The astronauts in the upper left of this photo are working on the space shuttle. As they orbit the Earth—at a rather high speed—they experience apparent weightlessness. The Moon, in the background, also is orbiting the Earth at high speed. Both the Moon and the space shuttle move in nearly circular orbits, and each undergoes a centripetal acceleration. What keeps the Moon and the space shuttle (and its astronauts) from moving off in a straight line away from Earth? It is the force of gravity. Newton's law of universal gravitation states that all objects attract all other objects with a force proportional to their masses and inversely proportional to the square of the distance between them.

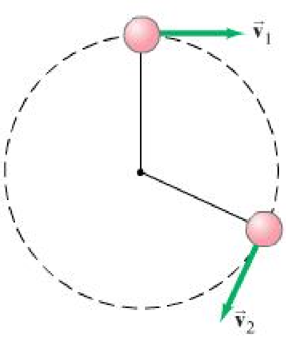
CHAPTER 5

Circular Motion; Gravitation

An object moves in a straight line if the net force on it acts in the direction of motion, or the net force is zero. If the net force acts at an angle to the direction of motion at any moment, then the object moves in a curved path. An example of the latter is projectile motion, which we discussed in Chapter 3. Another important case is that of an object moving in a circle, such as a ball at the end of a string revolving around one's head, or the nearly circular motion of the Moon about the Earth.

In this Chapter, we study the circular motion of objects, and how Newton's laws of motion apply. We also discuss how Newton conceived of another great law by applying the concepts of circular motion to the motion of the Moon and the planets. This is the law of universal gravitation, which was the capstone of Newton's analysis of the physical world.

FIGURE 5–1 A small object moving in a circle, showing how the velocity changes. At each point, the instantaneous velocity is in a direction tangent to the circular path.



5–1 Kinematics of Uniform Circular Motion

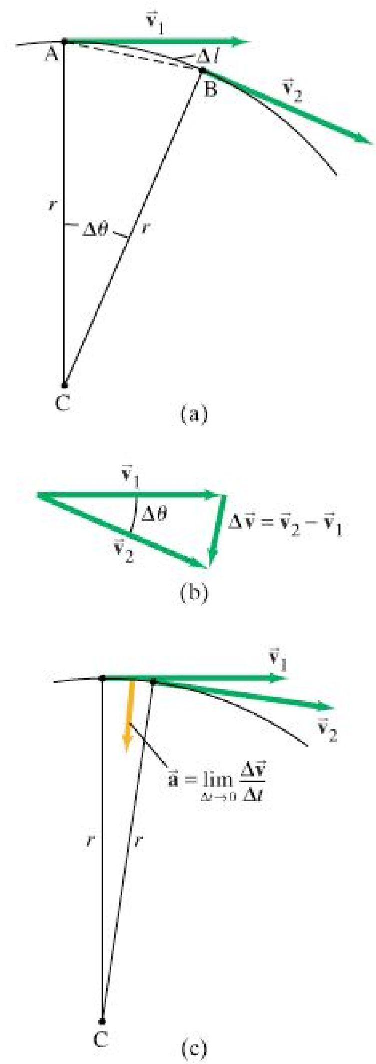
An object that moves in a circle at constant speed v is said to experience **uniform circular motion**. The *magnitude* of the velocity remains constant in this case, but the *direction* of the velocity continuously changes as the object moves around the circle (Fig. 5–1). Because acceleration is defined as the rate of change of velocity, a change in direction of velocity constitutes an acceleration, just as a change in magnitude of velocity does. Thus, an object revolving in a circle is continuously accelerating, even when the speed remains constant $(v_1 = v_2 = v)$. We now investigate this acceleration quantitatively.

Acceleration is defined as

$$\vec{\mathbf{a}} = \frac{\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1}{\Delta t} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t},$$

where $\Delta\vec{\mathbf{v}}$ is the change in velocity during the short time interval Δt. We will eventually consider the situation in which Δt approaches zero and thus obtain the instantaneous acceleration. But for purposes of making a clear drawing, Fig. 5–2, we consider a nonzero time interval. During the time interval Δt, the particle in Fig. 5–2a moves from point A to point B, covering a distance Δl *along the arc* which subtends an angle $\Delta\theta$. The change in the velocity vector is $\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1 = \Delta\vec{\mathbf{v}}$, and is shown in Fig. 5–2b.

FIGURE 5–2 Determining the change in velocity, $\Delta\vec{\mathbf{v}}$, for a particle moving in a circle. The length Δl is the distance along the arc, from A to B.



If we let Δt be very small (approaching zero), then Δl and $\Delta\theta$ are also very small, and $\vec{\mathbf{v}}_2$ will be almost parallel to $\vec{\mathbf{v}}_1$; $\Delta\vec{\mathbf{v}}$ will be essentially perpendicular to them (Fig. 5–2c). Thus $\Delta\vec{\mathbf{v}}$ points toward the center of the circle. Since $\vec{\mathbf{a}}$, by definition, is in the same direction as $\Delta\vec{\mathbf{v}}$, it too must point toward the center of the circle. Therefore, this acceleration is called **centripetal acceleration** ("center-pointing" acceleration) or **radial acceleration** (since it is directed along the radius, toward the center of the circle), and we denote it by $\vec{\mathbf{a}}_{\mathrm{R}}$.

We next determine the magnitude of the centripetal (radial) acceleration, a_{R}. Because CA in Fig. 5–2a is perpendicular to $\vec{\mathbf{v}}_1$, and CB is perpendicular to $\vec{\mathbf{v}}_2$, it follows that the angle $\Delta\theta$, defined as the angle between CA and CB, is also the angle between $\vec{\mathbf{v}}_1$ and $\vec{\mathbf{v}}_2$. Hence the vectors $\vec{\mathbf{v}}_1$, $\vec{\mathbf{v}}_2$, and $\Delta\vec{\mathbf{v}}$ in Fig. 5–2b form a triangle that is geometrically similar† to triangle CAB in Fig. 5–2a. If we take $\Delta\theta$ to be very small (letting Δt be very small) and setting $v = v_1 = v_2$ because the magnitude of the velocity is assumed not to change, we can write

$$\frac{\Delta v}{v} \approx \frac{\Delta l}{r}.$$

This is an exact equality when Δt approaches zero, for then the arc length Δl equals the cord length AB. We want to find the instantaneous acceleration, so we let Δt approach zero, write the above expression as an equality, and then solve for Δv:

$$\Delta v = \frac{v}{r}\,\Delta l.$$

To get the centripetal acceleration, a_{R}, we divide Δv by Δt:

$$a_{\mathrm{R}} = \frac{\Delta v}{\Delta t} = \frac{v}{r}\frac{\Delta l}{\Delta t}.$$

But $\Delta l/\Delta t$ is just the linear speed, v, of the object, so

$$a_{\mathrm{R}} = \frac{v^2}{r}. \qquad \textbf{(5–1)}$$

Centripetal (radial) acceleration

Equation 5–1 is valid even when v is not constant.

To summarize, *an object moving in a circle of radius r at constant speed v has an acceleration whose direction is toward the center of the circle and whose magnitude is* $a_{\mathrm{R}} = v^2/r$. It is not surprising that this acceleration depends on v and r. The greater the speed v, the faster the velocity changes direction; and the larger the radius, the less rapidly the velocity changes direction.

CAUTION

In uniform circular motion, the speed is constant, but the acceleration is not zero

† Appendix A contains a review of geometry.

The acceleration vector points toward the center of the circle. But the velocity vector always points in the direction of motion, which is tangential to the circle. Thus the velocity and acceleration vectors are perpendicular to each other at every point in the path for uniform circular motion (Fig. 5–3). This is another example that illustrates the error in thinking that acceleration and velocity are always in the same direction. For an object falling vertically, $\vec{\mathbf{a}}$ and $\vec{\mathbf{v}}$ are indeed parallel. But in circular motion, $\vec{\mathbf{a}}$ and $\vec{\mathbf{v}}$ are perpendicular, not parallel (nor were they parallel in projectile motion, Section 3–5).

CAUTION

The direction of motion ($\vec{\mathbf{v}}$) and the acceleration ($\vec{\mathbf{a}}$) are not in the same direction; instead, $\vec{\mathbf{a}} \perp \vec{\mathbf{v}}$

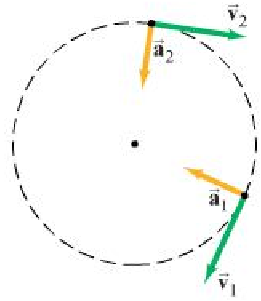


FIGURE 5–3 For uniform circular motion, $\vec{\mathbf{a}}$ is always perpendicular to $\vec{\mathbf{v}}$.

Period and frequency

Circular motion is often described in terms of the **frequency** f, the number of revolutions per second. The **period** T of an object revolving in a circle is the time required for one complete revolution. Period and frequency are related by

$$T = \frac{1}{f}. \tag{5–2}$$

For example, if an object revolves at a frequency of 3 rev/s, then each revolution takes $\frac{1}{3}$s. For an object revolving in a circle (of circumference $2\pi r$) at constant speed v, we can write

$$v = \frac{2\pi r}{T},$$

since in one revolution the object travels one circumference.

EXAMPLE 5–1 **Acceleration of a revolving ball.** A 150-g ball at the end of a string is revolving uniformly in a horizontal circle of radius 0.600 m, as in Fig. 5–1 or 5–3. The ball makes 2.00 revolutions in a second. What is its centripetal acceleration?

APPROACH The centripetal acceleration is $a_R = v^2/r$. We are given r, and we can find the speed of the ball, v, from the given radius and frequency.

SOLUTION If the ball makes two complete revolutions per second, then the ball travels in a complete circle in a time interval equal to 0.500 s, which is its period T. The distance traveled in this time is the circumference of the circle, $2\pi r$, where r is the radius of the circle. Therefore, the ball has speed

$$v = \frac{2\pi r}{T} = \frac{2(3.14)(0.600\,\text{m})}{(0.500\,\text{s})} = 7.54\,\text{m/s}.$$

The centripetal acceleration† is

$$a_R = \frac{v^2}{r} = \frac{(7.54\,\text{m/s})^2}{(0.600\,\text{m})} = 94.7\,\text{m/s}^2.$$

EXERCISE A If the string is doubled in length to 1.20 m but all else stays the same, by what factor will the centripetal acceleration change?

†Differences in the final digit can depend on whether you keep all digits in your calculator for v (which gives $a_R = 94.7\,\text{m/s}^2$), or if you use $v = 7.54\,\text{m/s}$ in which case you get $a_R = 94.8\,\text{m/s}^2$. Both results are valid since our assumed accuracy is about ± 0.1 m/s (see Section 1–4).

EXAMPLE 5–2 **Moon's centripetal acceleration.** The Moon's nearly circular orbit about the Earth has a radius of about 384,000 km and a period T of 27.3 days. Determine the acceleration of the Moon toward the Earth.

APPROACH Again we need to find the velocity v in order to find a_R. We will need to convert to SI units to get v in m/s.

SOLUTION In one orbit around the Earth, the Moon travels a distance $2\pi r$, where $r = 3.84 \times 10^8\,\text{m}$ is the radius of its circular path. The time required for one complete orbit is the Moon's period of 27.3 d. The speed of the Moon in its orbit about the Earth is $v = 2\pi r/T$. The period T in seconds is $T = (27.3\,\text{d})(24.0\,\text{h/d})(3600\,\text{s/h}) = 2.36 \times 10^6\,\text{s}$. Therefore,

$$a_R = \frac{v^2}{r} = \frac{(2\pi r)^2}{T^2 r} = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2(3.84 \times 10^8\,\text{m})}{(2.36 \times 10^6\,\text{s})^2}$$
$$= 0.00272\,\text{m/s}^2 = 2.72 \times 10^{-3}\,\text{m/s}^2.$$

We can write this acceleration in terms of $g = 9.80\,\text{m/s}^2$ (the acceleration of gravity at the Earth's surface) as

$$a = 2.72 \times 10^{-3}\,\text{m/s}^2\left(\frac{g}{9.80\,\text{m/s}^2}\right) = 2.78 \times 10^{-4}\,g.$$

NOTE The centripetal acceleration of the Moon, $a = 2.78 \times 10^{-4}\,g$, is *not* the acceleration of gravity for objects at the Moon's surface due to the Moon's gravity. Rather, it is the acceleration due to the *Earth's* gravity for any object (such as the Moon) that is 384,000 km from the Earth. Notice how small this acceleration is compared to the acceleration of objects near the Earth's surface.

CAUTION

Distinguish Moon's gravity on objects at its surface, from Earth's gravity acting on Moon (this Example)

5–2 Dynamics of Uniform Circular Motion

Force is needed to provide centripetal acceleration

According to Newton's second law ($\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$), an object that is accelerating must have a net force acting on it. An object moving in a circle, such as a ball on the end of a string, must therefore have a force applied to it to keep it moving in that circle. That is, a net force is necessary to give it centripetal acceleration. The magnitude of the required force can be calculated using Newton's second law for the radial component, $\Sigma F_R = ma_R$, where a_R is the centripetal acceleration, $a_R = v^2/r$, and ΣF_R is the total (or net) force in the radial direction:

$$\Sigma F_R = ma_R = m\frac{v^2}{r}. \qquad \text{[circular motion]} \quad \textbf{(5–3)}$$

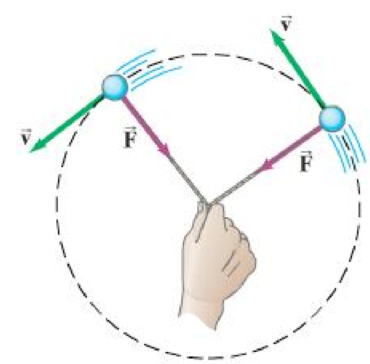


FIGURE 5–4 A force is required to keep an object moving in a circle. If the speed is constant, the force is directed toward the circle's center.

For uniform circular motion (v = constant), the acceleration is a_R, which is directed toward the center of the circle at any moment. Thus the *net force too must be directed toward the center of the circle* (Fig. 5–4). A net force is necessary because otherwise, if no net force were exerted on the object, it would not move in a circle but in a straight line, as Newton's first law tells us. The direction of the net force is continually changing so that it is always directed toward the center of the circle. This force is sometimes called a centripetal ("pointing toward the center") force. But be aware that "centripetal force" does not indicate some new kind of force. The term merely describes the *direction* of the net force needed to provide a circular path: the net force is directed toward the circle's center. The force *must be applied by other objects*. For example, to swing a ball in a circle on the end of a string, you pull on the string and the string exerts the force on the ball. (Try it.)

CAUTION

Centripetal force is not a new kind of force
(Every force must be exerted by an object)

CAUTION

There is no real "centrifugal force"

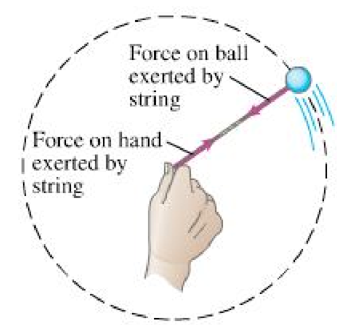
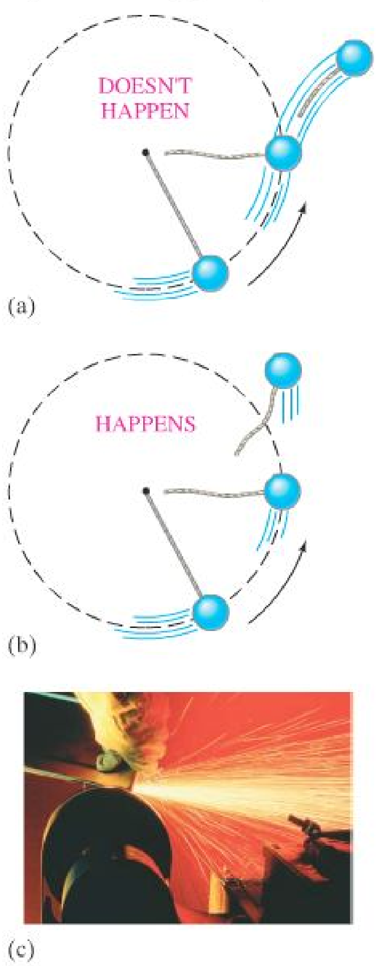


FIGURE 5–5 Swinging a ball on the end of a string.

There is a common misconception that an object moving in a circle has an outward force acting on it, a so-called centrifugal ("center-fleeing") force. This is incorrect: *there is no outward force* on the revolving object. Consider, for example, a person swinging a ball on the end of a string around her head (Fig. 5–5). If you have ever done this yourself, you know that you feel a force pulling outward on your hand. The misconception arises when this pull is interpreted as an outward "centrifugal" force pulling on the ball that is transmitted along the string to your hand. This is not what is happening at all. To keep the ball moving in a circle, you pull *inwardly* on the string, and the string exerts this force on the ball. The ball exerts an equal and opposite force on the string (Newton's third law), and *this* is the outward force your hand feels (see Fig. 5–5).

The force *on the ball* is the one exerted *inwardly* on it by you, via the string. To see even more convincing evidence that a "centrifugal force" does not act on the ball, consider what happens when you let go of the string. If a centrifugal force were acting, the ball would fly outward, as shown in Fig. 5–6a. But it doesn't; the ball flies off tangentially (Fig. 5–6b), in the direction of the velocity it had at the moment it was released, because the inward force no longer acts. Try it and see!

FIGURE 5–6 If centrifugal force existed, the revolving ball would fly outward as in (a) when released. In fact, it flies off tangentially as in (b). For example, in (c) sparks fly in straight lines tangentially from the edge of a rotating grinding wheel.



EXAMPLE 5–3 ESTIMATE Force on revolving ball (horizontal). Estimate the force a person must exert on a string attached to a 0.150-kg ball to make the ball revolve in a horizontal circle of radius 0.600 m. The ball makes 2.00 revolutions per second ($T = 0.500\,\mathrm{s}$), as in Example 5–1.

APPROACH First we need to draw the free-body diagram for the ball. The forces acting on the ball are the force of gravity, $m\vec{\mathbf{g}}$ downward, and the tension force $\vec{\mathbf{F}}_{\mathrm{T}}$ that the string exerts toward the hand at the center (which occurs because the person exerts that same force on the string). The free-body diagram for the ball is as shown in Fig. 5–7. The ball's weight complicates matters and makes it impossible to revolve a ball with the cord perfectly horizontal. We assume the weight is small, and put $\phi \approx 0$ in Fig. 5–7. Thus $\vec{\mathbf{F}}_{\mathrm{T}}$ will act nearly horizontally and, in any case, provides the force necessary to give the ball its centripetal acceleration.

SOLUTION We apply Newton's second law to the radial direction, which we assume is horizontal:

$$(\Sigma F)_{\mathrm{R}} = ma_{\mathrm{R}},$$

where $a_{\mathrm{R}} = v^2/r$ and $v = 2\pi r/T = 2\pi(0.600\,\mathrm{m})/(0.500\,\mathrm{s}) = 7.54\,\mathrm{m/s}$. Thus

$$F_{\mathrm{T}} = m\frac{v^2}{r} = (0.150\,\mathrm{kg})\frac{(7.54\,\mathrm{m/s})^2}{(0.600\,\mathrm{m})} \approx 14\,\mathrm{N}.$$

NOTE We keep only two significant figures in the answer because $mg = (0.150\,\mathrm{kg})(9.80\,\mathrm{m/s^2}) = 1.5\,\mathrm{N}$, being about $\frac{1}{10}$ of our result, is small but not so small as to justify stating a more precise answer since we ignored the effect of mg.

NOTE To include the effect of $m\vec{\mathbf{g}}$, resolve $\vec{\mathbf{F}}_{\mathrm{T}}$ in Fig. 5–7 into components, and set the horizontal component of $\vec{\mathbf{F}}_{\mathrm{T}}$ equal to mv^2/r and its vertical component equal to mg.

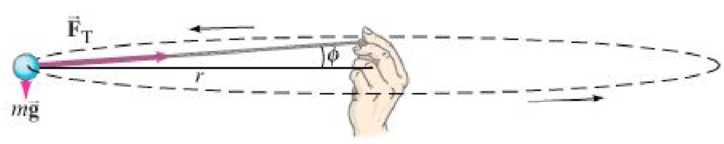


FIGURE 5–7 Example 5–3.

EXAMPLE 5–4 **Revolving ball (vertical circle).** A 0.150-kg ball on the end of a 1.10-m-long cord (negligible mass) is swung in a *vertical* circle. (*a*) Determine the minimum speed the ball must have at the top of its arc so that the ball continues moving in a circle. (*b*) Calculate the tension in the cord at the bottom of the arc, assuming the ball is moving at twice the speed of part (*a*).

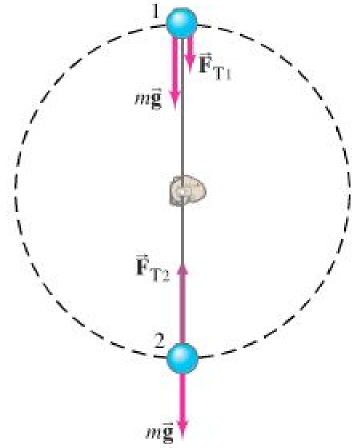


FIGURE 5–8 Example 5–4. Free-body diagrams for positions 1 and 2.

APPROACH The ball moves in a vertical circle and is *not* undergoing uniform circular motion. The radius is assumed constant, but the speed v changes because of gravity. Nonetheless, Eq. 5–1 is valid at each point along the circle, and we use it at points 1 and 2. The free-body diagram is shown in Fig. 5–8 for both positions 1 and 2.

SOLUTION (*a*) At the top (point 1), two forces act on the ball: $m\vec{\mathbf{g}}$, the force of gravity, and $\vec{\mathbf{F}}_{T1}$, the tension force the cord exerts at point 1. Both act downward, and their vector sum acts to give the ball its centripetal acceleration a_R. We apply Newton's second law, for the vertical direction, choosing downward as positive since the acceleration is downward (toward the center):

$$(\Sigma F)_R = ma_R$$

$$F_{T1} + mg = m\frac{v_1^2}{r}. \qquad \text{[at top]}$$

Cord tension and gravity together provide centripetal acceleration

From this equation we can see that the tension force F_{T1} at point 1 will get larger if v_1 (ball's speed at top of circle) is made larger, as expected. But we are asked for the *minimum* speed to keep the ball moving in a circle. The cord will remain taut as long as there is tension in it. But if the tension disappears (because v_1 is too small) the cord can go limp, and the ball will fall out of its circular path. Thus, the minimum speed will occur if $F_{T1} = 0$, for which we have

$$mg = m\frac{v_1^2}{r}. \qquad \text{[minimum speed at top]}$$

Gravity provides centripetal acceleration

We solve for v_1:

$$v_1 = \sqrt{gr} = \sqrt{(9.80\,\text{m/s}^2)(1.10\,\text{m})} = 3.28\,\text{m/s}.$$

This is the minimum speed at the top of the circle if the ball is to continue moving in a circular path.

(*b*) When the ball is at the bottom of the circle (point 2 in Fig. 5–8), the cord exerts its tension force F_{T2} upward, whereas the force of gravity, $m\vec{\mathbf{g}}$, still acts downward. So we apply Newton's second law, this time choosing *upward* as positive since the acceleration is upward (toward the center):

$$(\Sigma F)_R = ma_R$$

$$F_{T2} - mg = m\frac{v_2^2}{r}. \qquad \text{[at bottom]}$$

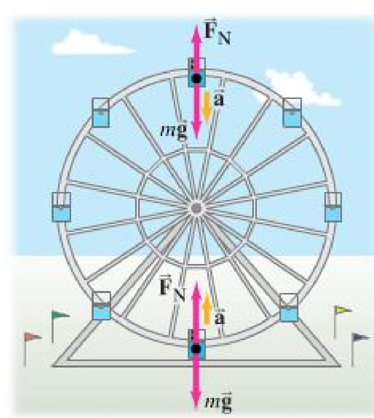
String tension and gravity acting in opposite directions provide centripetal acceleration

The speed v_2 is given as twice that in (*a*), namely 6.56 m/s. We solve for F_{T2}:

$$F_{T2} = m\frac{v_2^2}{r} + mg$$

$$= (0.150\,\text{kg})\frac{(6.56\,\text{m/s})^2}{(1.10\,\text{m})} + (0.150\,\text{kg})(9.80\,\text{m/s}^2) = 7.34\,\text{N}.$$

FIGURE 5–9 Exercise C.



EXERCISE B In a tumble dryer, the speed of the drum should be just large enough so that the clothes are carried nearly to the top of the drum and then fall away, rather than being pressed against the drum for the whole revolution. Determine whether this speed will be different for heavier wet clothes than for lighter dry clothes.

EXERCISE C A rider on a Ferris wheel moves in a vertical circle of radius r at constant speed v (Fig. 5–9). Is the normal force that the seat exerts on the rider at the top of the wheel (*a*) less than, (*b*) more than, or (*c*) the same as, the force the seat exerts at the bottom of the wheel?

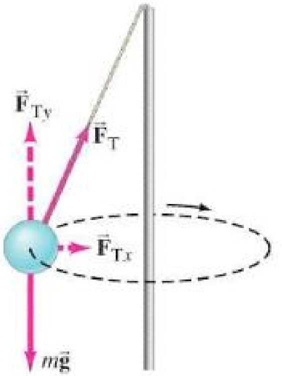


FIGURE 5–10 Example 5–5.

CONCEPTUAL EXAMPLE 5–5 **Tetherball.** The game of tetherball is played with a ball tied to a pole with a string. After the ball is struck, it revolves around the pole as shown in Fig. 5–10. In what direction is the acceleration of the ball, and what force causes the acceleration?

RESPONSE If the ball revolves in a horizontal plane as shown, then the acceleration points horizontally toward the center of the ball's circular path (not toward the top of the pole). The force responsible for the acceleration may not be obvious at first, since there seems to be no force pointing directly horizontally. But it is the *net* force (the sum of $m\vec{g}$ and $\vec{F}_T$ here) that must point in the direction of the acceleration. The vertical component of the string tension, F_{Ty}, balances the ball's weight, $m\vec{g}$. The horizontal component of the string tension, F_{Tx}, is the force that produces the centripetal acceleration toward the center.

PROBLEM SOLVING Uniform Circular Motion

1. **Draw a free-body diagram**, showing all the forces acting on each object under consideration. Be sure you can identify the source of each force (tension in a cord, Earth's gravity, friction, normal force, and so on). Don't put in something that doesn't belong (like a centrifugal force).
2. **Determine** which of the forces, or which of their components, act to provide the centripetal acceleration—that is, all the **forces or components that act radially**, toward or away from the center of the circular path. The sum of these forces (or components) provides the centripetal acceleration, $a_R = v^2/r$.
3. **Choose a convenient coordinate system**, preferably with one axis along the acceleration direction.
4. **Apply Newton's second law** to the radial component:

$$(\Sigma F)_R = ma_R = m\frac{v^2}{r}. \qquad \text{[radial direction]}$$

5–3 Highway Curves, Banked and Unbanked

PHYSICS APPLIED
Driving around a curve

An example of circular dynamics occurs when an automobile rounds a curve, say to the left. In such a situation, you may feel that you are thrust outward toward the right side door. But there is no mysterious centrifugal force pulling on you. What is happening is that you tend to move in a straight line, whereas the car has begun to follow a curved path. To make you go in the curved path, the seat (friction) or the door of the car (direct contact) exerts a force on you (Fig. 5–11). The car also must have a force exerted on it toward the center of the curve if it is to move in that curve. On a flat road, this force is supplied by friction between the tires and the pavement.

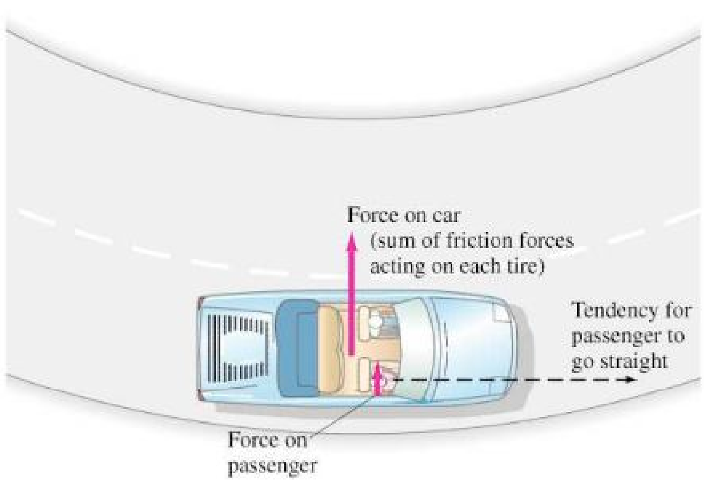


FIGURE 5–11 The road exerts an inward force (friction against the tires) on a car to make it move in a circle. The car exerts an inward force on the passenger.

If the wheels and tires of the car are rolling normally without slipping or sliding, the bottom of the tire is at rest against the road at each instant; so the friction force the road exerts on the tires is static friction. But if the static friction force is not great enough, as under icy conditions, sufficient friction force cannot be applied and the car will skid out of a circular path into a more nearly straight path. See Fig. 5–12. Once a car skids or slides, the friction force becomes kinetic friction, which is less than static friction.

FIGURE 5–12 Race car heading into a curve. From the tire marks we see that most cars experienced a sufficient friction force to give them the needed centripetal acceleration for rounding the curve safely. But, we also see tire tracks of cars on which there was not sufficient force—and which followed more nearly straight-line paths.

EXAMPLE 5–6 **Skidding on a curve.** A 1000-kg car rounds a curve on a flat road of radius 50 m at a speed of 50 km/h (14 m/s). Will the car follow the curve, or will it skid? Assume: (*a*) the pavement is dry and the coefficient of static friction is $\mu_s = 0.60$; (*b*) the pavement is icy and $\mu_s = 0.25$.

APPROACH The forces on the car are gravity mg downward, the normal force F_N exerted upward by the road, and a horizontal friction force due to the road. They are shown in Fig. 5–13, which is the free-body diagram for the car. The car will follow the curve if the maximum static friction force is greater than the mass times the centripetal acceleration.

SOLUTION In the vertical direction there is no acceleration. Newton's second law tells us that the normal force F_N on the car is equal to the weight mg since the road is flat:

$$F_N = mg = (1000\,\text{kg})(9.8\,\text{m/s}^2) = 9800\,\text{N}.$$

In the horizontal direction the only force is friction, and we must compare it to the force needed to produce the centripetal acceleration to see if it is sufficient. The net horizontal force required to keep the car moving in a circle around the curve is

$$(\Sigma F)_R = ma_R = m\frac{v^2}{r} = (1000\,\text{kg})\frac{(14\,\text{m/s})^2}{(50\,\text{m})} = 3900\,\text{N}.$$

Now we compute the maximum total static friction force (the sum of the friction forces acting on each of the four tires) to see if it can be large enough to provide a safe centripetal acceleration. For (*a*), $\mu_s = 0.60$, and the maximum friction force attainable (recall from Section 4–8 that $F_{fr} \leq \mu_s F_N$) is

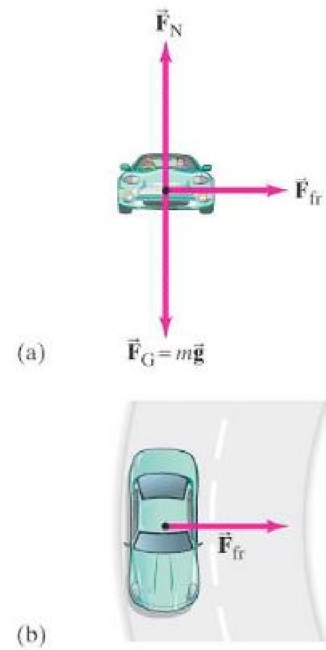
$$(F_{fr})_{max} = \mu_s F_N = (0.60)(9800\,\text{N}) = 5900\,\text{N}.$$

Since a force of only 3900 N is needed, and that is, in fact, how much will be exerted by the road as a static friction force, the car can follow the curve. But in (*b*) the maximum static friction force possible is

$$(F_{fr})_{max} = \mu_s F_N = (0.25)(9800\,\text{N}) = 2500\,\text{N}.$$

The car will skid because the ground cannot exert sufficient force (3900 N is needed) to keep it moving in a curve of radius 50 m at a speed of 50 km/h.

FIGURE 5–13 Example 5–6. Forces on a car rounding a curve on a flat road. (a) Front view, (b) top view.



The possibility of skidding is worse if the wheels lock (stop rotating) when the brakes are applied too hard. When the tires are rolling, static friction exists. But if the wheels lock (stop rotating), the tires slide and the friction force, which is now kinetic friction, is less. More importantly, the *direction* of the friction force changes suddenly if the wheels lock. Static friction can point perpendicular to the velocity, as in Fig. 5–13b, but if the car slides, kinetic friction points *opposite* to the velocity. The force no longer points toward the center of the circle, and the car cannot continue in a curved path (see Fig. 5–12). Even worse, if the road is wet or icy, locking of the wheels occurs with less force on the brake pedal since there is less road friction to keep the wheels turning rather than sliding. Antilock brakes (ABS) are designed to limit brake pressure just before the point where sliding would occur, by means of delicate sensors and a fast computer.

PHYSICS APPLIED
Antilock brakes

PHYSICS APPLIED

Banked curves

The banking of curves can reduce the chance of skidding. The normal force exerted by a banked road, acting perpendicular to the road, will have a component toward the center of the circle (Fig. 5–14), thus reducing the reliance on friction. For a given banking angle θ, there will be one speed for which no friction at all is required. This will be the case when the horizontal component of the normal force toward the center of the curve, $F_N \sin\theta$ (see Fig. 5–14), is just equal to the force required to give a vehicle its centripetal acceleration—that is, when

$$F_N \sin\theta = m\frac{v^2}{r}. \qquad \text{[no friction required]}$$

The banking angle of a road, θ, is chosen so that this condition holds for a particular speed, called the "design speed."

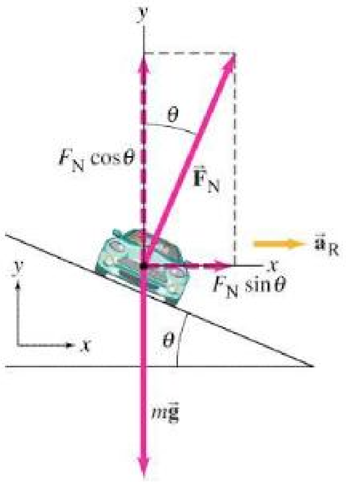


FIGURE 5–14 Normal force on a car rounding a banked curve, resolved into its horizontal and vertical components. The centripetal acceleration is horizontal (*not* parallel to the sloping road). The friction force on the tires, not shown, could point up or down along the slope, depending on the car's speed. The friction force will be zero for one particular speed.

EXAMPLE 5–7 **Banking angle.** (*a*) For a car traveling with speed v around a curve of radius r, determine a formula for the angle at which a road should be banked so that no friction is required. (*b*) What is this angle for an expressway off-ramp curve of radius 50 m at a design speed of 50 km/h?

APPROACH Even though the road is banked, the car is still moving along a horizontal circle, so the centripetal acceleration needs to be horizontal. We choose our x and y axes as horizontal and vertical so that a_R, which is horizontal, is along the x axis. The forces on the car are the Earth's gravity mg downward, and the normal force F_N exerted by the road perpendicular to its surface. See Fig. 5–14, where the components of F_N are also shown. We don't need to consider the friction of the road because we are designing a road to be banked so as to eliminate dependence on friction.

SOLUTION (*a*) For the horizontal direction, $\Sigma F_R = ma_R$ gives

$$F_N \sin\theta = \frac{mv^2}{r}.$$

Since there is no vertical motion, the y component of the acceleration is zero, so $\Sigma F_y = ma_y$ gives us

$$F_N \cos\theta - mg = 0.$$

Thus,

$$F_N = \frac{mg}{\cos\theta}.$$

CAUTION

F_N is not always equal to mg

[Note in this case that $F_N \geq mg$ since $\cos\theta \leq 1$.]
We substitute this relation for F_N into the equation for the horizontal motion,

Horizontal component of normal force acts to provide centripetal acceleration (friction is desired to be zero—otherwise it too would contribute)

$$F_N \sin\theta = m\frac{v^2}{r},$$

and obtain

$$\frac{mg}{\cos\theta}\sin\theta = m\frac{v^2}{r}$$

or

$$mg\tan\theta = m\frac{v^2}{r},$$

so

Banking angle (friction not needed)

$$\tan\theta = \frac{v^2}{rg}.$$

This is the formula for the banking angle θ: no friction needed at speed v.
(*b*) For $r = 50$ m and $v = 50$ km/h (or 14 m/s),

$$\tan\theta = \frac{(14\,\text{m/s})^2}{(50\,\text{m})(9.8\,\text{m/s}^2)} = 0.40,$$

so $\theta = 22°$.

EXERCISE D To negotiate an unbanked curve at a *faster* speed, a driver puts a couple of sand bags in his van aiming to increase the force of friction between the tires and the road. Will the sand bags help?

EXERCISE E Can a heavy truck and a small car travel safely at the same speed around an icy, banked-curve road?

* 5–4 Nonuniform Circular Motion

Circular motion at constant speed occurs when the net force on an object is exerted toward the center of the circle. If the net force is not directed toward the center but is at an angle, as shown in Fig. 5–15a, the force has two components. The component directed toward the center of the circle, F_R, gives rise to the centripetal acceleration, a_R, and keeps the object moving in a circle. The component tangent to the circle, F_{tan}, acts to increase (or decrease) the speed, and thus gives rise to a component of the acceleration tangent to the circle, a_{tan}. When the speed of the object is changing, a tangential component of force is acting.

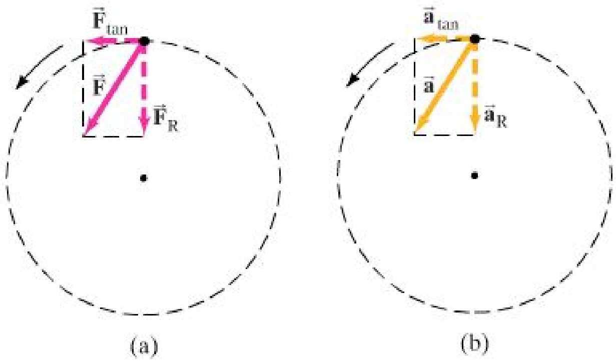


FIGURE 5–15 The speed of an object moving in a circle changes if the force on it has a tangential component, F_{tan}. Part (a) shows the force $\vec{F}$ and its vector components; part (b) shows the acceleration vector and its vector components.

When you first start revolving a ball on the end of a string around your head, you must give it tangential acceleration. You do this by pulling on the string with your hand displaced from the center of the circle. In athletics, a hammer thrower accelerates the hammer tangentially in a similar way so that it reaches a high speed before release.

The tangential component of the acceleration, a_{tan}, is equal to the rate of change of the *magnitude* of the object's velocity:

$$a_{tan} = \frac{\Delta v}{\Delta t}.$$

The radial (centripetal) acceleration arises from the change in *direction* of the velocity and, as we have seen (Eq. 5–1), is given by

$$a_R = \frac{v^2}{r}.$$

The tangential acceleration always points in a direction tangent to the circle, and is in the direction of motion (parallel to $\vec{v}$, which is always tangent to the circle) if the speed is increasing, as shown in Fig. 5–15b. If the speed is decreasing, $\vec{a}_{tan}$ points antiparallel to $\vec{v}$. In either case, $\vec{a}_{tan}$ and $\vec{a}_R$ are always perpendicular to each other; and *their directions change* continually as the object moves along its circular path. The total vector acceleration $\vec{a}$ is the sum of these two:

$$\vec{a} = \vec{a}_{tan} + \vec{a}_R.$$

Since $\vec{a}_R$ and $\vec{a}_{tan}$ are always perpendicular to each other, the magnitude of $\vec{a}$ at any moment is

$$a = \sqrt{a_{tan}^2 + a_R^2}.$$

EXAMPLE 5–8 **Two components of acceleration.** A race car starts from rest in the pit area and accelerates at a uniform rate to a speed of 35 m/s in 11 s, moving on a circular track of radius 500 m. Assuming constant tangential acceleration, find (*a*) the tangential acceleration, and (*b*) the radial acceleration, at the instant when the speed is $v = 15\ \mathrm{m/s}$.

APPROACH The tangential acceleration relates to the change in speed of the car, and can be calculated as $a_{\tan} = \Delta v/\Delta t$. The centripetal acceleration relates to the change in the *direction* of the velocity vector and is calculated using $a_{\mathrm{R}} = v^2/r$.

SOLUTION (*a*) During the 11-s time interval, we assume the tangential acceleration $a_{\tan}$ is constant. Its magnitude is

$$a_{\tan} = \frac{\Delta v}{\Delta t} = \frac{(35\ \mathrm{m/s} - 0\ \mathrm{m/s})}{11\ \mathrm{s}} = 3.2\ \mathrm{m/s^2}.$$

(*b*) When $v = 15\ \mathrm{m/s}$, the centripetal acceleration is

$$a_{\mathrm{R}} = \frac{v^2}{r} = \frac{(15\ \mathrm{m/s})^2}{(500\ \mathrm{m})} = 0.45\ \mathrm{m/s^2}.$$

EXERCISE F When the speed of the race car in Example 5–8 is 30 m/s, how are (*a*) $a_{\tan}$ and (*b*) a_{R} changed?

These concepts can be used for an object moving along any curved path, such as that shown in Fig. 5–16. We can treat any portion of the curve as an arc of a circle with a radius of curvature r. The velocity at any point is always tangent to the path. The acceleration can be written, in general, as a vector sum of two components: the tangential component $a_{\tan} = \Delta v/\Delta t$, and the radial (centripetal) component $a_{\mathrm{R}} = v^2/r$.

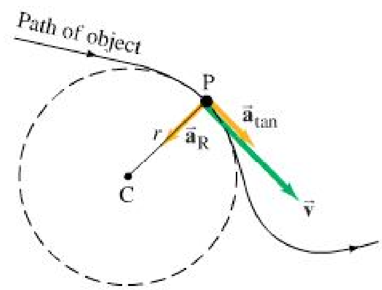


FIGURE 5–16 Object following a curved path (solid line). At point P the path has a radius of curvature r. The object has velocity $\vec{\mathbf{v}}$, tangential acceleration $\vec{\mathbf{a}}_{\tan}$ (the object is increasing in speed), and radial (centripetal) acceleration $\vec{\mathbf{a}}_{\mathrm{R}}$ (magnitude $a_{\mathrm{R}} = v^2/r$) which points toward the center of curvature C.

* 5–5 Centrifugation

PHYSICS APPLIED
Centrifuge

A useful device that nicely illustrates circular motion is the centrifuge, or the very high speed ultracentrifuge. These devices are used to sediment materials quickly or to separate materials. Test tubes are held in the centrifuge rotor, which is accelerated to very high rotational speeds: see Fig. 5–17, where one test tube is shown in two positions as the rotor turns. The small green dot represents a small particle, perhaps a macromolecule, in a fluid-filled test tube. When the tube is at position A and the rotor is turning, the particle has a tendency to move in a straight line in the direction of the dashed arrow. But the fluid, resisting the motion of the particles, exerts a centripetal force that keeps the particles moving nearly in a circle. Usually, the resistance of the fluid (a liquid, a gas, or a gel, depending on the application) does not quite equal mv^2/r, and the particles eventually reach the bottom of the tube. The purpose of a centrifuge is to provide an "effective gravity" much larger than normal gravity because of the high rotational speeds, thus causing more rapid sedimentation.

EXAMPLE 5–9 **Ultracentrifuge.** The rotor of an ultracentrifuge rotates at 50,000 rpm (revolutions per minute). The top of a 4.00-cm-long test tube (Fig. 5–17) is 6.00 cm from the rotation axis and is perpendicular to it. The bottom of the tube is 10.00 cm from the axis of rotation. Calculate the centripetal acceleration, in "g's," at the top and the bottom of the tube.

APPROACH We can calculate the centripetal acceleration from $a_R = v^2/r$. We divide by $g = 9.80\ \mathrm{m/s^2}$ to find a_R in g's.

SOLUTION At the top of the tube, a particle revolves in a circle of circumference $2\pi r$, which is a distance

$$2\pi r = (2\pi)(0.0600\ \mathrm{m}) = 0.377\ \mathrm{m\ per\ revolution.}$$

It makes 5.00×10^4 such revolutions each minute, or, dividing by 60 s/min, 833 rev/s. The time to make one revolution, the period T, is

$$T = \frac{1}{(833\ \mathrm{rev/s})} = 1.20 \times 10^{-3}\ \mathrm{s/rev.}$$

The speed of the particle is then

$$v = \frac{2\pi r}{T} = \left(\frac{0.377\ \mathrm{m/rev}}{1.20 \times 10^{-3}\ \mathrm{s/rev}}\right) = 3.14 \times 10^2\ \mathrm{m/s.}$$

The centripetal acceleration is

$$a_R = \frac{v^2}{r} = \frac{(3.14 \times 10^2\ \mathrm{m/s})^2}{0.0600\ \mathrm{m}} = 1.64 \times 10^6\ \mathrm{m/s^2},$$

which, dividing by $g = 9.80\ \mathrm{m/s^2}$, is 1.67×10^5 g's.

At the bottom of the tube ($r = 0.1000$ m), the speed is

$$v = \frac{2\pi r}{T} = \frac{(2\pi)(0.1000\ \mathrm{m})}{1.20 \times 10^{-3}\ \mathrm{s/rev}} = 523.6\ \mathrm{m/s.}$$

Then

$$a_R = \frac{v^2}{r} = \frac{(523.6\ \mathrm{m/s})^2}{(0.1000\ \mathrm{m})} = 2.74 \times 10^6\ \mathrm{m/s^2} = 2.80 \times 10^5\ g\text{'s},$$

or 280,000 g's.

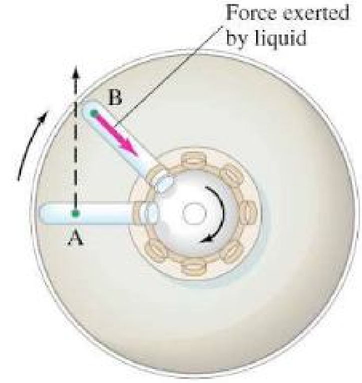


FIGURE 5–17 Two positions of a rotating test tube in a centrifuge (top view). At A, the green dot represents a macromolecule or other particle being sedimented. It would tend to follow the dashed line, heading toward the bottom of the tube, but the fluid resists this motion by exerting a force on the particle as shown at point B.

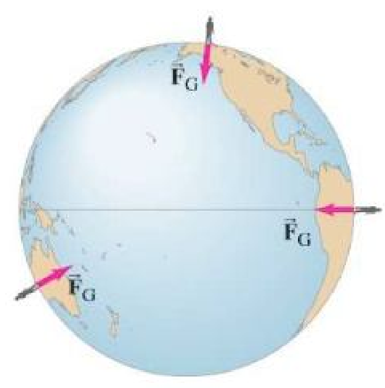
5–6 Newton's Law of Universal Gravitation

Besides developing the three laws of motion, Sir Isaac Newton also examined the motion of the planets and the Moon. In particular, he wondered about the nature of the force that must act to keep the Moon in its nearly circular orbit around the Earth.

Newton was also thinking about the problem of gravity. Since falling objects accelerate, Newton had concluded that they must have a force exerted on them, a force we call the force of gravity. Whenever an object has a force exerted *on* it, that force is exerted *by* some other object. But what *exerts* the force of gravity? Every object on the surface of the Earth feels the force of gravity, and no matter where the object is, the force is directed toward the center of the Earth (Fig. 5–18). Newton concluded that it must be the Earth itself that exerts the gravitational force on objects at its surface.

According to legend, Newton noticed an apple drop from a tree. He is said to have been struck with a sudden inspiration: If gravity acts at the tops of trees, and even at the tops of mountains, then perhaps it acts all the way to the Moon!

FIGURE 5–18 Anywhere on Earth, whether in Alaska, Peru, or Australia, the force of gravity acts downward toward the Earth's center.



With this idea that it is the Earth's gravity that holds the Moon in its orbit, Newton developed his great theory of gravitation. But there was controversy at the time. Many thinkers had trouble accepting the idea of a force "acting at a distance." Typical forces act through contact—your hand pushes a cart and pulls a wagon, a bat hits a ball, and so on. But gravity acts without contact, said Newton: the Earth exerts a force on a falling apple and on the Moon, even though there is no contact, and the two objects may even be very far apart.

Newton set about determining the magnitude of the gravitational force that the Earth exerts on the Moon as compared to the gravitational force on objects at the Earth's surface. The centripetal acceleration of the Moon, as we calculated in Example 5–2, is $a_R = 0.00272\,\mathrm{m/s^2}$. In terms of the acceleration of gravity at the Earth's surface, $g = 9.80\,\mathrm{m/s^2}$,

The Moon's acceleration toward Earth

$$a_R = \frac{0.00272\,\mathrm{m/s^2}}{9.8\,\mathrm{m/s^2}} \approx \frac{1}{3600}\,g.$$

That is, the acceleration of the Moon toward the Earth is about $\frac{1}{3600}$ as great as the acceleration of objects at the Earth's surface. The Moon is 384,000 km from the Earth, which is about 60 times the Earth's radius of 6380 km. That is, the Moon is 60 times farther from the Earth's center than are objects at the Earth's surface. But $60 \times 60 = 60^2 = 3600$. Again that number 3600. Newton concluded that the gravitational force exerted by the Earth on any object decreases with the square of its distance r from the Earth's center:

$$\text{force of gravity} \propto \frac{1}{r^2}.$$

The Moon is 60 Earth radii away, so it feels a gravitational force only $\frac{1}{60^2} = \frac{1}{3600}$ times as strong as an equal mass would at the Earth's surface.

Newton realized that the force of gravity on an object depends not only on distance but also on the object's mass. In fact, it is directly proportional to its mass, as we have seen. According to Newton's third law, when the Earth exerts its gravitational force on any object, such as the Moon, that object exerts an equal and opposite force on the Earth (Fig. 5–19). Because of this symmetry, Newton reasoned, the magnitude of the force of gravity must be proportional to *both* the masses. Thus

$$F \propto \frac{m_E\, m_{Obj}}{r^2},$$

where m_E is the mass of the Earth, m_{Obj} the mass of the other object, and r the distance from the Earth's center to the center of the other object.

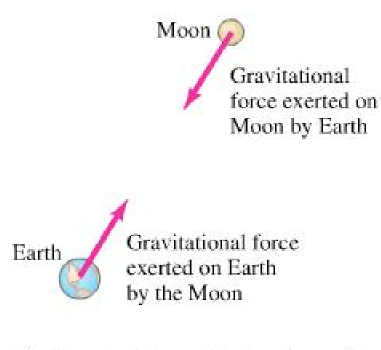


FIGURE 5–19 The gravitational force one object exerts on a second object is directed toward the first object, and (by Newton's third law) is equal and opposite to the force exerted by the second object on the first.

Newton went a step further in his analysis of gravity. In his examination of the orbits of the planets, he concluded that the force required to hold the planets in their orbits around the Sun seems to diminish as the inverse square of their distance from the Sun. This led him to believe that it is also the gravitational force that acts between the Sun and each of the planets to keep them in their orbits. And if gravity acts between these objects, why not between all objects?

Thus he proposed his **law of universal gravitation**, which we can state as follows:

Every particle in the universe attracts every other particle with a force that is proportional to the product of their masses and inversely proportional to the square of the distance between them. This force acts along the line joining the two particles.

NEWTON'S LAW OF UNIVERSAL GRAVITATION

The magnitude of the gravitational force can be written as

$$F = G\frac{m_1 m_2}{r^2}, \tag{5–4}$$

where m_1 and m_2 are the masses of the two particles, r is the distance between them, and G is a universal constant which must be measured experimentally and has the same numerical value for all objects.

The value of G must be very small, since we are not aware of any force of attraction between ordinary-sized objects, such as between two baseballs. The force between two ordinary objects was first measured by Henry Cavendish in 1798, over 100 years after Newton published his law. To detect and measure the incredibly small force between ordinary objects, he used an apparatus like that shown in Fig. 5–20. Cavendish confirmed Newton's hypothesis that two objects attract one another, and that Eq. 5–4 accurately describes this force. In addition, because Cavendish could measure F, m_1, m_2, and r accurately, he was able to determine the value of the constant G as well. The accepted value today is

$$G = 6.67 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2}.$$

[Strictly speaking, Eq. 5–4 gives the magnitude of the gravitational force that one particle exerts on a second particle that is a distance r away. For an extended object (that is, not a point), we must consider how to measure the distance r. This is often best done using integral calculus, which Newton himself invented. Newton showed that for two uniform spheres, Eq. 5–4 gives the correct force where r is the distance between their centers. When extended objects are small compared to the distance between them (as for the Earth–Sun system), little inaccuracy results from considering them as point particles.]

EXAMPLE 5–10 ESTIMATE Can you attract another person gravitationally? A 50-kg person and a 75-kg person are sitting on a bench. Estimate the magnitude of the gravitational force each exerts on the other.

APPROACH This is an estimate: we let the distance between the people be $\frac{1}{2}$ m, and round off G to $10^{-10}\,\mathrm{N \cdot m^2/kg^2}$.

SOLUTION We use Eq. 5–4:

$$F = G\frac{m_1 m_2}{r^2} \approx \frac{(10^{-10}\,\mathrm{N \cdot m^2/kg^2})(50\,\mathrm{kg})(75\,\mathrm{kg})}{(0.5\,\mathrm{m})^2} \approx 10^{-6}\,\mathrm{N},$$

which is unnoticeably small unless very delicate instruments are used.

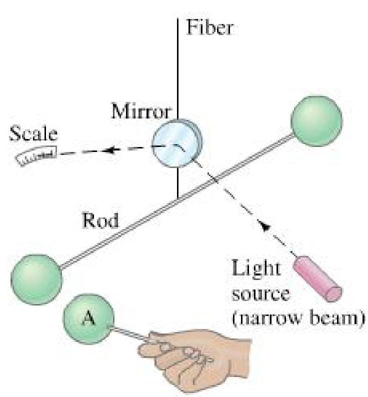


FIGURE 5–20 Schematic diagram of Cavendish's apparatus. Two spheres are attached to a lightweight horizontal rod, which is suspended at its center by a thin fiber. When a third sphere labeled A is brought close to one of the suspended spheres, the gravitational force causes the latter to move, and this twists the fiber slightly. The tiny movement is magnified by the use of a narrow light beam directed at a mirror mounted on the fiber. The beam reflects onto a scale. Previous determination of how large a force will twist the fiber a given amount then allows one to determine the magnitude of the gravitational force between two objects.

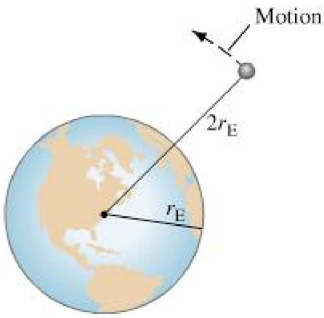


FIGURE 5–21 Example 5–11.

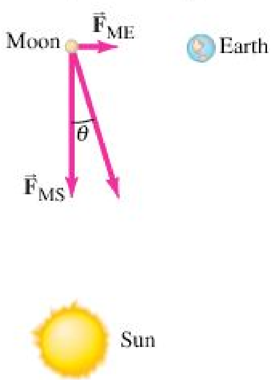
EXAMPLE 5–11 **Spacecraft at $2r_E$.** What is the force of gravity acting on a 2000-kg spacecraft when it orbits two Earth radii from the Earth's center (that is, a distance $r_E = 6380\,\text{km}$ above the Earth's surface, Fig. 5–21)? The mass of the Earth is $M_E = 5.98 \times 10^{24}\,\text{kg}$.

APPROACH We could plug all the numbers into Eq. 5–4, but there is a simpler approach. The spacecraft is twice as far from the Earth's center as when it is at the surface of the Earth. Therefore, since the force of gravity decreases as the square of the distance (and $\frac{1}{2^2} = \frac{1}{4}$), the force of gravity on the satellite will be only one-fourth its weight at the Earth's surface.

SOLUTION At the surface of the Earth, $F_G = mg$. At a distance from the Earth's center of $2r_E$, F_G is $\frac{1}{4}$ as great:

$$F_G = \tfrac{1}{4}mg = \tfrac{1}{4}(2000\,\text{kg})(9.80\,\text{m/s}^2)$$
$$= 4900\,\text{N}.$$

FIGURE 5–22 Example 5–12. Orientation of Sun (S), Earth (E), and Moon (M) at right angles to each other (not to scale).



EXAMPLE 5–12 **Force on the Moon.** Find the net force on the Moon $(m_M = 7.35 \times 10^{22}\,\text{kg})$ due to the gravitational attraction of both the Earth $(m_E = 5.98 \times 10^{24}\,\text{kg})$ and the Sun $(m_S = 1.99 \times 10^{30}\,\text{kg})$, assuming they are at right angles to each other as in Fig. 5–22.

APPROACH The forces on our object, the Moon, are the gravitational force exerted on the Moon by the Earth F_{ME} and that exerted by the Sun F_{MS}, as shown in the free-body diagram of Fig. 5–22. We use the law of universal gravitation to find the magnitude of each force, and then add the two forces as vectors.

SOLUTION The Earth is $3.84 \times 10^5\,\text{km} = 3.84 \times 10^8\,\text{m}$ from the Moon, so F_{ME} (the gravitational force on the Moon due to the Earth) is

$$F_{ME} = \frac{(6.67 \times 10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2)(7.35 \times 10^{22}\,\text{kg})(5.98 \times 10^{24}\,\text{kg})}{(3.84 \times 10^8\,\text{m})^2}$$
$$= 1.99 \times 10^{20}\,\text{N}.$$

The Sun is $1.50 \times 10^8\,\text{km}$ from the Earth and the Moon, so F_{MS} (the gravitational force on the Moon due to the Sun) is

$$F_{MS} = \frac{(6.67 \times 10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2)(7.35 \times 10^{22}\,\text{kg})(1.99 \times 10^{30}\,\text{kg})}{(1.50 \times 10^{11}\,\text{m})^2}$$
$$= 4.34 \times 10^{20}\,\text{N}.$$

The two forces act at right angles in the case we are considering (Fig. 5–22), so we can apply the Pythagorean theorem to find the magnitude of the total force:

$$F = \sqrt{(1.99 \times 10^{20}\,\text{N})^2 + (4.34 \times 10^{20}\,\text{N})^2} = 4.77 \times 10^{20}\,\text{N}.$$

The force acts at an angle θ (Fig. 5–22) given by $\theta = \tan^{-1}(1.99/4.34) = 24.6°$.

CAUTION

Distinguish between Newton's second law and the law of universal gravitation

Don't confuse the law of universal gravitation with Newton's second law of motion, $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$. The former describes a particular force, gravity, and how its strength varies with the distance and masses involved. Newton's second law, on the other hand, relates the net force on an object (i.e., the vector sum of all the different forces acting on the object, whatever their sources) to the mass and acceleration of that object.

5–7 Gravity Near the Earth's Surface; Geophysical Applications

When Eq. 5–4 is applied to the gravitational force between the Earth and an object at its surface, m_1 becomes the mass of the Earth m_E, m_2 becomes the mass of the object m, and r becomes the distance of the object from the Earth's center,[†] which is the radius of the Earth r_E. This force of gravity due to the Earth is the weight of the object, which we have been writing as mg. Thus,

$$mg = G\frac{mm_E}{r_E^2}.$$

We can solve this for g, the acceleration of gravity at the Earth's surface:

$$g = G\frac{m_E}{r_E^2}. \qquad (5\text{–}5)$$

g in terms of G

Thus, the acceleration of gravity at the surface of the Earth, g, is determined by m_E and r_E. (Don't confuse G with g; they are very different quantities, but are related by Eq. 5–5.)

CAUTION
Distinguish G from g

Until G was measured, the mass of the Earth was not known. But once G was measured, Eq. 5–5 could be used to calculate the Earth's mass, and Cavendish was the first to do so. Since $g = 9.80\,\text{m/s}^2$ and the radius of the Earth is $r_E = 6.38 \times 10^6\,\text{m}$, then, from Eq. 5–5, we obtain

$$m_E = \frac{gr_E^2}{G} = \frac{(9.80\,\text{m/s}^2)(6.38\times10^6\,\text{m})^2}{6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2} = 5.98\times10^{24}\,\text{kg}$$

Mass of the Earth

for the mass of the Earth.

Equation 5–5 can be applied to other planets, where g, m, and r would refer to that planet.

EXAMPLE 5–13 ESTIMATE Gravity on Everest. Estimate the effective value of g on the top of Mt. Everest, 8850 m (29,035 ft) above sea level. That is, what is the acceleration due to gravity of objects allowed to fall freely at this altitude?

APPROACH The force of gravity (and the acceleration due to gravity g) depends on the distance from the center of the Earth, so there will be an effective value g' on top of Mt. Everest which will be smaller than g at sea level. We assume the Earth is a uniform sphere (a reasonable "estimate").

SOLUTION We use Eq. 5–5, with r_E replaced by $r = 6380\,\text{km} + 8.9\,\text{km} = 6389\,\text{km} = 6.389\times10^6\,\text{m}$:

$$g = G\frac{m_E}{r^2} = \frac{(6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\,\text{kg})}{(6.389\times10^6\,\text{m})^2} = 9.77\,\text{m/s}^2,$$

which is a reduction of about 3 parts in a thousand (0.3%).

NOTE This is an estimate because, among other things, we ignored the mass accumulated under the mountaintop.

Note that Eq. 5–5 does not give precise values for g at different locations because the Earth is not a perfect sphere. The Earth not only has mountains and valleys, and bulges at the equator, but also its mass is not distributed precisely uniformly (see Table 5–1). The Earth's rotation also affects the value of g. However, for most practical purposes, when an object is near the Earth's surface, we will simply use $g = 9.80\,\text{m/s}^2$ and write the weight of an object as mg.

TABLE 5–1 Acceleration Due to Gravity at Various Locations on Earth

Location	Elevation (m)	g (m/s^2)
New York	0	9.803
San Francisco	0	9.800
Denver	1650	9.796
Pikes Peak	4300	9.789
Sydney, Australia	0	9.798
Equator	0	9.780
North Pole (calculated)	0	9.832

[†] That the distance is measured from the Earth's center does not imply that the force of gravity somehow emanates from that one point. Rather, all parts of the Earth attract gravitationally, but the net effect is a force acting toward the Earth's center.

The value of g can vary locally on the Earth's surface because of the presence of irregularities and rocks of different densities. Such variations in g, known as "gravity anomalies," are very small—on the order of 1 part per 10^6 or 10^7 in the value of g. But they can be measured by "gravimeters" which detect variations in g to 1 part in 10^9. Geophysicists use such measurements as part of their investigations into the structure of the Earth's crust, and in mineral and oil exploration. Mineral deposits, for example, often have a greater density than does surrounding material. Because of the greater mass in a given volume, g can have a slightly greater value on top of such a deposit than at its flanks. "Salt domes," under which petroleum is often found, have a lower than average density; searches for a slight reduction in the value of g in certain locales have led to the discovery of oil.

PHYSICS APPLIED
Geology—mineral and oil exploration

5–8 Satellites and "Weightlessness"

Satellite Motion

PHYSICS APPLIED
Artificial Earth satellites

Artificial satellites circling the Earth are now commonplace (Fig. 5–23). A satellite is put into orbit by accelerating it to a sufficiently high tangential speed with the use of rockets, as shown in Fig. 5–24. If the speed is too high, the spacecraft will not be confined by the Earth's gravity and will escape, never to return. If the speed is too low, it will return to Earth. Satellites are usually put into circular (or nearly circular) orbits, because such orbits require the least takeoff speed.

FIGURE 5–23 A satellite circling the Earth.

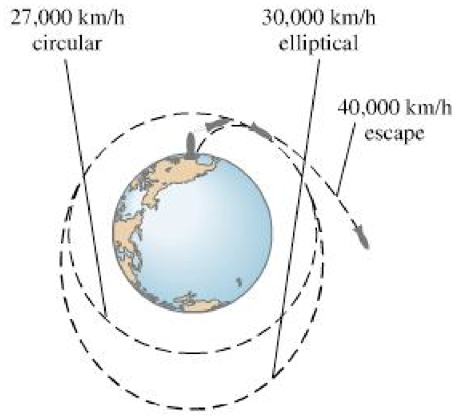


FIGURE 5–24 Artificial satellites launched at different speeds.

It is sometimes asked: "What keeps a satellite up?" The answer is: its high speed. If a satellite stopped moving, it would fall directly to Earth. But at the very high speed a satellite has, it would quickly fly out into space (Fig. 5–25) if it weren't for the gravitational force of the Earth pulling it into orbit. In fact, a satellite *is* falling (accelerating toward Earth), but its high tangential speed keeps it from hitting Earth.

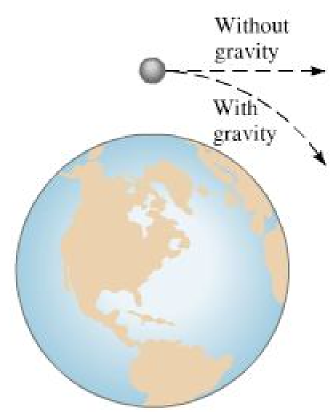


FIGURE 5–25 A moving satellite "falls" out of a straight-line path toward the Earth.

EXAMPLE 5–14 Geosynchronous satellite. A *geosynchronous* satellite is one that stays above the same point on the Earth, which is possible only if it is above a point on the equator. Such satellites are used for TV and radio transmission, for weather forecasting, and as communication relays. Determine (*a*) the height above the Earth's surface such a satellite must orbit, and (*b*) such a satellite's speed. (*c*) Compare to the speed of a satellite orbiting 200 km above Earth's surface.

PHYSICS APPLIED
Geosynchronous satellites

APPROACH To remain above the same point on Earth as the Earth rotates, the satellite must have a period of one day. We can apply Newton's second law, $F = ma$, where $a = v^2/r$ if we assume the orbit is circular.

SOLUTION (*a*) The only force on the satellite is the force of universal gravitation. So Eq. 5–4 gives us the force F, which we insert into Newton's second law:

$$F = ma$$

$$G\frac{m_{\text{Sat}}m_{\text{E}}}{r^2} = m_{\text{Sat}}\frac{v^2}{r}. \qquad \text{[satellite equation]}$$

This equation has two unknowns, r and v. But the satellite revolves around the Earth with the same period that the Earth rotates on its axis, namely once in 24 hours. Thus the speed of the satellite must be

$$v = \frac{2\pi r}{T},$$

where $T = 1\text{ day} = (24\text{ h})(3600\text{ s/h}) = 86{,}400\text{ s}$. We substitute this into the "satellite equation" above and obtain (after canceling m_{Sat} on both sides)

$$G\frac{m_{\text{E}}}{r^2} = \frac{(2\pi r)^2}{rT^2}.$$

After cancelling an r, we can solve for r^3:

$$r^3 = \frac{Gm_{\text{E}}T^2}{4\pi^2} = \frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\text{ kg})(86{,}400\text{ s})^2}{4\pi^2}$$
$$= 7.54\times10^{22}\text{ m}^3.$$

Taking the cube root, we get $r = 4.23\times10^7\text{ m}$, or 42,300 km from the Earth's center. We subtract the Earth's radius of 6380 km to find that a geosynchronous satellite must orbit about 36,000 km (about 6 r_{E}) above the Earth's surface.
(*b*) We solve for v in the satellite equation given in part (*a*):

$$v = \sqrt{\frac{Gm_{\text{E}}}{r}} = \sqrt{\frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\text{ kg})}{(4.23\times10^7\text{ m})}} = 3070\text{ m/s}.$$

We get the same result if we use $v = 2\pi r/T$.
(*c*) The equation in part (*b*) for v shows $v \propto \sqrt{1/r}$. So for $r = r_{\text{E}} + h = 6380\text{ km} + 200\text{ km} = 6580\text{ km}$, we get

$$v' = v\sqrt{\frac{r}{r'}} = (3070\text{ m/s})\sqrt{\frac{(42{,}300\text{ km})}{(6580\text{ km})}} = 7780\text{ m/s}.$$

NOTE The center of a satellite orbit is always at the center of the Earth; so it is not possible to have a satellite orbiting above a fixed point on the Earth at any latitude other than 0°.

EXERCISE G Two satellites orbit the Earth in circular orbits of the same radius. One satellite is twice as massive as the other. Which of the following statements is true about the speeds of these satellites? (*a*) The heavier satellite moves twice as fast as the lighter one. (*b*) The two satellites have the same speed. (*c*) The lighter satellite moves twice as fast as the heavier one. (*d*) The heavier satellite moves four times as fast as the lighter one.

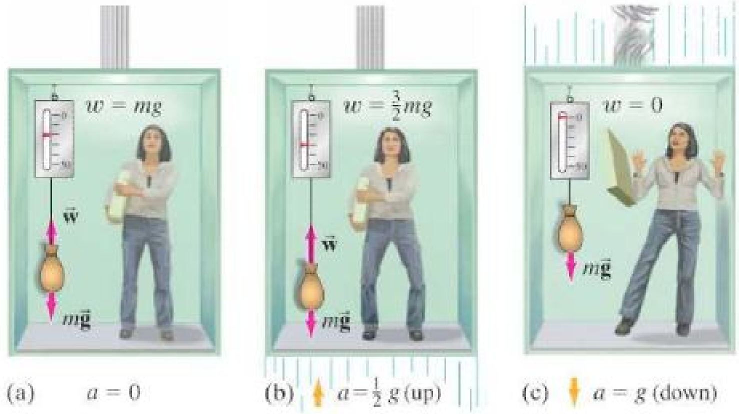


FIGURE 5–26 (a) An object in an elevator at rest exerts a force on a spring scale equal to its weight. (b) In an elevator accelerating upward at $\frac{1}{2}g$, the object's apparent weight is $1\frac{1}{2}$ times larger than its true weight. (c) In a freely falling elevator, the object experiences "weightlessness": the scale reads zero.

Weightlessness

"Weightlessness" in a falling elevator

People and other objects in a satellite circling the Earth are said to experience apparent weightlessness. Let us first look at a simpler case, that of a falling elevator. In Fig. 5–26a, an elevator is at rest with a bag hanging from a spring scale. The scale reading indicates the downward force exerted on it by the bag. This force, exerted *on* the scale, is equal and opposite to the force exerted *by* the scale upward on the bag, and we call its magnitude w. Two forces act on the bag: the downward gravitational force and the upward force exerted by the scale (Newton's third law) equal to w. Because the bag is not accelerating, when we apply $\Sigma F = ma$ to the bag in Fig. 5–26a we obtain

$$w - mg = 0,$$

where mg is the weight of the bag. Thus, $w = mg$, and since the scale indicates the force w exerted on it by the bag, it registers a force equal to the weight of the bag, as we expect.

If, now, the elevator has an acceleration, a, then applying $\Sigma F = ma$ to the bag, we have

$$w - mg = ma.$$

Solving for w, we have

$$w = mg + ma. \qquad [a \text{ is } + \text{ upward}]$$

We have chosen the positive direction up. Thus, if the acceleration a is up, a is positive; and the scale, which measures w, will read more than mg. We call w the *apparent weight* of the bag, which in this case would be greater than its actual weight (mg). If the elevator accelerates downward, a will be negative and w, the apparent weight, will be less than mg. The direction of the velocity $\vec{\mathbf{v}}$ doesn't matter. Only the direction of the acceleration $\vec{\mathbf{a}}$ influences the scale reading.

Suppose, for example, the elevator's acceleration is $\frac{1}{2}g$ upward; then we find

$$w = mg + m\left(\tfrac{1}{2}g\right) = \tfrac{3}{2}mg.$$

That is, the scale reads $1\frac{1}{2}$ times the actual weight of the bag (Fig. 5–26b). The apparent weight of the bag is $1\frac{1}{2}$ times its real weight. The same is true of the person: her apparent weight (equal to the normal force exerted on her by the elevator floor) is $1\frac{1}{2}$ times her real weight. We can say that she is experiencing $1\frac{1}{2}g$'s, just as astronauts experience so many g's at a rocket's launch.

If, instead, the elevator's acceleration is $a = -\frac{1}{2}g$ (downward), then $w = mg - \frac{1}{2}mg = \frac{1}{2}mg$. That is, the scale reads half the actual weight. If the elevator is in *free fall* (for example, if the cables break), then $a = -g$ and $w = mg - mg = 0$. The scale reads zero. See Fig. 5–26c. The bag appears weightless. If the person in the elevator accelerating at $-g$ let go of a pencil, say, it would not fall to the floor. True, the pencil would be falling with acceleration g. But so would the floor of the elevator and the person. The pencil would hover right in front of the person. This phenomenon is called *apparent weightlessness* because in the reference frame of the person, objects don't fall or seem to have weight—yet gravity does not disappear. It is still acting on the object, whose

FIGURE 5–27 Experiencing weightlessness on Earth.

(a) (b) (c)

weight is still mg. The objects seem weightless only because the elevator is in free fall, and there is no contact force to make us feel the weight.

The "weightlessness" experienced by people in a satellite orbit close to the Earth is the same apparent weightlessness experienced in a freely falling elevator. It may seem strange, at first, to think of a satellite as freely falling. But a satellite is indeed falling toward the Earth, as was shown in Fig. 5–25. The force of gravity causes it to "fall" out of its natural straight-line path. The acceleration of the satellite must be the acceleration due to gravity at that point, since the only force acting on it is gravity. Thus, although the force of gravity acts on objects within the satellite, the objects experience an apparent weightlessness because they, and the satellite, are all accelerating as in free fall.

"Weightlessness" in a satellite

Figure 5–27 shows some examples of "free fall," or apparent weightlessness, experienced by people on Earth for brief moments.

A different situation occurs when a spacecraft is out in space far from the Earth, the Moon, and other attracting objects. The force of gravity due to the Earth and other heavenly bodies will then be quite small because of the distances involved, and people in such a spacecraft will experience real weightlessness.

* 5–9 Kepler's Laws and Newton's Synthesis

More than a half century before Newton proposed his three laws of motion and his law of universal gravitation, the German astronomer Johannes Kepler (1571–1630) had worked out a detailed description of the motion of the planets about the Sun: three empirical findings that we now refer to as **Kepler's laws of planetary motion**. They are summarized as follows, with additional explanation in Figs. 5–28 and 5–29.

Kepler's first law: The path of each planet about the Sun is an ellipse with the Sun at one focus (Fig. 5–28).

Kepler's second law: Each planet moves so that an imaginary line drawn from the Sun to the planet sweeps out equal areas in equal periods of time (Fig. 5–29).

Kepler's third law: The ratio of the squares of the periods T of any two planets revolving about the Sun is equal to the ratio of the cubes of their mean distances s from the Sun: $(T_1/T_2)^2 = (s_1/s_2)^3$. [Actually, s is the semimajor axis, defined as half the long (major) axis of the orbit, as shown in Fig. 5–28. We can also call it the mean distance of the planet from the Sun.] Present-day data are given in Table 5–2: see the last column.

Kepler arrived at his laws through careful analysis of experimental data. Fifty years later, Newton was able to show that Kepler's laws could be derived mathematically from the law of universal gravitation and the laws of motion. Newton also showed that for any reasonable form for the gravitational force law, only one that depends on the inverse square of the distance is fully consistent with Kepler's laws. He thus used Kepler's laws as evidence in favor of his law of universal gravitation, Eq. 5–4.

TABLE 5–2 Planetary Data Applied to Kepler's Third Law

Planet	Mean Distance from Sun, s (10^6 km)	Period, T (Earth years)	s^3/T^2 (10^{24} km^3/y^2)
Mercury	57.9	0.241	3.34
Venus	108.2	0.615	3.35
Earth	149.6	1.0	3.35
Mars	227.9	1.88	3.35
Jupiter	778.3	11.86	3.35
Saturn	1427	29.5	3.34
Uranus	2870	84.0	3.35
Neptune	4497	165	3.34
Pluto	5900	248	3.34

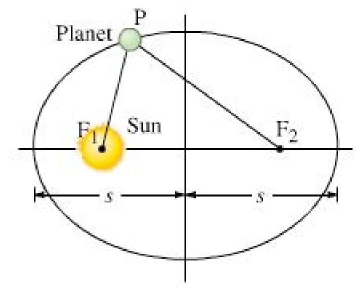
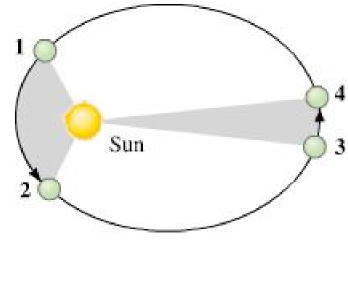


FIGURE 5–28 (a) *Kepler's first law*. An ellipse is a closed curve such that the sum of the distances from any point P on the curve to two fixed points (called the foci, F_1 and F_2) remains constant. That is, the sum of the distances, $F_1P + F_2P$, is the same for all points on the curve. A circle is a special case of an ellipse in which the two foci coincide, at the center of the circle.

FIGURE 5–29 *Kepler's second law*. The two shaded regions have equal areas. The planet moves from point 1 to point 2 in the same time as it takes to move from point 3 to point 4. Planets move fastest in that part of their orbit where they are closest to the Sun. Exaggerated scale.



Derivation of Kepler's third law

We will derive Kepler's third law for the special case of a circular orbit. (Most planetary orbits are close to a circle.) First, we write Newton's second law of motion, $\Sigma F = ma$. For F we use the gravitational force (Eq. 5–4) between the Sun and a planet of mass m_1, and for a the centripetal acceleration, v^2/r. We assume the mass of the Sun, M_S, is much greater than the mass of its planets. Then

$$\Sigma F = ma$$

$$G\frac{m_1 M_S}{r_1^2} = m_1 \frac{v_1^2}{r_1}.$$

Here r_1 is the distance of one planet from the Sun, and v_1 is its average speed in orbit; M_S is the mass of the Sun, since it is the gravitational attraction of the Sun that keeps each planet in its orbit. The period T_1 of the planet is the time required for one complete orbit, a distance equal to the circumference of its orbit, $2\pi r_1$. Thus

$$v_1 = \frac{2\pi r_1}{T_1}.$$

We substitute this formula for v_1 into the equation above:

$$G\frac{m_1 M_S}{r_1^2} = m_1 \frac{4\pi^2 r_1}{T_1^2}.$$

We rearrange this to get

$$\frac{T_1^2}{r_1^3} = \frac{4\pi^2}{GM_S}. \tag{5–6a}$$

We derived this for planet 1 (say, Mars). The same derivation would apply for a second planet (say, Saturn) orbiting the Sun,

$$\frac{T_2^2}{r_2^3} = \frac{4\pi^2}{GM_S},$$

where T_2 and r_2 are the period and orbit radius, respectively, for the second planet. Since the right sides of the two previous equations are equal, we have $T_1^2/r_1^3 = T_2^2/r_2^3$ or, rearranging,

Kepler's third law

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3, \tag{5–6b}$$

which is Kepler's third law.

The derivations of Eqs. 5–6a and 5–6b (Kepler's third law) compared two planets revolving around the Sun; but they are general enough to be applied to other systems. For example, we could apply Eq. 5–6a to our Moon revolving around Earth (then M_S would be M_E, the mass of the Earth). Or we could apply Eq. 5–6b to compare two moons revolving around Jupiter. But Kepler's third law applies only to objects orbiting the same attracting center. Do not use Eq. 5–6b to compare, say, the Moon's orbit around the Earth to the orbit of Mars around the Sun because they depend on different attracting centers.

CAUTION
Compare orbits of objects only around the same center

In the following Examples, we assume the orbits are circles, although it is not quite true in general.

EXAMPLE 5–15 **Where is Mars?** Mars' period (its "year") was noted by Kepler to be about 687 days (Earth days), which is (687 d/365 d) = 1.88 yr. Determine the distance of Mars from the Sun using the Earth as a reference.

APPROACH We know the periods of Earth and Mars, and the distance from the Sun to Earth. We can use Kepler's third law to obtain the distance from the Sun to Mars.

SOLUTION The period of the Earth is $T_E = 1$ yr, and the distance of Earth from the Sun is $r_{ES} = 1.50 \times 10^{11}$ m. From Kepler's third law (Eq. 5–6b):

$$\frac{r_{MS}}{r_{ES}} = \left(\frac{T_M}{T_E}\right)^{\frac{2}{3}} = \left(\frac{1.88\text{ yr}}{1\text{ yr}}\right)^{\frac{2}{3}} = 1.52.$$

So Mars is 1.52 times the Earth's distance from the Sun, or 2.28×10^{11} m.

EXAMPLE 5–16 **The Sun's mass determined.** Determine the mass of the Sun given the Earth's distance from the Sun as $r_{ES} = 1.5 \times 10^{11}$ m.

PHYSICS APPLIED

Determining the Sun's mass

APPROACH Equation 5–6a relates the mass of the Sun M_S to the period and distance of any planet. We use the Earth.

SOLUTION The Earth's period is $T_E = 1\text{ yr} = (365\tfrac{1}{4}\text{ d})(24\text{ h/d})(3600\text{ s/h}) = 3.16 \times 10^7$ s. We solve Eq. 5–6a for M_S:

$$M_S = \frac{4\pi^2 r_{ES}^3}{G T_E^2} = \frac{4\pi^2(1.5 \times 10^{11}\text{ m})^3}{(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(3.16 \times 10^7\text{ s})^2} = 2.0 \times 10^{30}\text{ kg}.$$

Accurate measurements on the orbits of the planets indicated that they did not precisely follow Kepler's laws. For example, slight deviations from perfectly elliptical orbits were observed. Newton was aware that this was to be expected because any planet would be attracted gravitationally not only by the Sun but also (to a much lesser extent) by the other planets. Such deviations, or **perturbations**, in the orbit of Saturn were a hint that helped Newton formulate the law of universal gravitation, that all objects attract gravitationally. Observation of other perturbations later led to the discovery of Neptune and Pluto. Deviations in the orbit of Uranus, for example, could not all be accounted for by perturbations due to the other known planets. Careful calculation in the nineteenth century indicated that these deviations could be accounted for if another planet existed farther out in the solar system. The position of this planet was predicted from the deviations in the orbit of Uranus, and telescopes focused on that region of the sky quickly found it; the new planet was called Neptune. Similar but much smaller perturbations of Neptune's orbit led to the discovery of Pluto in 1930.

Perturbations and discovery of planets

Starting in the mid–1990s, planets revolving about distant stars (Fig. 5–30) were inferred from the regular "wobble" of each star due to the gravitational attraction of the revolving planet(s).

Planets around other stars

The development by Newton of the law of universal gravitation and the three laws of motion was a major intellectual achievement: with these laws, he was able to describe the motion of objects on Earth and in the heavens. The motions of heavenly bodies and objects on Earth were seen to follow the same laws. For this reason, and also because Newton integrated the results of earlier scientists into his system, we sometimes speak of *Newton's synthesis*.

Newton's synthesis

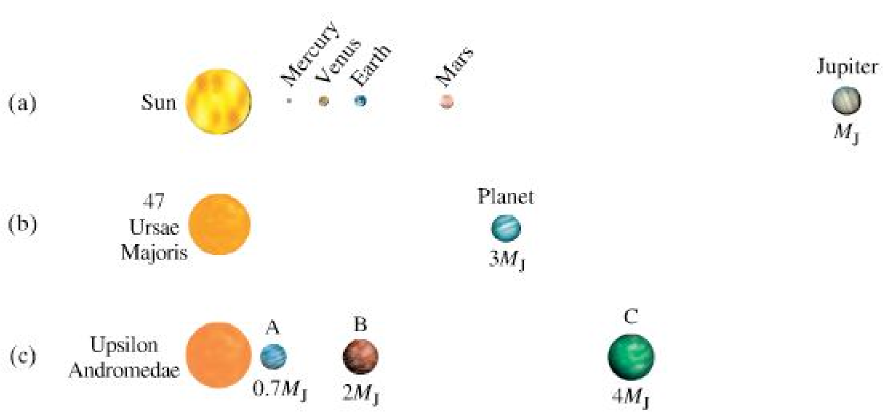


FIGURE 5–30 Our solar system (a) is compared to recently discovered planets orbiting (b) the star 47 Ursae Majoris and (c) the star Upsilon Andromedae with at least three planets. M_J is the mass of Jupiter. (Sizes not to scale.)

Causality

The laws formulated by Newton are referred to as **causal laws**. By **causality** we mean the idea that one occurrence can cause another. When a rock strikes a window, we infer that the rock *causes* the window to break. This idea of "cause and effect" relates to Newton's laws: the acceleration of an object was seen to be *caused* by the net force acting on it.

5–10 Types of Forces in Nature

We have already discussed that Newton's law of universal gravitation, Eq. 5–4, describes how a particular type of force—gravity—depends on the masses of the objects involved and the distance between them. Newton's second law, $\Sigma \vec{\mathbf{F}} = m\vec{\mathbf{a}}$, on the other hand, tells how an object will accelerate due to *any* type of force. But what are the types of forces that occur in nature besides gravity?

In the twentieth century, physicists came to recognize four fundamental forces in nature: (1) the gravitational force; (2) the electromagnetic force (we shall see later that electric and magnetic forces are intimately related); (3) the strong nuclear force; and (4) the weak nuclear force. In this Chapter, we discussed the gravitational force in detail. The nature of the electromagnetic force will be discussed in Chapters 16 to 22. The strong and weak nuclear forces, which are discussed in Chapters 30 to 32, operate at the level of the atomic nucleus; although they manifest themselves in such phenomena as radioactivity and nuclear energy, they are much less obvious in our daily lives.

Electroweak and GUT

Physicists have been working on theories that would unify these four forces—that is, to consider some or all of these forces as different manifestations of the same basic force. So far, the electromagnetic and weak nuclear forces have been theoretically united to form *electroweak* theory, in which the electromagnetic and weak forces are seen as two different manifestations of a single *electroweak force*. Attempts to further unify the forces, such as in *grand unified theories* (GUT), are hot research topics today.

Everyday forces are gravity and electromagnetic

But where do everyday forces fit into this scheme? Ordinary forces, other than gravity, such as pushes, pulls, and other contact forces like the normal force and friction, are today considered to be due to the electromagnetic force acting at the atomic level. For example, the force your fingers exert on a pencil is the result of electrical repulsion between the outer electrons of the atoms of your finger and those of the pencil.

Summary

An object moving in a circle of radius r with constant speed v is said to be in **uniform circular motion**. It has a **centripetal acceleration** a_R that is directed radially toward the center of the circle (also called **radial acceleration**), and has magnitude

$$a_R = \frac{v^2}{r}. \qquad (5\text{–}1)$$

The direction of the velocity vector and that of the acceleration $\vec{\mathbf{a}}_R$ are continually changing in direction, but are perpendicular to each other at each moment.

A force is needed to keep a particle revolving in a circle, and the direction of this force is toward the center of the circle. This force may be due to gravity, to tension in a cord, to a component of the normal force, to another type of force, or to a combination of forces.

[*When the speed of circular motion is not constant, the acceleration has two components, tangential as well as centripetal.]

Newton's **law of universal gravitation** states that every particle in the universe attracts every other particle with a force proportional to the product of their masses and inversely proportional to the square of the distance between them:

$$F = G\frac{m_1 m_2}{r^2}. \qquad (5\text{–}4)$$

The direction of this force is along the line joining the two particles. It is this gravitational force that keeps the Moon revolving around the Earth, and the planets revolving around the Sun.

Satellites revolving around the Earth are acted on by gravity, but "stay up" because of their high tangential speed.

[*Newton's three laws of motion, plus his law of universal gravitation, constituted a wide-ranging theory of the universe. With them, motion of objects on Earth and in the heavens could be accurately described. And they provided a theoretical base for **Kepler's laws** of planetary motion.]

The four fundamental forces in nature are (1) the gravitational force, (2) electromagnetic force, (3) strong nuclear force, and (4) weak nuclear force. The first two fundamental forces are responsible for nearly all "everyday" forces.

Questions

1. Sometimes people say that water is removed from clothes in a spin dryer by centrifugal force throwing the water outward. What is wrong with this statement?

2. Will the acceleration of a car be the same when the car travels around a sharp curve at a constant 60 km/h as when it travels around a gentle curve at the same speed? Explain.

3. Suppose a car moves at constant speed along a hilly road. Where does the car exert the greatest and least forces on the road: (*a*) at the top of a hill, (*b*) at a dip between two hills, (*c*) on a level stretch near the bottom of a hill?

4. Describe all the forces acting on a child riding a horse on a merry-go-round. Which of these forces provides the centripetal acceleration of the child?

5. A bucket of water can be whirled in a vertical circle without the water spilling out, even at the top of the circle when the bucket is upside down. Explain.

6. How many "accelerators" do you have in your car? There are at least three controls in the car which can be used to cause the car to accelerate. What are they? What accelerations do they produce?

7. A child on a sled comes flying over the crest of a small hill, as shown in Fig. 5–31. His sled does not leave the ground (he does not achieve "air"), but he feels the normal force between his chest and the sled decrease as he goes over the hill. Explain this decrease using Newton's second law.

FIGURE 5–31 Question 7.

8. Why do bicycle riders lean inward when rounding a curve at high speed?

9. Why do airplanes bank when they turn? How would you compute the banking angle given its speed and radius of the turn?

10. A girl is whirling a ball on a string around her head in a horizontal plane. She wants to let go at precisely the right time so that the ball will hit a target on the other side of the yard. When should she let go of the string?

11. Does an apple exert a gravitational force on the Earth? If so, how large a force? Consider an apple (*a*) attached to a tree, and (*b*) falling.

12. If the Earth's mass were double what it is, in what ways would the Moon's orbit be different?

13. Which pulls harder gravitationally, the Earth on the Moon, or the Moon on the Earth? Which accelerates more?

14. The Sun's gravitational pull on the Earth is much larger than the Moon's. Yet the Moon's is mainly responsible for the tides. Explain. [*Hint*: Consider the difference in gravitational pull from one side of the Earth to the other.]

15. Will an object weigh more at the equator or at the poles? What two effects are at work? Do they oppose each other?

16. The gravitational force on the Moon due to the Earth is only about half the force on the Moon due to the Sun. Why isn't the Moon pulled away from the Earth?

17. Is the centripetal acceleration of Mars in its orbit around the Sun larger or smaller than the centripetal acceleration of the Earth?

18. Would it require less speed to launch a satellite (*a*) toward the east or (*b*) toward the west? Consider the Earth's rotation direction.

19. When will your apparent weight be the greatest, as measured by a scale in a moving elevator: when the elevator (*a*) accelerates downward, (*b*) accelerates upward, (*c*) is in free fall, (*d*) moves upward at constant speed? In which case would your weight be the least? When would it be the same as when you are on the ground?

20. What keeps a satellite up in its orbit around the Earth?

21. Astronauts who spend long periods in outer space could be adversely affected by weightlessness. One way to simulate gravity is to shape the spaceship like a cylindrical shell that rotates, with the astronauts walking on the inside surface (Fig. 5–32). Explain how this simulates gravity. Consider (*a*) how objects fall, (*b*) the force we feel on our feet, and (*c*) any other aspects of gravity you can think of.

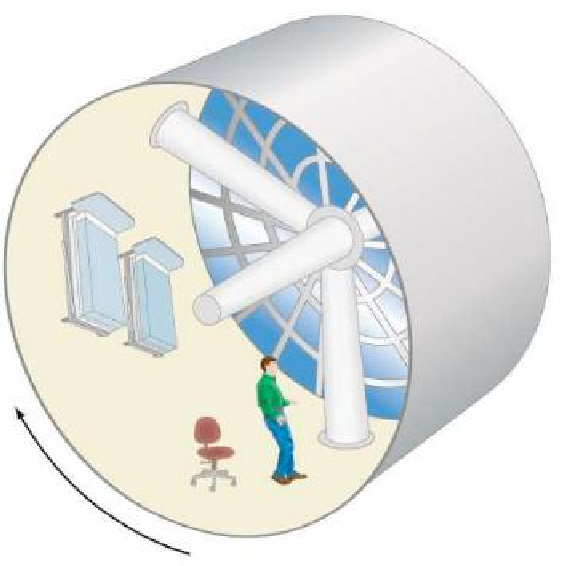


FIGURE 5–32 Question 21 and Problem 45.

22. Explain how a runner experiences "free fall" or "apparent weightlessness" between steps.

* 23. The Earth moves faster in its orbit around the Sun in January than in July. Is the Earth closer to the Sun in January, or in July? Explain. [*Note*: This is not much of a factor in producing the seasons—the main factor is the tilt of the Earth's axis relative to the plane of its orbit.]

* 24. The mass of Pluto was not known until it was discovered to have a moon. Explain how this discovery enabled an estimate of Pluto's mass.

Problems

5–1 to 5–3 Uniform Circular Motion; Highway Curves

1. (I) A child sitting 1.10 m from the center of a merry-go-round moves with a speed of 1.25 m/s. Calculate (*a*) the centripetal acceleration of the child, and (*b*) the net horizontal force exerted on the child (mass = 25.0 kg).

2. (I) A jet plane traveling 1890 km/h (525 m/s) pulls out of a dive by moving in an arc of radius 6.00 km. What is the plane's acceleration in *g*'s?

3. (I) Calculate the centripetal acceleration of the Earth in its orbit around the Sun, and the net force exerted on the Earth. What exerts this force on the Earth? Assume that the Earth's orbit is a circle of radius 1.50×10^{11} m. [*Hint*: see the Tables inside the front cover of this book.]

4. (I) A horizontal force of 210 N is exerted on a 2.0-kg discus as it rotates uniformly in a horizontal circle (at arm's length) of radius 0.90 m. Calculate the speed of the discus.

5. (II) Suppose the space shuttle is in orbit 400 km from the Earth's surface, and circles the Earth about once every 90 minutes. Find the centripetal acceleration of the space shuttle in its orbit. Express your answer in terms of *g*, the gravitational acceleration at the Earth's surface.

6. (II) What is the magnitude of the acceleration of a speck of clay on the edge of a potter's wheel turning at 45 rpm (revolutions per minute) if the wheel's diameter is 32 cm?

7. (II) A ball on the end of a string is revolved at a uniform rate in a vertical circle of radius 72.0 cm, as shown in Fig. 5–33. If its speed is 4.00 m/s and its mass is 0.300 kg, calculate the tension in the string when the ball is (*a*) at the top of its path, and (*b*) at the bottom of its path.

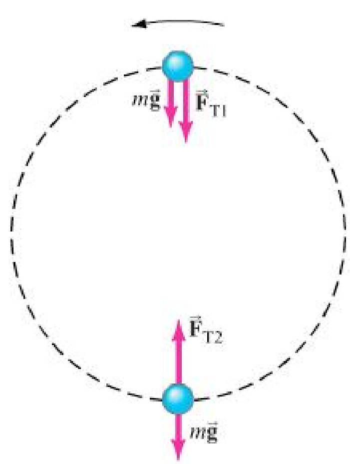


FIGURE 5–33 Problem 7.

8. (II) A 0.45-kg ball, attached to the end of a horizontal cord, is rotated in a circle of radius 1.3 m on a frictionless horizontal surface. If the cord will break when the tension in it exceeds 75 N, what is the maximum speed the ball can have?

9. (II) What is the maximum speed with which a 1050-kg car can round a turn of radius 77 m on a flat road if the coefficient of static friction between tires and road is 0.80? Is this result independent of the mass of the car?

10. (II) How large must the coefficient of static friction be between the tires and the road if a car is to round a level curve of radius 85 m at a speed of 95 km/h?

11. (II) A device for training astronauts and jet fighter pilots is designed to rotate a trainee in a horizontal circle of radius 12.0 m. If the force felt by the trainee on her back is 7.85 times her own weight, how fast is she rotating? Express your answer in both m/s and rev/s.

12. (II) A coin is placed 11.0 cm from the axis of a rotating turntable of variable speed. When the speed of the turntable is slowly increased, the coin remains fixed on the turntable until a rate of 36 rpm is reached and the coin slides off. What is the coefficient of static friction between the coin and the turntable?

13. (II) At what minimum speed must a roller coaster be traveling when upside down at the top of a circle (Fig. 5–34) so that the passengers will not fall out? Assume a radius of curvature of 7.4 m.

FIGURE 5–34 Problem 13.

14. (II) A sports car of mass 950 kg (including the driver) crosses the rounded top of a hill (radius = 95 m) at 22 m/s. Determine (*a*) the normal force exerted by the road on the car, (*b*) the normal force exerted by the car on the 72-kg driver, and (*c*) the car speed at which the normal force on the driver equals zero.

15. (II) How many revolutions per minute would a 15-m-diameter Ferris wheel need to make for the passengers to feel "weightless" at the topmost point?

16. (II) A bucket of mass 2.00 kg is whirled in a vertical circle of radius 1.10 m. At the lowest point of its motion the tension in the rope supporting the bucket is 25.0 N. (*a*) Find the speed of the bucket. (*b*) How fast must the bucket move at the top of the circle so that the rope does not go slack?

17. (II) How fast (in rpm) must a centrifuge rotate if a particle 9.00 cm from the axis of rotation is to experience an acceleration of 115,000 *g*'s?

18. (II) In a "Rotor-ride" at a carnival, people are rotated in a cylindrically walled "room." (See Fig. 5–35.) The room radius is 4.6 m, and the rotation frequency is 0.50 revolutions per second when the floor drops out. What is the minimum coefficient of static friction so that the people will not slip down? People on this ride say they were "pressed against the wall." Is there really an outward force pressing them against the wall? If so, what is its source? If not, what is the proper description of their situation (besides "scary")? [*Hint*: First draw the free-body diagram for a person.]

FIGURE 5–35 Problem 18.

19. (II) A flat puck (mass M) is rotated in a circle on a frictionless air-hockey tabletop, and is held in this orbit by a light cord connected to a dangling block (mass m) through a central hole as shown in Fig. 5–36. Show that the speed of the puck is given by

$$v = \sqrt{\frac{mgR}{M}}.$$

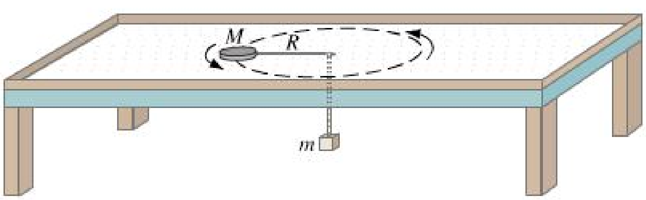


FIGURE 5–36 Problem 19.

20. (II) Redo Example 5–3, precisely this time, by not ignoring the weight of the ball which revolves on a string 0.600 m long. In particular, find the magnitude of $\vec{\mathbf{F}}_T$, and the angle it makes with the horizontal. [*Hint*: Set the horizontal component of $\vec{\mathbf{F}}_T$ equal to ma_R; also, since there is no vertical motion, what can you say about the vertical component of $\vec{\mathbf{F}}_T$?]

21. (III) If a curve with a radius of 88 m is perfectly banked for a car traveling 75 km/h, what must be the coefficient of static friction for a car not to skid when traveling at 95 km/h?

22. (III) A 1200-kg car rounds a curve of radius 67 m banked at an angle of 12°. If the car is traveling at 95 km/h, will a friction force be required? If so, how much and in what direction?

23. (III) Two blocks, of masses m_1 and m_2, are connected to each other and to a central post by cords as shown in Fig. 5–37. They rotate about the post at a frequency f (revolutions per second) on a frictionless horizontal surface at distances r_1 and r_2 from the post. Derive an algebraic expression for the tension in each segment of the cord.

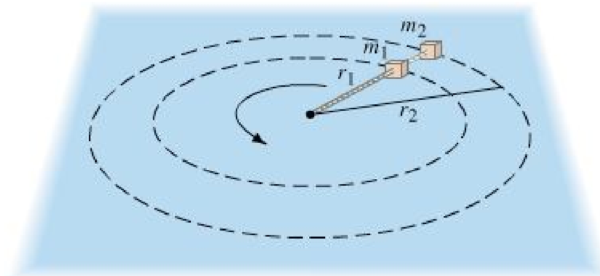


FIGURE 5–37 Problem 23.

24. (III) A pilot performs an evasive maneuver by diving vertically at 310 m/s. If he can withstand an acceleration of 9.0 *g*'s without blacking out, at what altitude must he begin to pull out of the dive to avoid crashing into the sea?

* 5–4 Nonuniform Circular Motion

* 25. (I) Determine the tangential and centripetal components of the net force exerted on the car (by the ground) in Example 5–8 when its speed is 15 m/s. The car's mass is 1100 kg.

* 26. (II) A car at the Indianapolis 500 accelerates uniformly from the pit area, going from rest to 320 km/h in a semicircular arc with a radius of 220 m. Determine the tangential and radial acceleration of the car when it is halfway through the turn, assuming constant tangential acceleration. If the curve were flat, what would the coefficient of static friction have to be between the tires and the road to provide this acceleration with no slipping or skidding?

* 27. (III) A particle revolves in a horizontal circle of radius 2.90 m. At a particular instant, its acceleration is 1.05 m/s^2, in a direction that makes an angle of 32.0° to its direction of motion. Determine its speed (*a*) at this moment, and (*b*) 2.00 s later, assuming constant tangential acceleration.

5–6 and 5–7 Law of Universal Gravitation

28. (I) Calculate the force of Earth's gravity on a spacecraft 12,800 km (2 Earth radii) above the Earth's surface if its mass is 1350 kg.

29. (I) At the surface of a certain planet, the gravitational acceleration g has a magnitude of 12.0 m/s^2. A 21.0-kg brass ball is transported to this planet. What is (*a*) the mass of the brass ball on the Earth and on the planet, and (*b*) the weight of the brass ball on the Earth and on the planet?

30. (II) Calculate the acceleration due to gravity on the Moon. The Moon's radius is 1.74×10^6 m and its mass is 7.35×10^{22} kg.

31. (II) A hypothetical planet has a radius 1.5 times that of Earth, but has the same mass. What is the acceleration due to gravity near its surface?

32. (II) A hypothetical planet has a mass 1.66 times that of Earth, but the same radius. What is g near its surface?

33. (II) Two objects attract each other gravitationally with a force of 2.5×10^{-10} N when they are 0.25 m apart. Their total mass is 4.0 kg. Find their individual masses.

34. (II) Calculate the effective value of g, the acceleration of gravity, at (*a*) 3200 m, and (*b*) 3200 km, above the Earth's surface.

35. (II) What is the distance from the Earth's center to a point outside the Earth where the gravitational acceleration due to the Earth is $\frac{1}{10}$ of its value at the Earth's surface?

36. (II) A certain neutron star has five times the mass of our Sun packed into a sphere about 10 km in radius. Estimate the surface gravity on this monster.

37. (II) A typical white-dwarf star, which once was an average star like our Sun but is now in the last stage of its evolution, is the size of our Moon but has the mass of our Sun. What is the surface gravity on this star?

38. (II) You are explaining why astronauts feel weightless while orbiting in the space shuttle. Your friends respond that they thought gravity was just a lot weaker up there. Convince them and yourself that it isn't so by calculating the acceleration of gravity 250 km above the Earth's surface in terms of g.

39. (II) Four 9.5-kg spheres are located at the corners of a square of side 0.60 m. Calculate the magnitude and direction of the total gravitational force exerted on one sphere by the other three.

40. (II) Every few hundred years most of the planets line up on the same side of the Sun. Calculate the total force on the Earth due to Venus, Jupiter, and Saturn, assuming all four planets are in a line (Fig. 5–38). The masses are $M_V = 0.815 M_E$, $M_J = 318 M_E$, $M_S = 95.1 M_E$, and their mean distances from the Sun are 108, 150, 778, and 1430 million km, respectively. What fraction of the Sun's force on the Earth is this?

FIGURE 5–38 Problem 40. (Not to scale.)

41. (II) Given that the acceleration of gravity at the surface of Mars is 0.38 of what it is on Earth, and that Mars' radius is 3400 km, determine the mass of Mars.

42. (III) Determine the mass of the Sun using the known value for the period of the Earth and its distance from the Sun. [*Note*: Compare your answer to that obtained using Kepler's laws, Example 5–16.]

5–8 Satellites; Weightlessness

43. (I) Calculate the speed of a satellite moving in a stable circular orbit about the Earth at a height of 3600 km.

44. (I) The space shuttle releases a satellite into a circular orbit 650 km above the Earth. How fast must the shuttle be moving (relative to Earth) when the release occurs?

45. (II) At what rate must a cylindrical spaceship rotate if occupants are to experience simulated gravity of 0.60 g? Assume the spaceship's diameter is 32 m, and give your answer as the time needed for one revolution. (See Question 21, Fig 5–32.)

46. (II) Determine the time it takes for a satellite to orbit the Earth in a circular "near-Earth" orbit. A "near-Earth" orbit is one at a height above the surface of the Earth which is very small compared to the radius of the Earth. Does your result depend on the mass of the satellite?

47. (II) At what horizontal velocity would a satellite have to be launched from the top of Mt. Everest to be placed in a circular orbit around the Earth?

48. (II) During an *Apollo* lunar landing mission, the command module continued to orbit the Moon at an altitude of about 100 km. How long did it take to go around the Moon once?

49. (II) The rings of Saturn are composed of chunks of ice that orbit the planet. The inner radius of the rings is 73,000 km, while the outer radius is 170,000 km. Find the period of an orbiting chunk of ice at the inner radius and the period of a chunk at the outer radius. Compare your numbers with Saturn's mean rotation period of 10 hours and 39 minutes. The mass of Saturn is 5.7×10^{26} kg.

50. (II) A Ferris wheel 24.0 m in diameter rotates once every 15.5 s (see Fig. 5–9). What is the ratio of a person's apparent weight to her real weight (*a*) at the top, and (*b*) at the bottom?

51. (II) What is the apparent weight of a 75-kg astronaut 4200 km from the center of the Earth's Moon in a space vehicle (*a*) moving at constant velocity, and (*b*) accelerating toward the Moon at 2.9 m/s^2? State the "direction" in each case.

52. (II) Suppose that a binary-star system consists of two stars of equal mass. They are observed to be separated by 360 million km and take 5.7 Earth years to orbit about a point midway between them. What is the mass of each?

53. (II) What will a spring scale read for the weight of a 55-kg woman in an elevator that moves (*a*) upward with constant speed of 6.0 m/s, (*b*) downward with constant speed of 6.0 m/s, (*c*) upward with acceleration of 0.33 g, (*d*) downward with acceleration 0.33 g, and (*e*) in free fall?

54. (II) A 17.0-kg monkey hangs from a cord suspended from the ceiling of an elevator. The cord can withstand a tension of 220 N and breaks as the elevator accelerates. What was the elevator's minimum acceleration (magnitude and direction)?

55. (III) (*a*) Show that if a satellite orbits very near the surface of a planet with period T, the density (mass/volume) of the planet is $\rho = m/V = 3\pi/GT^2$. (*b*) Estimate the density of the Earth, given that a satellite near the surface orbits with a period of about 85 min.

* 5–9 Kepler's Laws

* 56. (I) Use Kepler's laws and the period of the Moon (27.4 d) to determine the period of an artificial satellite orbiting very near the Earth's surface.

* 57. (I) The asteroid Icarus, though only a few hundred meters across, orbits the Sun like the planets. Its period is 410 d. What is its mean distance from the Sun?

* **58.** (I) Neptune is an average distance of 4.5×10^9 km from the Sun. Estimate the length of the Neptunian year given that the Earth is 1.50×10^8 km from the Sun on the average.

* **59.** (II) Halley's comet orbits the Sun roughly once every 76 years. It comes very close to the surface of the Sun on its closest approach (Fig. 5–39). Estimate the greatest distance of the comet from the Sun. Is it still "in" the Solar System? What planet's orbit is nearest when it is out there? [*Hint*: The mean distance s in Kepler's third law is half the sum of the nearest and farthest distance from the Sun.]

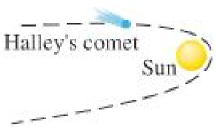


FIGURE 5–39
Problem 59.

* **60.** (II) Our Sun rotates about the center of the Galaxy $(M_G \approx 4 \times 10^{41}\,\text{kg})$ at a distance of about 3×10^4 light-years $(1\,\text{ly} = 3 \times 10^8\,\text{m/s} \times 3.16 \times 10^7\,\text{s/y} \times 1\,\text{y})$. What is the period of our orbital motion about the center of the Galaxy?

* **61.** (II) Table 5–3 gives the mass, period, and mean distance for the four largest moons of Jupiter (those discovered by Galileo in 1609). (*a*) Determine the mass of Jupiter using the data for Io. (*b*) Determine the mass of Jupiter using data for each of the other three moons. Are the results consistent?

TABLE 5–3 Principal Moons of Jupiter

Moon	Mass (kg)	Period (Earth days)	Mean distance from Jupiter (km)
Io	8.9×10^{22}	1.77	422×10^3
Europa	4.9×10^{22}	3.55	671×10^3
Ganymede	15×10^{22}	7.16	1070×10^3
Callisto	11×10^{22}	16.7	1883×10^3

* **62.** (II) Determine the mass of the Earth from the known period and distance of the Moon.

* **63.** (II) Determine the mean distance from Jupiter for each of Jupiter's moons, using Kepler's third law. Use the distance of Io and the periods given in Table 5–3. Compare to the values in the Table.

* **64.** (II) The asteroid belt between Mars and Jupiter consists of many fragments (which some space scientists think came from a planet that once orbited the Sun but was destroyed). (*a*) If the center of mass of the asteroid belt (where the planet would have been) is about three times farther from the Sun than the Earth is, how long would it have taken this hypothetical planet to orbit the Sun? (*b*) Can we use these data to deduce the mass of this planet?

* **65.** (III) A science-fiction tale describes an artificial "planet" in the form of a band completely encircling a sun (Fig. 5–40). The inhabitants live on the inside surface (where it is always noon). Imagine that this sun is exactly like our own, that the distance to the band is the same as the Earth–Sun distance (to make the climate temperate), and that the ring rotates quickly enough to produce an apparent gravity of g as on Earth. What will be the period of revolution, this planet's year, in Earth days?

FIGURE 5–40
Problem 65.

General Problems

66. Tarzan plans to cross a gorge by swinging in an arc from a hanging vine (Fig. 5–41). If his arms are capable of exerting a force of 1400 N on the vine, what is the maximum speed he can tolerate at the lowest point of his swing? His mass is 80 kg, and the vine is 5.5 m long.

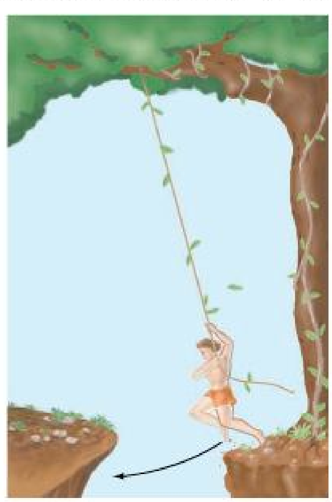


FIGURE 5–41
Problem 66.

67. How far above the Earth's surface will the acceleration of gravity be half what it is on the surface?

68. On an ice rink, two skaters of equal mass grab hands and spin in a mutual circle once every 2.5 s. If we assume their arms are each 0.80 m long and their individual masses are 60.0 kg, how hard are they pulling on one another?

69. Because the Earth rotates once per day, the apparent acceleration of gravity at the equator is slightly less than it would be if the Earth didn't rotate. Estimate the magnitude of this effect. What fraction of g is this?

70. At what distance from the Earth will a spacecraft traveling directly from the Earth to the Moon experience zero net force because the Earth and Moon pull with equal and opposite forces?

71. You know your mass is 65 kg, but when you stand on a bathroom scale in an elevator, it says your mass is 82 kg. What is the acceleration of the elevator, and in which direction?

72. A projected space station consists of a circular tube that will rotate about its center (like a tubular bicycle tire) (Fig. 5–42). The circle formed by the tube has a diameter of about 1.1 km. What must be the rotation speed (revolutions per day) if an effect equal to gravity at the surface of the Earth (1.0 g) is to be felt?

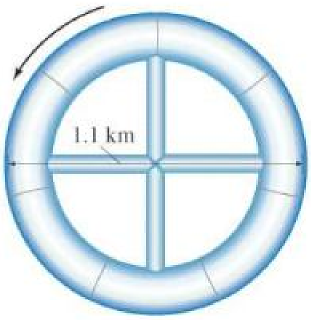


FIGURE 5–42 Problem 72.

73. A jet pilot takes his aircraft in a vertical loop (Fig. 5–43). (*a*) If the jet is moving at a speed of 1300 km/h at the lowest point of the loop, determine the minimum radius of the circle so that the centripetal acceleration at the lowest point does not exceed 6.0 g's. (*b*) Calculate the 78-kg pilot's effective weight (the force with which the seat pushes up on him) at the bottom of the circle, and (*c*) at the top of the circle (assume the same speed).

FIGURE 5–43 Problem 73.

74. Derive a formula for the mass of a planet in terms of its radius r, the acceleration due to gravity at its surface g_P, and the gravitational constant G.

75. A plumb bob (a mass m hanging on a string) is deflected from the vertical by an angle θ due to a massive mountain nearby (Fig. 5–44). (*a*) Find an approximate formula for θ in terms of the mass of the mountain, m_M, the distance to its center, D_M, and the radius and mass of the Earth. (*b*) Make a rough estimate of the mass of Mt. Everest, assuming it has the shape of a cone 4000 m high and base of diameter 4000 m. Assume its mass per unit volume is 3000 kg per m^3. (*c*) Estimate the angle θ of the plumb bob if it is 5 km from the center of Mt. Everest.

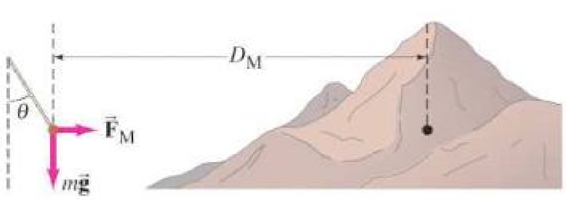


FIGURE 5–44 Problem 75.

76. A curve of radius 67 m is banked for a design speed of 95 km/h. If the coefficient of static friction is 0.30 (wet pavement), at what range of speeds can a car safely handle the curve?

77. How long would a day be if the Earth were rotating so fast that objects at the equator were apparently weightless?

78. Two equal-mass stars maintain a constant distance apart of 8.0×10^{10} m and rotate about a point midway between them at a rate of one revolution every 12.6 yr. (*a*) Why don't the two stars crash into one another due to the gravitational force between them? (*b*) What must be the mass of each star?

79. A train traveling at a constant speed rounds a curve of radius 235 m. A lamp suspended from the ceiling swings out to an angle of 17.5° throughout the curve. What is the speed of the train?

80. Jupiter is about 320 times as massive as the Earth. Thus, it has been claimed that a person would be crushed by the force of gravity on a planet the size of Jupiter since people can't survive more than a few g's. Calculate the number of g's a person would experience at the equator of such a planet. Use the following data for Jupiter: mass $= 1.9 \times 10^{27}$ kg, equatorial radius $= 7.1 \times 10^4$ km, rotation period = 9 hr 55 min. Take the centripetal acceleration into account.

81. Astronomers using the Hubble Space Telescope deduced the presence of an extremely massive core in the distant galaxy M87, so dense that it could be a black hole (from which no light escapes). They did this by measuring the speed of gas clouds orbiting the core to be 780 km/s at a distance of 60 light-years $(5.7 \times 10^{17}\,\text{m})$ from the core. Deduce the mass of the core, and compare it to the mass of our Sun.

82. A car maintains a constant speed v as it traverses the hill and valley shown in Fig. 5–45. Both the hill and valley have a radius of curvature R. (*a*) How do the normal forces acting on the car at A, B, and C compare? (Which is largest? Smallest?) Explain. (*b*) Where would the driver feel heaviest? Lightest? Explain. (*c*) How fast can the car go without losing contact with the road at A?

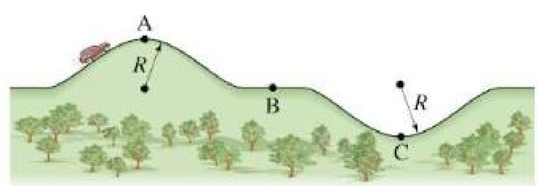


FIGURE 5–45 Problem 82.

83. The Navstar Global Positioning System (GPS) utilizes a group of 24 satellites orbiting the Earth. Using "triangulation" and signals transmitted by these satellites, the position of a receiver on the Earth can be determined to within an accuracy of a few centimeters. The satellite orbits are distributed evenly around the Earth, with four satellites in each of six orbits, allowing continuous navigational "fixes." The satellites orbit at an altitude of approximately 11,000 nautical miles [1 nautical mile = 1.852 km = 6076 ft]. (*a*) Determine the speed of each satellite. (*b*) Determine the period of each satellite.

84. The *Near Earth Asteroid Rendezvous* (*NEAR*), after traveling 2.1 billion km, is meant to orbit the asteroid Eros at a height of about 15 km. Eros is roughly 40 km × 6 km × 6 km. Assume Eros has a density (mass/volume) of about $2.3 \times 10^3\,\mathrm{kg/m^3}$. (*a*) What will be the period of *NEAR* as it orbits Eros? (*b*) If Eros were a sphere with the same mass and density, what would its radius be? (*c*) What would g be at the surface of this spherical Eros?

85. You are an astronaut in the space shuttle pursuing a satellite in need of repair. You are in a circular orbit of the same radius as the satellite (400 km above the Earth), but 25 km behind it. (*a*) How long will it take to overtake the satellite if you reduce your orbital radius by 1.0 km? (*b*) By how much must you reduce your orbital radius to catch up in 7.0 hours?

* **86.** The comet Hale-Bopp has a period of 3000 years. (*a*) What is its mean distance from the Sun? (*b*) At its closest approach, the comet is about 1 A.U. from the Sun (1 A.U. = distance from Earth to the Sun). What is the farthest distance? (*c*) What is the ratio of the speed at the closest point to the speed at the farthest point? [*Hint*: Use Kepler's second law and estimate areas by a triangle (as in Fig. 5–29, but smaller distance travelled; see also Hint for Problem 59.]

87. Estimate what the value of G would need to be if you could actually "feel" yourself gravitationally attracted to someone near you. Make reasonable assumptions, like $F \approx 1\,\mathrm{N}$.

* **88.** The Sun rotates around the center of the Milky Way Galaxy (Fig. 5–46) at a distance of about 30,000 light-years from the center $(1\,\mathrm{ly} = 9.5 \times 10^{15}\,\mathrm{m})$. If it takes about 200 million years to make one rotation, estimate the mass of our Galaxy. Assume that the mass distribution of our Galaxy is concentrated mostly in a central uniform sphere. If all the stars had about the mass of our Sun $(2 \times 10^{30}\,\mathrm{kg})$, how many stars would there be in our Galaxy?

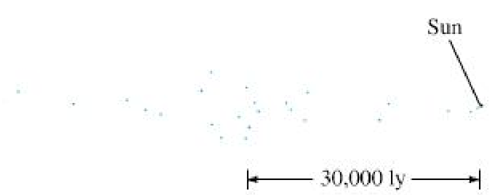


FIGURE 5–46 Problem 88. Edge-on view of our Galaxy.

89. Four 1.0-kg masses are located at the corners of a square 0.50 m on each side. Find the magnitude and direction of the gravitational force on a fifth 1.0-kg mass placed at the midpoint of the bottom side of the square.

90. A satellite of mass 5500 kg orbits the Earth (mass = $6.0 \times 10^{24}\,\mathrm{kg}$) and has a period of 6200 s. Find (*a*) the magnitude of the Earth's gravitational force on the satellite, (*b*) the altitude of the satellite.

91. What is the acceleration experienced by the tip of the 1.5-cm-long sweep second hand on your wrist watch?

92. While fishing, you get bored and start to swing a sinker weight around in a circle below you on a 0.25-m piece of fishing line. The weight makes a complete circle every 0.50 s. What is the angle that the fishing line makes with the vertical? [*Hint*: See Fig. 5–10.]

93. A circular curve of radius R in a new highway is designed so that a car traveling at speed v_0 can negotiate the turn safely on glare ice (zero friction). If a car travels too slowly, then it will slip toward the center of the circle. If it travels too fast, then it will slip away from the center of the circle. If the coefficient of static friction increases, a car can stay on the road while traveling at any speed within a range from v_{min} to v_{max}. Derive formulas for v_{min} and v_{max} as functions of μ_s, v_0, and R.

94. Amtrak's high speed train, the *Acela*, utilizes tilt of the cars when negotiating curves. The angle of tilt is adjusted so that the main force exerted on the passengers, to provide the centripetal acceleration, is the normal force. The passengers experience less friction force against the seat, thus feeling more comfortable. Consider an *Acela* train that rounds a curve with a radius of 620 m at a speed of 160 km/h (approximately 100 mi/h). (*a*) Calculate the friction force needed on a train passenger of mass 75 kg if the track is not banked and the train does not tilt. (*b*) Calculate the friction force on the passenger if the train tilts to its maximum tilt of 8.0° toward the center of the curve.

Answers to Exercises

A: A factor of two (doubles).
B: Speed is independent of the mass of the clothes.
C: (*a*).
D: No.
E: Yes.
F: (*a*) No change; (*b*) four times larger.
G: (*b*).