring (ignore its mass), whose spring stiffness constant is 950 N/m, is attached to a table and is compressed down 0.150 m. (*a*) What upward speed can it give to a 0.30-kg ball when released? (*b*) How high above its original position (spring compressed) will the ball fly?

40. (II) A block of mass m slides without friction along the looped track shown in Fig. 6–39. If the block is to remain on the track, even at the top of the circle (whose radius is r), from what minimum height h must it be released?

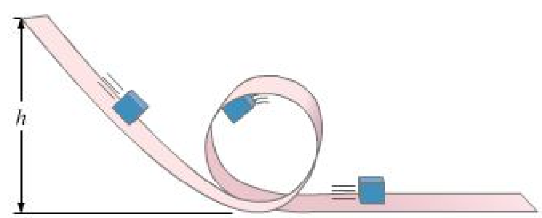


FIGURE 6–39 Problems 40 and 75.

41. (II) A block of mass m is attached to the end of a spring (spring stiffness constant k), Fig. 6–40. The block is given an initial displacement x_0, after which it oscillates back and forth. Write a formula for the total mechanical energy (ignore friction and the mass of the spring) in terms of x_0, position x, and speed v.

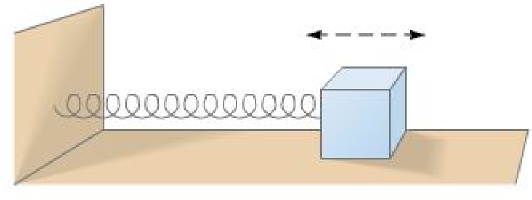


FIGURE 6–40 Problems 41, 55, and 56.

42. (II) A 62-kg bungee jumper jumps from a bridge. She is tied to a bungee cord whose unstretched length is 12 m, and falls a total of 31 m. (*a*) Calculate the spring stiffness constant k of the bungee cord, assuming Hooke's law applies. (*b*) Calculate the maximum acceleration she experiences.

43. (II) The roller-coaster car shown in Fig. 6–41 is dragged up to point 1 where it is released from rest. Assuming no friction, calculate the speed at points 2, 3, and 4.

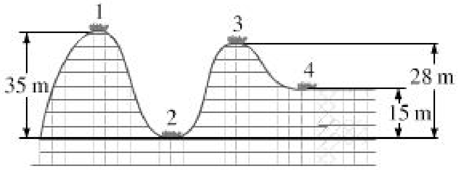


FIGURE 6–41 Problems 43 and 53.

44. (II) A 0.40-kg ball is thrown with a speed of 12 m/s at an angle of 33°. (*a*) What is its speed at its highest point, and (*b*) how high does it go? (Use conservation of energy, and ignore air resistance.)

45. (III) An engineer is designing a spring to be placed at the bottom of an elevator shaft. If the elevator cable should break when the elevator is at a height h above the top of the spring, calculate the value that the spring stiffness constant k should have so that passengers undergo an acceleration of no more than 5.0 g when brought to rest. Let M be the total mass of the elevator and passengers.

46. (III) A cyclist intends to cycle up a 7.8° hill whose vertical height is 150 m. Assuming the mass of bicycle plus cyclist is 75 kg, (*a*) calculate how much work must be done against gravity. (*b*) If each complete revolution of the pedals moves the bike 5.1 m along its path, calculate the average force that must be exerted on the pedals tangent to their circular path. Neglect work done by friction and other losses. The pedals turn in a circle of diameter 36 cm.

6–8 and 6–9 Law of Conservation of Energy

47. (I) Two railroad cars, each of mass 7650 kg and traveling 95 km/h in opposite directions, collide head-on and come to rest. How much thermal energy is produced in this collision?

48. (II) A 21.7-kg child descends a slide 3.5 m high and reaches the bottom with a speed of 2.2 m/s. How much thermal energy due to friction was generated in this process?

49. (II) A ski starts from rest and slides down a 22° incline 75 m long. (*a*) If the coefficient of friction is 0.090, what is the ski's speed at the base of the incline? (*b*) If the snow is level at the foot of the incline and has the same coefficient of friction, how far will the ski travel along the level? Use energy methods.

50. (II) A 145-g baseball is dropped from a tree 13.0 m above the ground. (*a*) With what speed would it hit the ground if air resistance could be ignored? (*b*) If it actually hits the ground with a speed of 8.00 m/s, what is the average force of air resistance exerted on it?

51. (II) You drop a ball from a height of 2.0 m, and it bounces back to a height of 1.5 m. (*a*) What fraction of its initial energy is lost during the bounce? (*b*) What is the ball's speed just as it leaves the ground after the bounce? (*c*) Where did the energy go?

52. (II) A 110-kg crate, starting from rest, is pulled across a floor with a constant horizontal force of 350 N. For the first 15 m the floor is frictionless, and for the next 15 m the coefficient of friction is 0.30. What is the final speed of the crate?

53. (II) Suppose the roller coaster in Fig. 6–41 passes point 1 with a speed of 1.70 m/s. If the average force of friction is equal to one-fifth of its weight, with what speed will it reach point 2? The distance traveled is 45.0 m.

54. (II) A skier traveling 12.0 m/s reaches the foot of a steady upward 18.0° incline and glides 12.2 m up along this slope before coming to rest. What was the average coefficient of friction?

55. (III) A 0.620-kg wood block is firmly attached to a very light horizontal spring ($k = 180$ N/m) as shown in Fig. 6–40. It is noted that the block–spring system, when compressed 5.0 cm and released, stretches out 2.3 cm beyond the equilibrium position before stopping and turning back. What is the coefficient of kinetic friction between the block and the table?

56. (III) A 280-g wood block is firmly attached to a very light horizontal spring, Fig. 6–40. The block can slide along a table where the coefficient of friction is 0.30. A force of 22 N compresses the spring 18 cm. If the spring is released from this position, how far beyond its equilibrium position will it stretch at its first maximum extension?

57. (III) Early test flights for the space shuttle used a "glider" (mass of 980 kg including pilot) that was launched horizontally at 500 km/h from a height of 3500 m. The glider eventually landed at a speed of 200 km/h. (*a*) What would its landing speed have been in the absence of air resistance? (*b*) What was the average force of air resistance exerted on it if it came in at a constant glide of 10° to the Earth?

6–10 Power

58. (I) How long will it take a 1750-W motor to lift a 315-kg piano to a sixth-story window 16.0 m above?

59. (I) If a car generates 18 hp when traveling at a steady 88 km/h, what must be the average force exerted on the car due to friction and air resistance?

60. (I) A 1400-kg sports car accelerates from rest to 95 km/h in 7.4 s. What is the average power delivered by the engine?

61. (I) (*a*) Show that one British horsepower (550 ft·lb/s) is equal to 746 W. (*b*) What is the horsepower rating of a 75-W lightbulb?

62. (II) Electric energy units are often expressed in the form of "kilowatt-hours." (*a*) Show that one kilowatt-hour (kWh) is equal to 3.6×10^6 J. (*b*) If a typical family of four uses electric energy at an average rate of 520 W, how many kWh would their electric bill be for one month, and (*c*) how many joules would this be? (*d*) At a cost of \$0.12 per kWh, what would their monthly bill be in dollars? Does the monthly bill depend on the *rate* at which they use the electric energy?

63. (II) A driver notices that her 1150-kg car slows down from 85 km/h to 65 km/h in about 6.0 s on the level when it is in neutral. Approximately what power (watts and hp) is needed to keep the car traveling at a constant 75 km/h?

64. (II) How much work can a 3.0-hp motor do in 1.0 h?

65. (II) A shot-putter accelerates a 7.3-kg shot from rest to 14 m/s. If this motion takes 1.5 s, what average power was developed?

66. (II) A pump is to lift 18.0 kg of water per minute through a height of 3.60 m. What output rating (watts) should the pump motor have?

67. (II) During a workout, the football players at State U. ran up the stadium stairs in 66 s. The stairs are 140 m long and inclined at an angle of 32°. If a typical player has a mass of 95 kg, estimate the average power output on the way up. Ignore friction and air resistance.

68. (II) How fast must a cyclist climb a 6.0° hill to maintain a power output of 0.25 hp? Neglect work done by friction, and assume the mass of cyclist plus bicycle is 68 kg.

69. (II) A 1200-kg car has a maximum power output of 120 hp. How steep a hill can it climb at a constant speed of 75 km/h if the frictional forces add up to 650 N?

70. (II) What minimum horsepower must a motor have to be able to drag a 310-kg box along a level floor at a speed of 1.20 m/s if the coefficient of friction is 0.45?

71. (III) A bicyclist coasts down a 7.0° hill at a steady speed of 5.0 m/s. Assuming a total mass of 75 kg (bicycle plus rider), what must be the cyclist's power output to climb the same hill at the same speed?

General Problems

72. Designers of today's cars have built "5 mi/h (8 km/h) bumpers" that are designed to compress and rebound elastically without any physical damage at speeds below 8 km/h. If the material of the bumpers permanently deforms after a compression of 1.5 cm, but remains like an elastic spring up to that point, what must the effective spring stiffness constant of the bumper be, assuming the car has a mass of 1300 kg and is tested by ramming into a solid wall?

73. In a certain library the first shelf is 10.0 cm off the ground, and the remaining four shelves are each spaced 30.0 cm above the previous one. If the average book has a mass of 1.5 kg with a height of 21 cm, and an average shelf holds 25 books, how much work is required to fill all the shelves, assuming the books are all laying flat on the floor to start?

74. A film of Jesse Owens's famous long jump (Fig. 6–42) in the 1936 Olympics shows that his center of mass rose 1.1 m from launch point to the top of the arc. What minimum speed did he need at launch if he was traveling at 6.5 m/s at the top of the arc?

FIGURE 6–42 Problem 74.

75. The block of mass m sliding without friction along the looped track shown in Fig. 6–39 is to remain on the track at all times, even at the very top of the loop of radius r. (a) In terms of the given quantities, determine the minimum release height h (as in Problem 40). Next, if the actual release height is $2h$, calculate (b) the normal force exerted by the track at the bottom of the loop, (c) the normal force exerted by the track at the top of the loop, and (d) the normal force exerted by the track after the block exits the loop onto the flat section.

76. An airplane pilot fell 370 m after jumping from an aircraft without his parachute opening. He landed in a snowbank, creating a crater 1.1 m deep, but survived with only minor injuries. Assuming the pilot's mass was 78 kg and his terminal velocity was 35 m/s, estimate (a) the work done by the snow in bringing him to rest; (b) the average force exerted on him by the snow to stop him; and (c) the work done on him by air resistance as he fell.

77. A ball is attached to a horizontal cord of length L whose other end is fixed (Fig. 6–43). (a) If the ball is released, what will be its speed at the lowest point of its path? (b) A peg is located a distance h directly below the point of attachment of the cord. If $h = 0.80L$, what will be the speed of the ball when it reaches the top of its circular path about the peg?

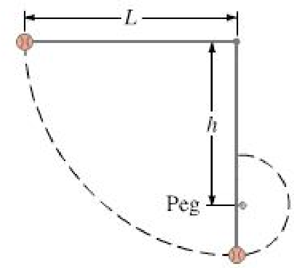


FIGURE 6–43 Problem 77.

78. A 65-kg hiker climbs to the top of a 3700-m-high mountain. The climb is made in 5.0 h starting at an elevation of 2300 m. Calculate (a) the work done by the hiker against gravity, (b) the average power output in watts and in horsepower, and (c) assuming the body is 15% efficient, what rate of energy input was required.

79. An elevator cable breaks when a 920-kg elevator is 28 m above a huge spring $(k = 2.2 \times 10^5\ \text{N/m})$ at the bottom of the shaft. Calculate (a) the work done by gravity on the elevator before it hits the spring, (b) the speed of the elevator just before striking the spring, and (c) the amount the spring compresses (note that work is done by both the spring and gravity in this part).

80. Squaw Valley ski area in California claims that its lifts can move 47,000 people per hour. If the average lift carries people about 200 m (vertically) higher, estimate the power needed.

81. Water flows ($v \approx 0$) over a dam at the rate of 650 kg/s and falls vertically 81 m before striking the turbine blades. Calculate (a) the speed of the water just before striking the turbine blades (neglect air resistance), and (b) the rate at which mechanical energy is transferred to the turbine blades, assuming 58% efficiency.

82. Show that on a roller coaster with a circular vertical loop (Fig. 6–44), the difference in your apparent weight at the top of the circular loop and the bottom of the circular loop is 6 g's—that is, six times your weight. Ignore friction. Show also that as long as your speed is above the minimum needed, this answer doesn't depend on the size of the loop or how fast you go through it.

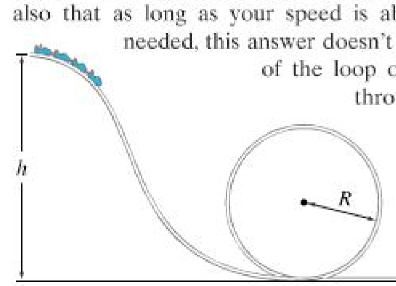


FIGURE 6–44 Problem 82.

83. (*a*) If the human body could convert a candy bar directly into work, how high could an 82-kg man climb a ladder if he were fueled by one bar (= 1100 kJ)? (*b*) If the man then jumped off the ladder, what will be his speed when he reaches the bottom?

84. A projectile is fired at an upward angle of 45.0° from the top of a 165-m cliff with a speed of 175 m/s. What will be its speed when it strikes the ground below? (Use conservation of energy and neglect air resistance.)

85. If you stand on a bathroom scale, the spring inside the scale compresses 0.60 mm, and it tells you your weight is 710 N. Now if you jump on the scale from a height of 1.0 m, what does the scale read at its peak?

86. A 65-kg student runs at 5.0 m/s, grabs a rope, and swings out over a lake (Fig. 6–45). He releases the rope when his velocity is zero. (*a*) What is the angle θ when he releases the rope? (*b*) What is the tension in the rope just before he releases it? (*c*) What is the maximum tension in the rope?

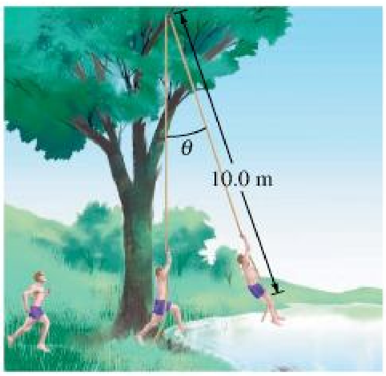


FIGURE 6–45 Problem 86.

87. In the rope climb, a 72-kg athlete climbs a vertical distance of 5.0 m in 9.0 s. What minimum power output was used to accomplish this feat?

88. Some electric-power companies use water to store energy. Water is pumped by reversible turbine pumps from a low to a high reservoir. To store the energy produced in 1.0 hour by a 120-MW $(120 \times 10^6\,\text{W})$ electric-power plant, how many cubic meters of water will have to be pumped from the lower to the upper reservoir? Assume the upper reservoir is 520 m above the lower and we can neglect the small change in depths within each. Water has a mass of 1000 kg for every $1.0\,\text{m}^3$.

89. A spring with spring stiffness constant k is cut in half. What is the spring stiffness constant for each of the two resulting springs?

90. A 6.0-kg block is pushed 8.0 m up a rough 37° inclined plane by a horizontal force of 75 N. If the initial speed of the block is 2.2 m/s up the plane and a constant kinetic friction force of 25 N opposes the motion, calculate (*a*) the initial kinetic energy of the block; (*b*) the work done by the 75-N force; (*c*) the work done by the friction force; (*d*) the work done by gravity; (*e*) the work done by the normal force; (*f*) the final kinetic energy of the block.

91. If a 1500-kg car can accelerate from 35 km/h to 55 km/h in 3.2 s, how long will it take to accelerate from 55 km/h to 75 km/h? Assume the power stays the same, and neglect frictional losses.

92. In a common test for cardiac function (the "stress test"), the patient walks on an inclined treadmill (Fig. 6–46). Estimate the power required from a 75-kg patient when the treadmill is sloping at an angle of 15° and the velocity is 3.3 km/h. (How does this power compare to the power rating of a lightbulb?)

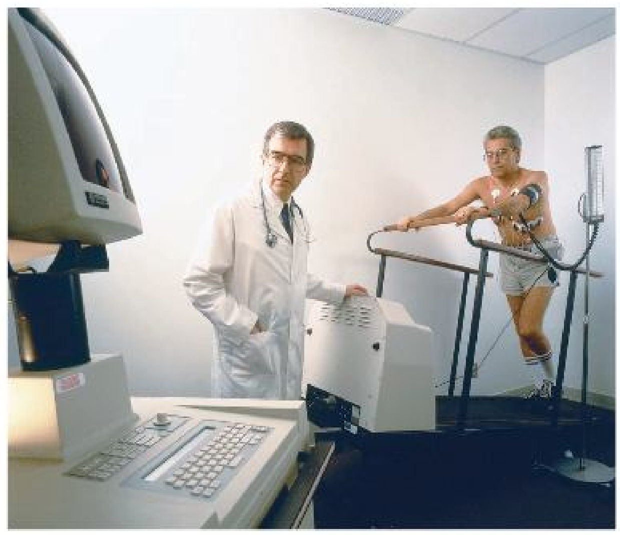


FIGURE 6–46 Problem 92.

93. (*a*) If a volcano spews a 500-kg rock vertically upward a distance of 500 m, what was its velocity when it left the volcano? (*b*) If the volcano spews the equivalent of 1000 rocks of this size every minute, what is its power output?

94. Water falls onto a water wheel from a height of 2.0 m at a rate of 95 kg/s. (*a*) If this water wheel is set up to provide electricity output, what is its maximum power output? (*b*) What is the speed of the water as it hits the wheel?

Answers to Exercises

A: (*c*).

B: No, because the speed v would be the square root of a negative number, which is not real.

C: It is nonconservative, because for a conservative force $W = 0$ in a round trip.

D: $W_{\text{net}} = \Delta\text{KE}$, where $W_{\text{net}} = mg(y_1 - y_2)$ and $\Delta\text{KE} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = \frac{1}{2}mv_2^2$. Then $v_2^2 = 2g(y_1 - y_2)$.

E: Equal speeds.