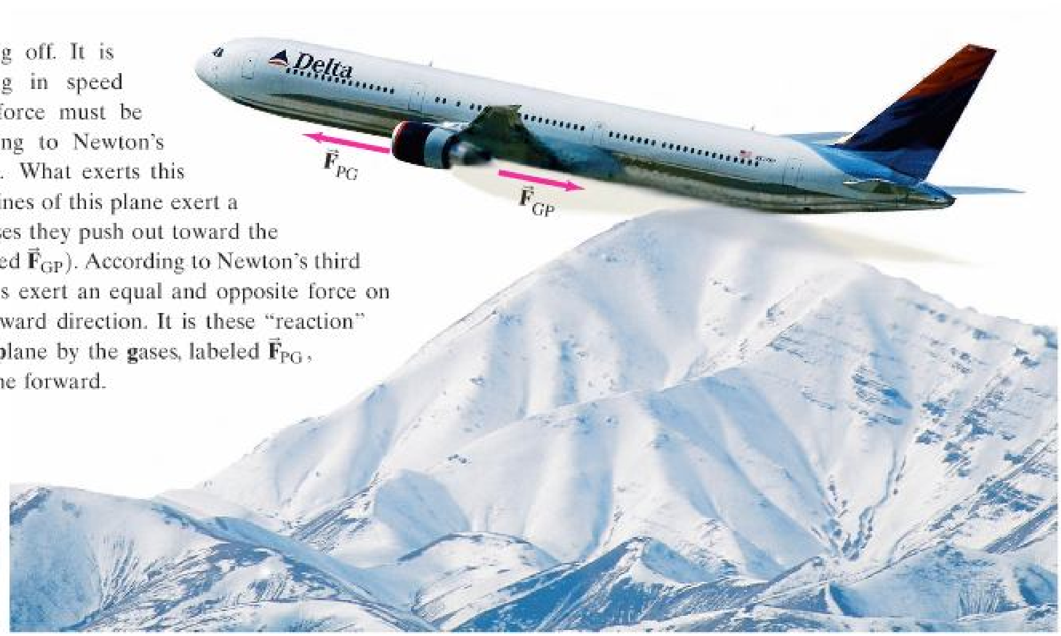
This airplane is taking off. It is accelerating, increasing in speed rapidly. To do so, a force must be exerted on it according to Newton's second law, $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$. What exerts this force? The two jet engines of this plane exert a strong force on the gases they push out toward the rear of the plane (labeled $\vec{\mathbf{F}}_{GP}$). According to Newton's third law, these ejected gases exert an equal and opposite force on the airplane in the forward direction. It is these "reaction" forces exerted on the **p**lane by the **g**ases, labeled $\vec{\mathbf{F}}_{PG}$, that accelerate the plane forward.



CHAPTER 4

Dynamics: Newton's Laws of Motion

We have discussed how motion is described in terms of velocity and acceleration. Now we deal with the question of *why* objects move as they do: What makes an object at rest begin to move? What causes an object to accelerate or decelerate? What is involved when an object moves in a circle? We can answer in each case that a force is required. In this Chapter, we will investigate the connection between force and motion, which is the subject called **dynamics**.

We begin with intuitive ideas of what a force is, and then discuss Newton's three laws of motion. We next look at several types of force, including friction and the force of gravity. We then apply Newton's laws to real problems.

4–1 Force

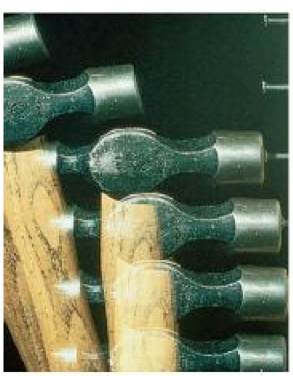


FIGURE 4–1 A force exerted on a grocery cart—in this case exerted by a child.

Intuitively, we experience **force** as any kind of a push or a pull on an object. When you push a stalled car or a grocery cart (Fig. 4–1), you are exerting a force on it. When a motor lifts an elevator, or a hammer hits a nail, or the wind blows the leaves of a tree, a force is being exerted. We say that an object falls because of the *force of gravity*.

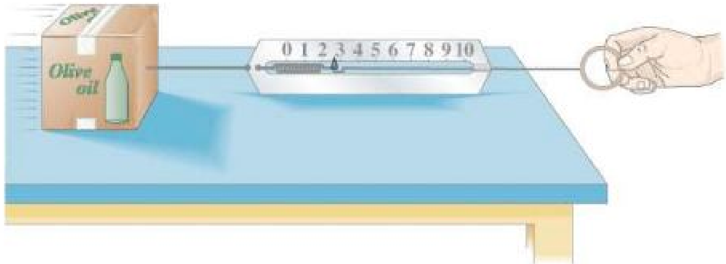


FIGURE 4–2 A spring scale used to measure a force.

If an object is at rest, to start it moving requires force—that is, a force is needed to accelerate an object from zero velocity to a nonzero velocity. For an object already moving, if you want to change its velocity—either in direction or in magnitude—again a force is required. In other words, to accelerate an object, a force is required.

Measuring force

One way to measure the magnitude (or strength) of a force is to use a spring scale (Fig. 4–2). Normally, such a spring scale is used to find the weight of an object; by weight we mean the force of gravity acting on the object (Section 4–6). The spring scale, once calibrated, can be used to measure other kinds of forces as well, such as the pulling force shown in Fig. 4–2.

A force exerted in different directions has a different effect. Clearly, force has direction as well as magnitude, and is indeed a vector that follows the rules of vector addition discussed in Chapter 3. We can represent any force on a diagram by an arrow, just as we did with velocity. The direction of the arrow is the direction of the push or pull, and its length is drawn proportional to the magnitude of the force.

4–2 Newton's First Law of Motion

Aristotle vs. Galileo

What is the relationship between force and motion? Aristotle (384–322 B.C.) believed that a force was required to keep an object moving along a horizontal plane. To Aristotle, the natural state of an object was at rest, and a force was believed necessary to keep an object in motion. Furthermore, Aristotle argued, the greater the force on the object, the greater its speed.

Some 2000 years later, Galileo disagreed: He maintained that it is just as natural for an object to be in motion with a constant velocity as it is for it to be at rest.

To understand Galileo's idea, consider the following observations involving motion along a horizontal plane. To push an object with a rough surface along a tabletop at constant speed requires a certain amount of force. To push an equally heavy object with a very smooth surface across the table at the same speed will require less force. If a layer of oil or other lubricant is placed between the surface of the object and the table, then almost no force is required to move the object. Notice that in each successive step, less force is required. As the next step, we imagine that the object does not rub against the table at all—or there is a perfect lubricant between the object and the table—and theorize that once started, the object would move across the table at constant speed with *no* force applied. A steel ball bearing rolling on a hard horizontal surface approaches this situation. So does a puck on an air table, in which a thin layer of air reduces friction almost to zero.

Friction as a force

It was Galileo's genius to imagine such an idealized world—in this case, one where there is no friction—and to see that it could lead to a more accurate and richer understanding of the real world. This idealization led him to his remarkable conclusion that if no force is applied to a moving object, it will continue to move with constant speed in a straight line. An object slows down only if a force is exerted on it. Galileo thus interpreted friction as a force akin to ordinary pushes and pulls.

To push an object across a table at constant speed requires a force from your hand that can balance out the force of friction (Fig. 4–3). When the object moves at constant speed, your pushing force is equal in magnitude to the friction force, but these two forces are in opposite directions, so the *net* force on the object (the vector sum of the two forces) is zero. This is consistent with Galileo's viewpoint, for the object moves with constant speed when no net force is exerted on it.

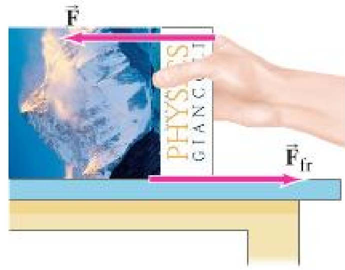


FIGURE 4–3 $\vec{\mathbf{F}}$ represents the force applied by the person and $\vec{\mathbf{F}}_{fr}$ represents the force of friction.

Upon this foundation laid by Galileo, Isaac Newton (Fig. 4–4) built his great theory of motion. Newton's analysis of motion is summarized in his famous "three laws of motion." In his great work, the *Principia* (published in 1687), Newton readily acknowledged his debt to Galileo. In fact, **Newton's first law of motion** is close to Galileo's conclusions. It states that

NEWTON'S FIRST LAW OF MOTION

Every object continues in its state of rest, or of uniform velocity in a straight line, as long as no net force acts on it.

Inertia

The tendency of an object to maintain its state of rest or of uniform motion in a straight line is called **inertia**. As a result, Newton's first law is often called the **law of inertia**.

FIGURE 4–4 Isaac Newton (1642–1727).

CONCEPTUAL EXAMPLE 4–1 **Newton's first law.** A school bus comes to a sudden stop, and all of the backpacks on the floor start to slide forward. What force causes them to do that?

RESPONSE It isn't "force" that does it. The backpacks continue their state of motion, maintaining their velocity (friction may slow them down), as the velocity of the bus decreases.

Inertial Reference Frames

Inertial reference frames

Newton's first law does not hold in every reference frame. For example, if your reference frame is fixed in an accelerating car, an object such as a cup resting on the dashboard may begin to move toward you (it stayed at rest as long as the car's velocity remained constant). The cup accelerated toward you, but neither you nor anything else exerted a force on it in that direction. Similarly, in the reference frame of the bus in Example 4–1, there was no force pushing the backpacks forward. In accelerating reference frames, Newton's first law does not hold. Reference frames in which Newton's first law does hold are called **inertial reference frames** (the law of inertia is valid in them). For most purposes, we can usually assume that reference frames fixed on the Earth are inertial frames. (This is not precisely true, due to the Earth's rotation, but usually it is close enough.) Any reference frame that moves with constant velocity (say, a car or an airplane) relative to an inertial frame is also an inertial reference frame. Reference frames where the law of inertia does *not* hold, such as the accelerating reference frames discussed above, are called **noninertial** reference frames. How can we be sure a reference frame is inertial or not? By checking to see if Newton's first law holds. Thus Newton's first law serves as the definition of inertial reference frames.

4–3 Mass

Newton's second law, which we come to in the next Section, makes use of the concept of mass. Newton used the term *mass* as a synonym for *quantity of matter*. This intuitive notion of the mass of an object is not very precise because the concept "quantity of matter" is not very well defined. More precisely, we can say that **mass** is a *measure of the inertia* of an object. The more mass an object has, the greater the force needed to give it a particular acceleration. It is harder to start it moving from rest, or to stop it when it is moving, or to change its velocity sideways out of a straight-line path. A truck has much more inertia than a baseball moving at the same speed, and it requires a much greater force to change the truck's velocity at the same rate as the ball's. The truck therefore has much more mass.

Mass as inertia

To quantify the concept of mass, we must define a standard. In SI units, the unit of mass is the **kilogram** (kg) as we discussed in Chapter 1, Section 1–5.

CAUTION
Distinguish mass from weight

The terms *mass* and *weight* are often confused with one another, but it is important to distinguish between them. Mass is a property of an object itself (a measure of an object's inertia, or its "quantity of matter"). Weight, on the other hand, is a force, the pull of gravity acting on an object. To see the difference, suppose we take an object to the Moon. The object will weigh only about one-sixth as much as it did on Earth, since the force of gravity is weaker. But its mass will be the same. It will have the same amount of matter as on Earth, and will have just as much inertia—for in the absence of friction, it will be just as hard to start it moving on the Moon as on Earth, or to stop it once it is moving. (More on weight in Section 4–6.)

4–4 Newton's Second Law of Motion

Newton's first law states that if no net force is acting on an object at rest, the object remains at rest; or if the object is moving, it continues moving with constant speed in a straight line. But what happens if a net force *is* exerted on an object? Newton perceived that the object's velocity will change (Fig. 4–5). A net force exerted on an object may make its velocity increase. Or, if the net force is in a direction opposite to the motion, the force will reduce the object's velocity. If the net force acts sideways on a moving object, the *direction* of the object's velocity changes (and the magnitude may as well). Since a change in velocity is an acceleration (Section 2–4), we can say that *a net force causes acceleration*.

FIGURE 4–5 The bobsled accelerates because the team exerts a force.

What precisely is the relationship between acceleration and force? Everyday experience can suggest an answer. Consider the force required to push a cart when friction is small enough to ignore. (If there is friction, consider the *net* force, which is the force you exert minus the force of friction.) Now if you push with a gentle but constant force for a certain period of time, you will make the cart accelerate from rest up to some speed, say 3 km/h. If you push with twice the force, the cart will reach 3 km/h in half the time. The acceleration will be twice as great. If you triple the force, the acceleration is tripled, and so on. Thus, the acceleration of an object is directly proportional† to the net applied force. But the acceleration depends on the mass of the object as well. If you push an empty grocery cart with the same force as you push one that is filled with groceries, you will find that the full cart accelerates more slowly. The greater the mass, the less the acceleration for the same net force. The mathematical relation, as Newton argued, is that the acceleration of an object is inversely proportional to its mass. These relationships are found to hold in general and can be summarized as follows:

> **The acceleration of an object is directly proportional to the net force acting on it, and is inversely proportional to its mass. The direction of the acceleration is in the direction of the net force acting on the object.**

NEWTON'S SECOND LAW OF MOTION

This is **Newton's second law of motion**.

† A review of proportionality is given in Appendix A, at the back of this book.

Newton's second law can be written as an equation:

$$\vec{\mathbf{a}} = \frac{\Sigma\vec{\mathbf{F}}}{m},$$

where $\vec{\mathbf{a}}$ stands for acceleration, m for the mass, and $\Sigma\vec{\mathbf{F}}$ for the *net force* on the object. The symbol Σ (Greek "sigma") stands for "sum of"; $\vec{\mathbf{F}}$ stands for force, so $\Sigma\vec{\mathbf{F}}$ means the *vector sum of all forces* acting on the object, which we define as the **net force**.

Net force

We rearrange this equation to obtain the familiar statement of Newton's second law:

NEWTON'S SECOND LAW OF MOTION

$$\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}. \tag{4–1}$$

Newton's second law relates the description of motion to the cause of motion, force. It is one of the most fundamental relationships in physics. From Newton's second law we can make a more precise definition of **force** as *an action capable of accelerating an object.*

Force defined

Every force $\vec{\mathbf{F}}$ is a vector, with magnitude and direction. Equation 4–1 is a vector equation valid in any inertial reference frame. It can be written in component form in rectangular coordinates as

$$\Sigma F_x = ma_x, \qquad \Sigma F_y = ma_y, \qquad \Sigma F_z = ma_z.$$

If the motion is all along a line (one-dimensional), we can leave out the subscripts and simply write $\Sigma F = ma$.

Unit of force: the newton

In SI units, with the mass in kilograms, the unit of force is called the **newton** (N). One newton, then, is the force required to impart an acceleration of $1\ \text{m/s}^2$ to a mass of 1 kg. Thus $1\ \text{N} = 1\ \text{kg}\cdot\text{m/s}^2$.

In cgs units, the unit of mass is the gram (g) as mentioned earlier.† The unit of force is the *dyne*, which is defined as the net force needed to impart an acceleration of $1\ \text{cm/s}^2$ to a mass of 1 g. Thus $1\ \text{dyne} = 1\ \text{g}\cdot\text{cm/s}^2$. It is easy to show that $1\ \text{dyne} = 10^{-5}\ \text{N}$.

In the British system, the unit of force is the *pound* (abbreviated lb), where $1\ \text{lb} = 4.44822\ \text{N} \approx 4.45\ \text{N}$. The unit of mass is the *slug*, which is defined as that mass which will undergo an acceleration of $1\ \text{ft/s}^2$ when a force of 1 lb is applied to it. Thus $1\ \text{lb} = 1\ \text{slug}\cdot\text{ft/s}^2$. Table 4–1 summarizes the units in the different systems.

PROBLEM SOLVING

Use a consistent set of units

It is very important that only one set of units be used in a given calculation or problem, with the SI being preferred. If the force is given in, say, newtons, and the mass in grams, then before attempting to solve for the acceleration in SI units, we must change the mass to kilograms. For example, if the force is given as 2.0 N along the x axis and the mass is 500 g, we change the latter to 0.50 kg, and the acceleration will then automatically come out in m/s^2 when Newton's second law is used (we set $1\ \text{N} = 1\ \text{kg}\cdot\text{m/s}^2$):

$$a_x = \frac{\Sigma F_x}{m} = \frac{2.0\ \text{N}}{0.50\ \text{kg}} = \frac{2.0\ \text{kg}\cdot\text{m/s}^2}{0.50\ \text{kg}} = 4.0\ \text{m/s}^2.$$

TABLE 4–1
Units for Mass and Force

System	Mass	Force
SI	kilogram (kg)	newton (N) ($=\text{kg}\cdot\text{m/s}^2$)
cgs	gram (g)	dyne ($=\text{g}\cdot\text{cm/s}^2$)
British	slug	pound (lb)

Conversion factors: $1\ \text{dyne} = 10^{-5}\ \text{N}$; $1\ \text{lb} \approx 4.45\ \text{N}$.

EXAMPLE 4–2 ESTIMATE Force to accelerate a fast car. Estimate the net force needed to accelerate (*a*) a 1000-kg car at $\frac{1}{2}g$; (*b*) a 200-g apple at the same rate.

APPROACH We can use Newton's second law to find the net force needed for each object, because we are given the mass and the acceleration. This is an estimate (the $\frac{1}{2}$ is not said to be precise) so we round off to one significant figure.

†Be careful not to confuse g for gram with g for the acceleration due to gravity. The latter is always italicized (or boldface when a vector).

SOLUTION (*a*) The car's acceleration is $a = \frac{1}{2}g = \frac{1}{2}(9.8\ \mathrm{m/s^2}) \approx 5\ \mathrm{m/s^2}$. We use Newton's second law to get the net force needed to achieve this acceleration:

$$\Sigma F = ma \approx (1000\ \mathrm{kg})(5\ \mathrm{m/s^2}) = 5000\ \mathrm{N}.$$

(If you are used to British units, to get an idea of what a 5000-N force is, you can divide by 4.45 N/lb and get a force of about 1000 lb.)
(*b*) For the apple, $m = 200\ \mathrm{g} = 0.200\ \mathrm{kg}$, so

$$\Sigma F = ma \approx (0.200\ \mathrm{kg})(5\ \mathrm{m/s^2}) = 1\ \mathrm{N}.$$

EXAMPLE 4–3 **Force to stop a car.** What average net force is required to bring a 1500-kg car to rest from a speed of 100 km/h within a distance of 55 m?

APPROACH We can use Newton's second law, $\Sigma F = ma$, to determine the force if we know the mass and acceleration of the car. We are given the mass, but we will have to calculate the acceleration a. We assume the acceleration is constant, so we can use the kinematic equations, Eqs. 2–11, to calculate it.

FIGURE 4–6 Example 4–3.

SOLUTION We assume the motion is along the $+x$ axis (Fig. 4–6). We are given the initial velocity $v_0 = 100\ \mathrm{km/h} = 28\ \mathrm{m/s}$ (Section 1–6), the final velocity $v = 0$, and the distance traveled $x - x_0 = 55\ \mathrm{m}$. From Eq. 2–11c, we have

$$v^2 = v_0^2 + 2a(x - x_0),$$

so

$$a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{0 - (28\ \mathrm{m/s})^2}{2(55\ \mathrm{m})} = -7.1\ \mathrm{m/s^2}.$$

The net force required is then

$$\Sigma F = ma = (1500\ \mathrm{kg})(-7.1\ \mathrm{m/s^2}) = -1.1 \times 10^4\ \mathrm{N}.$$

The force must be exerted in the direction *opposite* to the initial velocity, which is what the negative sign means.

NOTE When we assume the acceleration is constant, even though it may not be precisely true, we are determining an "average" acceleration and we obtain an "average" net force (or vice versa).

Newton's second law, like the first law, is valid only in inertial reference frames (Section 4–2). In the noninertial reference frame of an accelerating car, for example, a cup on the dashboard starts sliding—it accelerates—even though the net force on it is zero; thus $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$ doesn't work in such an accelerating reference frame.

4–5 Newton's Third Law of Motion

A force is exerted **on** *an object and is exerted* **by** *another object*

Newton's second law of motion describes quantitatively how forces affect motion. But where, we may ask, do forces come from? Observations suggest that a force applied to any object is always applied *by another object*. A horse pulls a wagon, a person pushes a grocery cart, a hammer pushes on a nail, a magnet attracts a paper clip. In each of these examples, a force is exerted *on* one object, and that force is exerted *by* another object. For example, the force exerted *on* the nail is exerted *by* the hammer.

But Newton realized that things are not so one-sided. True, the hammer exerts a force on the nail (Fig. 4–7). But the nail evidently exerts a force back on the hammer as well, for the hammer's speed is rapidly reduced to zero upon contact. Only a strong force could cause such a rapid deceleration of the hammer. Thus, said Newton, the two objects must be treated on an equal basis. The hammer exerts a force on the nail, and the nail exerts a force back on the hammer. This is the essence of **Newton's third law of motion**:

NEWTON'S THIRD LAW OF MOTION

Whenever one object exerts a force on a second object, the second exerts an equal force in the opposite direction on the first.

This law is sometimes paraphrased as "to every action there is an equal and opposite reaction." This is perfectly valid. But to avoid confusion, it is very important to remember that the "action" force and the "reaction" force are acting on *different* objects.

CAUTION
Action and reaction forces act on different objects

FIGURE 4–7 A hammer striking a nail. The hammer exerts a force on the nail and the nail exerts a force back on the hammer. The latter force decelerates the hammer and brings it to rest.

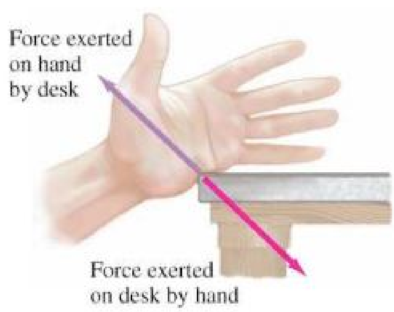


FIGURE 4–8 If your hand pushes against the edge of a desk (the force vector is shown in red), the desk pushes back against your hand (this force vector is shown in a different color, violet, to remind us that this force acts on a different object).

As evidence for the validity of Newton's third law, look at your hand when you push against the edge of a desk, Fig. 4–8. Your hand's shape is distorted, clear evidence that a force is being exerted on it. You can *see* the edge of the desk pressing into your hand. You can even *feel* the desk exerting a force on your hand; it hurts! The harder you push against the desk, the harder the desk pushes back on your hand. (You only feel forces exerted *on* you; when you exert a force on another object, what you feel is that object pushing back on you.)

As another demonstration of Newton's third law, consider the ice skater in Fig. 4–9. There is very little friction between her skates and the ice, so she will move freely if a force is exerted on her. She pushes against the wall; and then *she* starts moving backward. The force she exerts on the wall cannot make *her* start moving, for that force acts on the wall. Something had to exert a force *on her* to start her moving, and that force could only have been exerted by the wall. The force with which the wall pushes on her is, by Newton's third law, equal and opposite to the force she exerts on the wall.

FIGURE 4–9 An example of Newton's third law: when an ice skater pushes against the wall, the wall pushes back and this force causes her to accelerate away.

When a person throws a package out of a boat (initially at rest), the boat starts moving in the opposite direction. The person exerts a force on the package. The package exerts an equal and opposite force back on the person, and this force propels the person (and the boat) backward slightly.

Rocket acceleration

Rocket propulsion also is explained using Newton's third law (Fig. 4–10). A common misconception is that rockets accelerate because the gases rushing out the back of the engine push against the ground or the atmosphere. Not true. What happens, instead, is that a rocket exerts a strong force on the gases, expelling them; and the gases exert an equal and opposite force *on the rocket*. It is this latter force that propels the rocket forward—the force exerted *on* the rocket *by* the gases. Thus, a space vehicle is maneuvered in empty space by firing its rockets in the direction opposite to that in which it needs to accelerate. When the rocket pushes on the gases in one direction, the gases push back on the rocket in the opposite direction.

FIGURE 4–10 Another example of Newton's third law: the launch of a rocket. The rocket engine pushes the gases downward, and the gases exert an equal and opposite force upward on the rocket, accelerating it upward. (A rocket does *not* accelerate as a result of its propelling gases pushing against the ground.)

Consider how we walk. A person begins walking by pushing with the foot backward against the ground. The ground then exerts an equal and opposite force forward on the person (Fig. 4–11), and it is this force, *on* the person, that moves the person forward. (If you doubt this, try walking normally where there is no friction, such as on very smooth slippery ice.) In a similar way, a bird flies forward by exerting a backward force on the air, but it is the air pushing forward on the bird's wings that propels the bird forward.

How we can walk

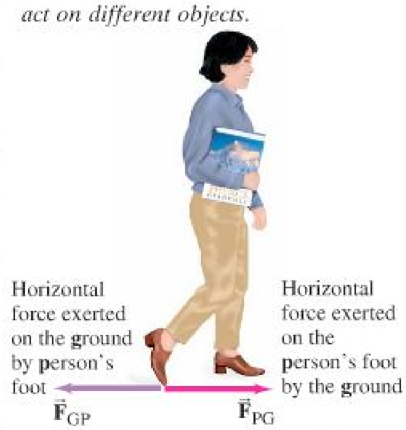


FIGURE 4–11 We can walk forward because, when one foot pushes backward against the ground, the ground pushes forward on that foot. The two forces shown *act on different objects.*

CONCEPTUAL EXAMPLE 4–4 **What exerts the force on a car?** What makes a car go forward?

RESPONSE A common answer is that the engine makes the car move forward. But it is not so simple. The engine makes the wheels go around. But if the tires are on slick ice or deep mud, they just spin. Friction is needed. On *solid* ground, the tires push backward against the ground because of friction. By Newton's third law, the ground pushes on the tires in the opposite direction, accelerating the car forward.

We tend to associate forces with active objects such as humans, animals, engines, or a moving object like a hammer. It is often difficult to see how an inanimate object at rest, such as a wall or a desk, or the wall of an ice rink (Fig. 4–9), can exert a force. The explanation is that every material, no matter how hard, is elastic (springy), at least to some degree. A stretched rubber band can exert a force on a wad of paper and accelerate it to fly across the room. Other materials may not stretch as readily as rubber, but they do stretch or compress when a force is applied to them. And just as a stretched rubber band exerts a force, so does a stretched (or compressed) wall, desk, or car fender.

Inanimate objects can exert a force (due to elasticity)

From the examples discussed above, we can see how important it is to remember *on* what object a given force is exerted and *by* what object that force is exerted. A force influences the motion of an object only when it is applied *on* that object. A force exerted *by* an object does not influence that same object; it only influences the other object *on* which it is exerted. Thus, to avoid confusion, the two prepositions *on* and *by* must always be used—and used with care.

PROBLEM SOLVING

For each force, be clear **on** *which object it acts, and* **by** *which object it is exerted.* $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$ *applies only to forces acting* **on** *an object.*

One way to keep clear which force acts on which object is to use double subscripts. For example, the force exerted on the **P**erson by the **G**round as the person walks in Fig. 4–11 can be labeled $\vec{\mathbf{F}}_{PG}$. And the force exerted on the ground by the person is $\vec{\mathbf{F}}_{GP}$. By Newton's third law

$$\vec{\mathbf{F}}_{GP} = -\vec{\mathbf{F}}_{PG}. \tag{4–2}$$

NEWTON'S THIRD LAW OF MOTION

$\vec{\mathbf{F}}_{GP}$ and $\vec{\mathbf{F}}_{PG}$ have the same magnitude (Newton's third law), and the minus sign reminds us that these two forces are in opposite directions.

Note carefully that the two forces shown in Fig. 4–11 act on different objects—hence we used slightly different colors for the vector arrows representing these forces. These two forces would never appear together in a sum of forces in Newton's second law, $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$. Why not? Because they act on different objects: $\vec{\mathbf{a}}$ is the acceleration of one particular object, and $\Sigma\vec{\mathbf{F}}$ must include *only* the forces on that *one* object.

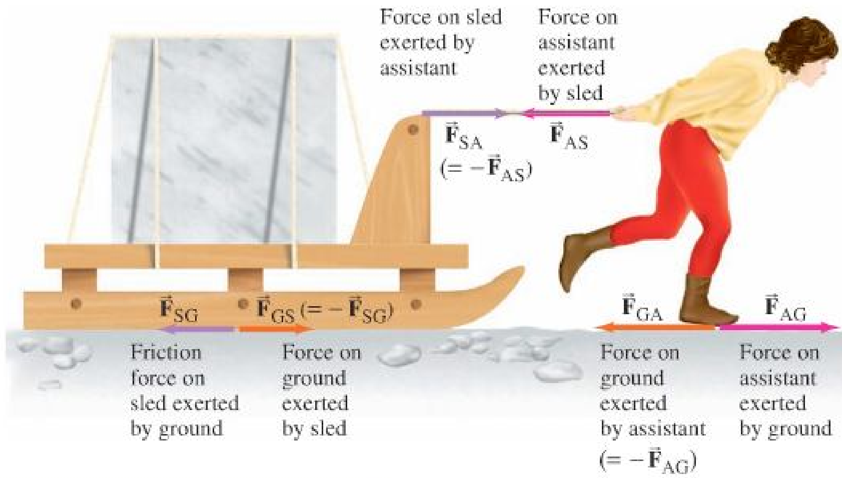


FIGURE 4–12 Example 4–5, showing only horizontal forces. Seventy-year-old Michelangelo has selected a fine block of marble for his next sculpture. Shown here is his assistant pulling it on a sled away from the quarry. Forces on the assistant are shown as red (magenta) arrows. Forces on the sled are purple arrows. Forces acting on the ground are orange arrows. Action–reaction forces that are equal and opposite are labeled by the same subscripts but reversed (such as $\vec{\mathbf{F}}_{GA}$ and $\vec{\mathbf{F}}_{AG}$) and are of different colors because they act on different objects.

CONCEPTUAL EXAMPLE 4–5 **Third law clarification.** Michelangelo's assistant has been assigned the task of moving a block of marble using a sled (Fig. 4–12). He says to his boss, "When I exert a forward force on the sled, the sled exerts an equal and opposite force backward. So how can I ever start it moving? No matter how hard I pull, the backward reaction force always equals my forward force, so the net force must be zero. I'll never be able to move this load." Is this a case of a little knowledge being dangerous? Explain.

RESPONSE Yes. Although it is true that the action and reaction forces are equal in magnitude, the assistant has forgotten that they are exerted on different objects. The forward ("action") force is exerted by the assistant on the sled (Fig. 4–12), whereas the backward "reaction" force is exerted by the sled on the assistant. To determine if the *assistant* moves or not, we must consider only the forces *on the assistant* and then apply $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$, where $\Sigma\vec{\mathbf{F}}$ is the net force *on the assistant*, $\vec{\mathbf{a}}$ is the acceleration of the assistant, and m is the assistant's mass. There are two forces on the assistant that affect his forward motion; they are shown as bright red (magenta) arrows in Figs. 4–12 and 4–13: they are (1) the horizontal force $\vec{\mathbf{F}}_{AG}$ exerted on the assistant by the ground (the harder he pushes backward against the ground, the harder the ground pushes forward on him—Newton's third law), and (2) the force $\vec{\mathbf{F}}_{AS}$ exerted on the assistant by the sled, pulling backward on him; see Fig. 4–13. If he pushes hard enough on the ground, the force on him exerted by the ground, $\vec{\mathbf{F}}_{AG}$, will be larger than the sled pulling back, $\vec{\mathbf{F}}_{AS}$, and the assistant accelerates forward (Newton's second law). The sled, on the other hand, accelerates forward when the force on *it* exerted by the assistant is greater than the frictional force exerted backward on it by the ground (that is, when $\vec{\mathbf{F}}_{SA}$ has greater magnitude than $\vec{\mathbf{F}}_{SG}$ in Fig. 4–12).

PROBLEM SOLVING
A study of Newton's second and third laws

FIGURE 4–13 Example 4–5. The horizontal forces on the assistant.

Using double subscripts to clarify Newton's third law can become cumbersome, and we won't usually use them in this way. Nevertheless, if there is any confusion in your mind about a given force, go ahead and use them to identify *on* what object and *by* what object the force is exerted. We will usually use a single subscript referring to what exerts the force on the object being discussed.

EXERCISE A A massive truck collides head-on with a small sports car. (*a*) Which vehicle experiences the greater force of impact? (*b*) Which experiences the greater acceleration? (*c*) Which of Newton's laws is useful to obtain the correct answer?

4–6 Weight—the Force of Gravity; and the Normal Force

As we saw in Chapter 2, Galileo claimed that all objects dropped near the surface of the Earth will fall with the same acceleration, $\vec{\mathbf{g}}$, if air resistance can be neglected. The force that causes this acceleration is called the *force of gravity* or *gravitational force*. What exerts the gravitational force on an object? It is the Earth, as we will discuss in Chapter 5, and the force acts vertically[†] downward, toward the center of the Earth. Let us apply Newton's second law to an object of mass m falling due to gravity; for the acceleration, $\vec{\mathbf{a}}$, we use the downward acceleration due to gravity, $\vec{\mathbf{g}}$. Thus, the **gravitational force** on an object, $\vec{\mathbf{F}}_G$, can be written as

$$\vec{\mathbf{F}}_G = m\vec{\mathbf{g}}. \quad (4\text{–}3)$$

Weight = gravitational force

The direction of this force is down toward the center of the Earth. The magnitude of the force of gravity on an object is commonly called the object's **weight**.

In SI units, $g = 9.80\ \text{m/s}^2 = 9.80\ \text{N/kg}$,[‡] so the weight of a 1.00-kg mass on Earth is $1.00\ \text{kg} \times 9.80\ \text{m/s}^2 = 9.80\ \text{N}$. We will mainly be concerned with the weight of objects on Earth, but we note that on the Moon, on other planets, or in space, the weight of a given mass will be different than it is on Earth. For example, on the Moon the acceleration due to gravity is about one-sixth what it is on Earth, and a 1.0-kg mass weighs only 1.7 N. Although we will not use British units, we note that for practical purposes on the Earth, a mass of 1 kg weighs about 2.2 lb. (On the Moon, 1 kg weighs only about 0.4 lb.)

CAUTION
Mass vs. weight

The force of gravity acts on an object when it is falling. When an object is at rest on the Earth, the gravitational force on it does not disappear, as we know if we weigh it on a spring scale. The same force, given by Eq. 4–3, continues to act. Why, then, doesn't the object move? From Newton's second law, the net force on an object that remains at rest is zero. There must be another force on the object to balance the gravitational force. For an object resting on a table, the table exerts this upward force; see Fig. 4–14a. The table is compressed slightly beneath the object, and due to its elasticity, it pushes up on the object as shown. The force exerted by the table is often called a **contact force**, since it occurs when two objects are in contact. (The force of your hand pushing on a cart is also a contact force.) When a contact force acts *perpendicular* to the common surface of contact, it is referred to as the **normal force** ("normal" means perpendicular); hence it is labeled $\vec{\mathbf{F}}_N$ in Fig. 4–14a.

Contact force

Normal force

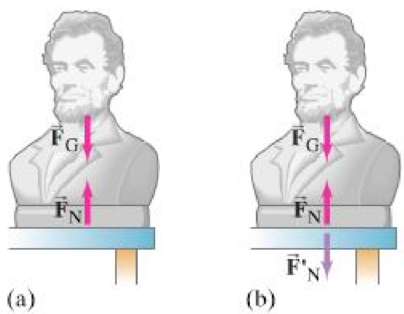


FIGURE 4–14 (a) The net force on an object at rest is zero according to Newton's second law. Therefore the downward force of gravity ($\vec{\mathbf{F}}_G$) on an object must be balanced by an upward force (the normal force $\vec{\mathbf{F}}_N$) exerted by the table in this case. (b) $\vec{\mathbf{F}}'_N$ is the force exerted on the table by the statue and is the reaction force to $\vec{\mathbf{F}}_N$ per Newton's third law. ($\vec{\mathbf{F}}'_N$ is shown in a different color to remind us it acts on a different object.) The reaction to $\vec{\mathbf{F}}_G$ is not shown.

The two forces shown in Fig. 4–14a are both acting on the statue, which remains at rest, so the vector sum of these two forces must be zero (Newton's second law). Hence $\vec{\mathbf{F}}_G$ and $\vec{\mathbf{F}}_N$ must be of equal magnitude and in opposite directions. But they are *not* the equal and opposite forces spoken of in Newton's third law. The action and reaction forces of Newton's third law act on *different objects*, whereas the two forces shown in Fig. 4–14a act on the *same* object. For each of the forces shown in Fig. 4–14a, we can ask, "What is the reaction force?" The upward force, $\vec{\mathbf{F}}_N$, on the statue is exerted by the table. The reaction to this force is a force exerted by the statue downward on the table. It is shown in Fig. 4–14b, where it is labeled $\vec{\mathbf{F}}'_N$. This force, $\vec{\mathbf{F}}'_N$, exerted on the table by the statue, is the reaction force to $\vec{\mathbf{F}}_N$ in accord with Newton's third law. What about the other force on the statue, the force of gravity $\vec{\mathbf{F}}_G$ exerted by the Earth? Can you guess what the reaction is to this force? We will see in Chapter 5 that the reaction force is also a gravitational force, exerted on the Earth by the statue.

CAUTION
Weight and normal force are **not** *action–reaction pairs*

[†]The concept of "vertical" is tied to gravity. The best definition of *vertical* is that it is the direction in which objects fall. A surface that is "horizontal," on the other hand, is a surface on which a round object won't start rolling: gravity has no effect. Horizontal is perpendicular to vertical.

[‡]Since $1\ \text{N} = 1\ \text{kg}\cdot\text{m/s}^2$ (Section 4–4), $1\ \text{m/s}^2 = 1\ \text{N/kg}$.

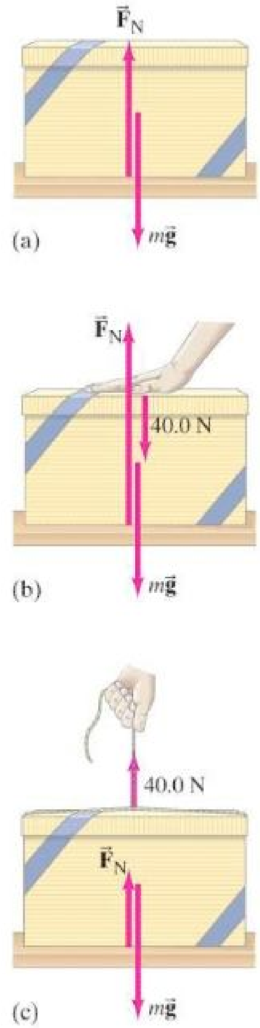


FIGURE 4–15 Example 4–6. (a) A 10-kg gift box is at rest on a table. (b) A person pushes down on the box with a force of 40.0 N. (c) A person pulls upward on the box with a force of 40.0 N. The forces are all assumed to act along a line; they are shown slightly displaced in order to be distinguishable. Only forces acting on the box are shown.

CAUTION
The normal force is not necessarily equal to the weight

EXAMPLE 4–6 **Weight, normal force, and a box.** A friend has given you a special gift, a box of mass 10.0 kg with a mystery surprise inside. The box is resting on the smooth (frictionless) horizontal surface of a table (Fig. 4–15a). (*a*) Determine the weight of the box and the normal force exerted on it by the table. (*b*) Now your friend pushes down on the box with a force of 40.0 N, as in Fig. 4–15b. Again determine the normal force exerted on the box by the table. (*c*) If your friend pulls upward on the box with a force of 40.0 N (Fig. 4–15c), what now is the normal force exerted on the box by the table?

APPROACH The box is at rest on the table, so the net force on the box in each case is zero (Newton's second law). The weight of the box equals mg in all three cases.

SOLUTION (*a*) The weight of the box is $mg = (10.0\,\text{kg})(9.80\,\text{m/s}^2) = 98.0\,\text{N}$, and this force acts downward. The only other force on the box is the normal force exerted upward on it by the table, as shown in Fig. 4–15a. We chose the upward direction as the positive y direction; then the net force ΣF_y on the box is $\Sigma F_y = F_N - mg$. The box is at rest, so the net force on it must be zero (Newton's second law, $\Sigma F_y = ma_y$, and $a_y = 0$). Thus

$$\Sigma F_y = F_N - mg = 0,$$

and we have in this case

$$F_N = mg.$$

The normal force on the box, exerted by the table, is 98.0 N upward, and has magnitude equal to the box's weight.

(*b*) Your friend is pushing down on the box with a force of 40.0 N. So instead of only two forces acting on the box, now there are three forces acting on the box, as shown in Fig. 4–15b. The weight of the box is still $mg = 98.0\,\text{N}$. The net force is $\Sigma F_y = F_N - mg - 40.0\,\text{N}$, and is equal to zero because the box remains at rest. Thus, since $a = 0$, Newton's second law gives

$$\Sigma F_y = F_N - mg - 40.0\,\text{N} = 0.$$

We solve this equation for the normal force:

$$F_N = mg + 40.0\,\text{N} = 98.0\,\text{N} + 40.0\,\text{N} = 138.0\,\text{N},$$

which is greater than in (*a*). The table pushes back with more force when a person pushes down on the box. The normal force is not always equal to the weight!

(*c*) The box's weight is still 98.0 N and acts downward. The force exerted by your friend and the normal force both act upward (positive direction), as shown in Fig. 4–15c. The box doesn't move since your friend's upward force is less than the weight. The net force, again set to zero in Newton's second law because $a = 0$, is

$$\Sigma F_y = F_N - mg + 40.0\,\text{N} = 0,$$

so

$$F_N = mg - 40.0\,\text{N} = 98.0\,\text{N} - 40.0\,\text{N} = 58.0\,\text{N}.$$

The table does not push against the full weight of the box because of the upward pull exerted by your friend.

NOTE The weight of the box ($= mg$) does not change as a result of your friend's push or pull. Only the normal force is affected.

CAUTION
The normal force, $\vec{F}_N$, is not necessarily vertical

Recall that the normal force is elastic in origin (the table in Fig. 4–15 sags slightly under the weight of the box). The normal force in Example 4–6 is vertical, perpendicular to the horizontal table. The normal force is not always vertical, however. When you push against a vertical wall, for example, the normal force with which the wall pushes back on you is horizontal. For an object on a plane inclined at an angle to the horizontal, such as a skier or car on a hill, the normal force acts perpendicular to the plane and so is not vertical.

EXAMPLE 4–7 **Accelerating the box.** What happens when a person pulls upward on the box in Example 4–6 (*c*) with a force equal to, or greater than, the box's weight, say $F_P = 100.0\,\mathrm{N}$ rather than the 40.0 N shown in Fig. 4–15c?

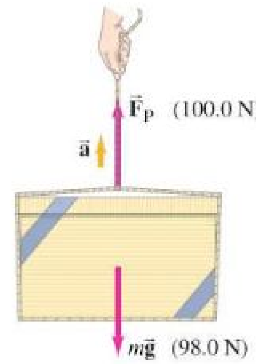


FIGURE 4–16 Example 4–7. The box accelerates upward because $F_P > mg$.

APPROACH We can start just as in Example 4–6, but be ready for a surprise.

SOLUTION The net force on the box is

$$\Sigma F_y = F_N - mg + F_P$$
$$= F_N - 98.0\,\mathrm{N} + 100.0\,\mathrm{N},$$

and if we set this equal to zero (thinking the acceleration might be zero), we would get $F_N = -2.0\,\mathrm{N}$. This is nonsense, since the negative sign implies F_N points downward, and the table surely cannot *pull* down on the box (unless there's glue on the table). The least F_N can be is zero, which it will be in this case. What really happens here is that the box accelerates upward because the net force is not zero. The net force (setting the normal force $F_N = 0$) is

$$\Sigma F_y = F_P - mg = 100.0\,\mathrm{N} - 98.0\,\mathrm{N}$$
$$= 2.0\,\mathrm{N}$$

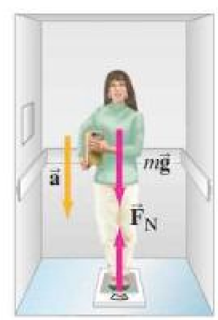
upward. See Fig. 4–16. We apply Newton's second law and see that the box moves upward with an acceleration

$$a_y = \frac{\Sigma F_y}{m} = \frac{2.0\,\mathrm{N}}{10.0\,\mathrm{kg}}$$
$$= 0.20\,\mathrm{m/s^2}.$$

Additional Example

EXAMPLE 4–8 **Apparent weight loss.** A 65-kg woman descends in an elevator that briefly accelerates at $0.20g$ downward when leaving a floor. She stands on a scale that reads in kg. (*a*) During this acceleration, what is her weight and what does the scale read? (*b*) What does the scale read when the elevator descends at a constant speed of 2.0 m/s?

FIGURE 4–17 Example 4–8.



APPROACH Figure 4–17 shows all the forces that act on the woman (and *only* those that act on her). The direction of the acceleration is downward, which we take as positive.

SOLUTION (*a*) From Newton's second law,

$$\Sigma F = ma$$
$$mg - F_N = m(0.20g).$$

We solve for F_N:

$$F_N = mg - 0.20mg = 0.80mg,$$

and it acts upward. The normal force $\vec{\mathbf{F}}_N$ is the force the scale exerts on the person, and is equal and opposite to the force she exerts on the scale: $F'_N = 0.80mg$ downward. Her weight (force of gravity on her) is still $mg = (65\,\mathrm{kg})(9.8\,\mathrm{m/s^2}) = 640\,\mathrm{N}$. But the scale, needing to exert a force of only $0.80mg$, will give a reading of $0.80m = 52\,\mathrm{kg}$.

(*b*) Now there is no acceleration, $a = 0$, so by Newton's second law, $mg - F_N = 0$ and $F_N = mg$. The scale reads her true mass of 65 kg.

NOTE The scale in (*a*) may give a reading of 52 kg (as an "apparent mass"), but her mass doesn't change as a result of the acceleration: it stays at 65 kg.

4–7 Solving Problems with Newton's Laws: Free-Body Diagrams

Newton's second law tells us that the acceleration of an object is proportional to the *net force* acting on the object. The **net force**, as mentioned earlier, is the *vector sum* of all forces acting on the object. Indeed, extensive experiments have shown that forces do add together as vectors precisely according to the rules we developed in Chapter 3. For example, in Fig. 4–18, two forces of equal magnitude (100 N each) are shown acting on an object at right angles to each other. Intuitively, we can see that the object will start moving at a 45° angle and thus the net force acts at a 45° angle. This is just what the rules of vector addition give. From the theorem of Pythagoras, the magnitude of the resultant force is $F_R = \sqrt{(100\text{ N})^2 + (100\text{ N})^2} = 141\text{ N}$.

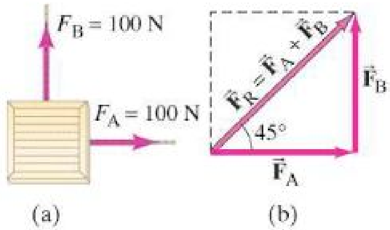
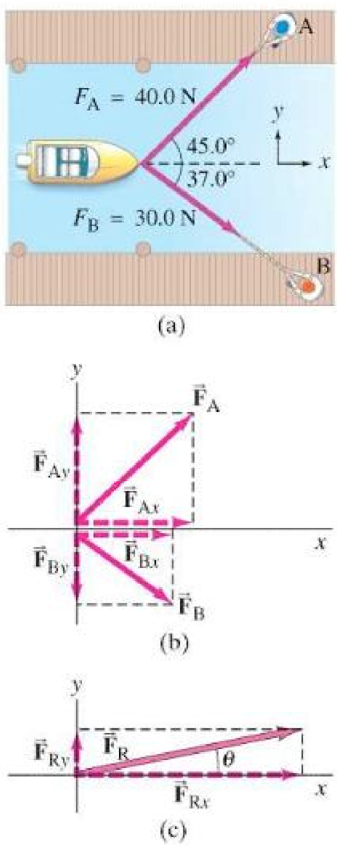


FIGURE 4–18 (a) Two forces, $\vec{F}_A$ and $\vec{F}_B$, exerted by workers A and B, act on a crate. (b) The sum, or resultant, of $\vec{F}_A$ and $\vec{F}_B$ is $\vec{F}_R$.

FIGURE 4–19 Example 4–9: Two force vectors act on a boat.



EXAMPLE 4–9 Adding force vectors. Calculate the sum of the two forces exerted on the boat by workers A and B in Fig. 4–19a.

APPROACH We add force vectors like any other vectors as described in Chapter 3. The first step is to choose an xy coordinate system, as in Fig. 4–19a, and then resolve vectors into their components.

SOLUTION The two force vectors are shown resolved into components in Fig. 4–19b. We add the forces using the method of components. The components of $\vec{F}_A$ are

$$F_{Ax} = F_A \cos 45.0° = (40.0\text{ N})(0.707) = 28.3\text{ N},$$
$$F_{Ay} = F_A \sin 45.0° = (40.0\text{ N})(0.707) = 28.3\text{ N}.$$

The components of $\vec{F}_B$ are

$$F_{Bx} = +F_B \cos 37.0° = +(30.0\text{ N})(0.799) = +24.0\text{ N},$$
$$F_{By} = -F_B \sin 37.0° = -(30.0\text{ N})(0.602) = -18.1\text{ N}.$$

F_{By} is negative because it points along the negative y axis. The components of the resultant force are (see Fig. 4–19c)

$$F_{Rx} = F_{Ax} + F_{Bx} = 28.3\text{ N} + 24.0\text{ N} = 52.3\text{ N},$$
$$F_{Ry} = F_{Ay} + F_{By} = 28.3\text{ N} - 18.1\text{ N} = 10.2\text{ N}.$$

To find the magnitude of the resultant force, we use the Pythagorean theorem:

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{(52.3)^2 + (10.2)^2}\text{ N} = 53.3\text{ N}.$$

The only remaining question is the angle θ that the net force $\vec{F}_R$ makes with the x axis. We use:

$$\tan\theta = \frac{F_{Ry}}{F_{Rx}} = \frac{10.2\text{ N}}{52.3\text{ N}} = 0.195,$$

and $\tan^{-1}(0.195) = 11.0°$. The net force on the boat has magnitude 53.3 N and acts at an 11.0° angle to the x axis.

PROBLEM SOLVING

Free-body diagram

Identifying every force

When solving problems involving Newton's laws and force, it is very important to draw a diagram showing all the forces acting *on* each object involved. Such a diagram is called a **free-body diagram**, or **force diagram**: choose one object, and draw an arrow to represent each force acting on it. Include *every* force acting on that object. Do not show forces that the chosen object exerts on *other* objects. To help you identify each and every force that is exerted on your chosen object, ask yourself what other objects could exert a force on it. If your problem involves more than one object, a separate free-body diagram is needed for each object.

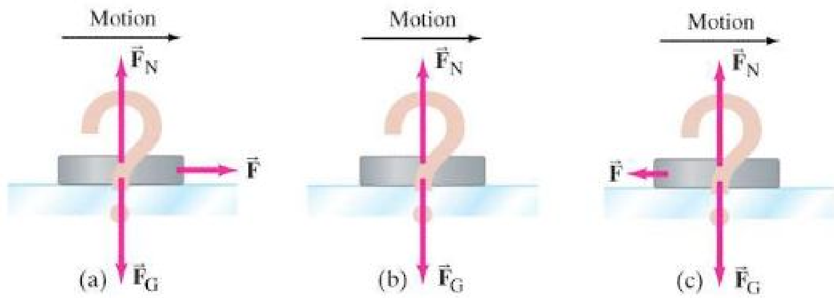


FIGURE 4–20 Example 4–10. Which is the correct free-body diagram for a hockey puck sliding across frictionless ice?

CONCEPTUAL EXAMPLE 4–10 **The hockey puck.** A hockey puck is sliding at constant velocity across a flat horizontal ice surface that is assumed to be frictionless. Which of the sketches in Fig. 4–20 is the correct free-body diagram for this puck? What would your answer be if the puck slowed down?

RESPONSE Did you choose (*a*)? If so, can you answer the question: what exerts the horizontal force labeled $\vec{F}$ on the puck? If you say that it is the force needed to maintain the motion, ask yourself: what exerts this force? Remember that another object must exert any force—and there simply isn't any possibility here. Therefore, (*a*) is wrong. Besides, the force $\vec{F}$ in Fig. 4–20a would give rise to an acceleration by Newton's second law. It is (*b*) that is correct, as long as there is no friction. No net force acts on the puck, and the puck slides at constant velocity across the ice.

In the real world, where even smooth ice exerts at least a tiny friction force, then (*c*) is the correct answer. The tiny friction force is in the direction opposite to the motion, and the puck's velocity decreases, even if very slowly.

Here now is a brief summary of how to approach solving problems involving Newton's laws.

PROBLEM SOLVING Newton's Laws; Free-Body Diagrams

1. **Draw a sketch** of the situation.
2. Consider only one object (at a time), and draw a **free-body diagram** for that object, showing *all* the forces acting *on* that object. Include any unknown forces that you have to solve for. Do not show any forces that the chosen object exerts on other objects. Draw the arrow for each force vector reasonably accurately for direction and magnitude. Label each force, including forces you must solve for, as to its source (gravity, person, friction, and so on).

   If several objects are involved, draw a free-body diagram for each object *separately*, showing all the forces acting *on that object* (and *only* forces acting on that object). For each (and every) force, you must be clear about: *on* what object that force acts, and *by* what object that force is exerted. Only forces acting *on* a given object can be included in $\Sigma\vec{F} = m\vec{a}$ for that object.
3. Newton's second law involves vectors, and it is usually important to **resolve vectors** into components. **Choose** x and y **axes** in a way that simplifies the calculation. For example, it often saves work if you choose one coordinate axis to be in the direction of the acceleration.
4. For each object, **apply Newton's second law** to the x and y components separately. That is, the x component of the net force on that object is related to the x component of that object's acceleration: $\Sigma F_x = ma_x$, and similarly for the y direction.
5. **Solve** the equation or equations for the unknown(s).

This Problem Solving Box should not be considered a prescription. Rather it is a summary of things to do that will start you thinking and getting involved in the problem at hand.

Force arrow placement on diagrams

When we are concerned only about translational motion, all the forces on a given object can be drawn as acting at the center of the object, thus treating the object as a point particle. However, for problems involving rotation or statics, the place *where* each force acts is also important, as we shall see in Chapters 8 and 9.

In the Examples that follow, we assume that all surfaces are very smooth so that friction can be ignored. (Friction, and Examples using it, are discussed in Section 4–8.)

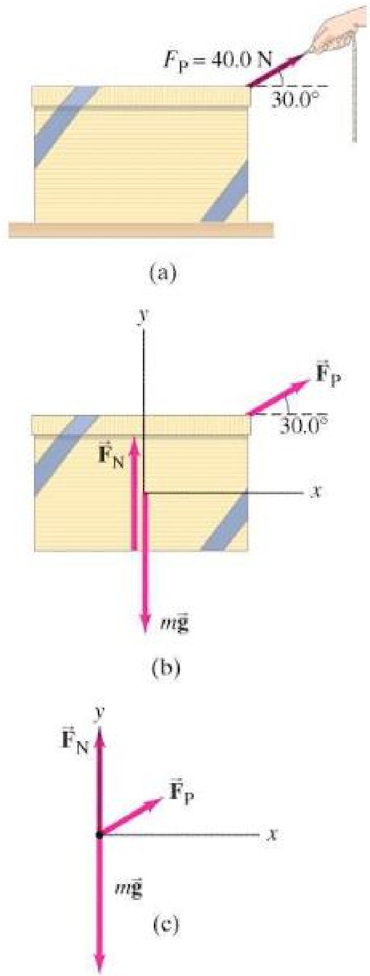


FIGURE 4–21 (a) Pulling the box, Example 4–11; (b) is the free-body diagram for the box, and (c) is the free-body diagram considering all the forces to act at a point (translational motion only, which is what we have here).

EXAMPLE 4–11 Pulling the mystery box. Suppose a friend asks to examine the 10.0-kg box you were given (Example 4–6, Fig. 4–15), hoping to guess what is inside; and you respond, "Sure, pull the box over to you." She then pulls the box by the attached cord, as shown in Fig. 4–21a, along the smooth surface of the table. The magnitude of the force exerted by the person is $F_P = 40.0\,\text{N}$, and it is exerted at a 30.0° angle as shown. Calculate (*a*) the acceleration of the box, and (*b*) the magnitude of the upward force F_N exerted by the table on the box. Assume that friction can be neglected.

APPROACH We follow the Problem Solving Box on the previous page.

SOLUTION

1. **Draw a sketch**: The situation is shown in Fig. 4–21a; it shows the box and the force applied by the person, F_P.
2. **Free-body diagram**: Figure 4–21b shows the free-body diagram of the box. To draw it correctly, we show *all* the forces acting on the box and *only* the forces acting on the box. They are: the force of gravity $m\vec{g}$; the normal force exerted by the table $\vec{F}_N$; and the force exerted by the person $\vec{F}_P$. We are interested only in translational motion, so we can show the three forces acting at a point, Fig. 4–21c.
3. **Choose axes and resolve vectors**: We expect the motion to be horizontal, so we choose the x axis horizontal and the y axis vertical. The pull of 40.0 N has components

$$F_{Px} = (40.0\,\text{N})(\cos 30.0^\circ) = (40.0\,\text{N})(0.866) = 34.6\,\text{N},$$
$$F_{Py} = (40.0\,\text{N})(\sin 30.0^\circ) = (40.0\,\text{N})(0.500) = 20.0\,\text{N}.$$

In the horizontal (x) direction, $\vec{F}_N$ and $m\vec{g}$ have zero components. Thus the horizontal component of the net force is F_{Px}.

4. (*a*) **Apply Newton's second law** to determine the x component of the acceleration:

$$F_{Px} = ma_x.$$

5. (*a*) **Solve**:

$$a_x = \frac{F_{Px}}{m} = \frac{(34.6\,\text{N})}{(10.0\,\text{kg})} = 3.46\,\text{m/s}^2.$$

The acceleration of the box is $3.46\,\text{m/s}^2$ to the right.

(*b*) Next we want to find F_N.

4. (*b*) **Apply Newton's second law** to the vertical (y) direction, with upward as positive:

$$\Sigma F_y = ma_y$$
$$F_N - mg + F_{Py} = ma_y.$$

5. (*b*) **Solve**: We have $mg = (10.0\,\text{kg})(9.80\,\text{m/s}^2) = 98.0\,\text{N}$ and, from point 3 above, $F_{Py} = 20.0\,\text{N}$. Furthermore, since $F_{Py} < mg$, the box does not move vertically, so $a_y = 0$. Thus

$$F_N - 98.0\,\text{N} + 20.0\,\text{N} = 0,$$

so

$$F_N = 78.0\,\text{N}.$$

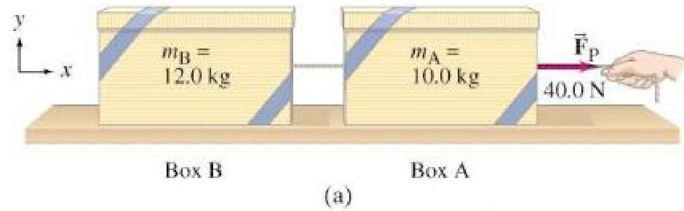
NOTE F_N is less than mg: the table does not push against the full weight of the box because part of the pull exerted by the person is in the upward direction.

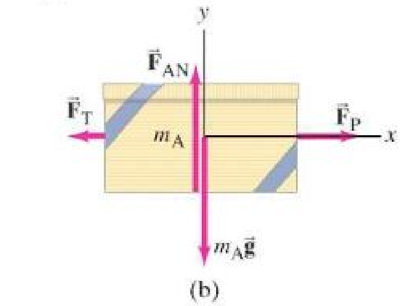
Tension in a Flexible Cord

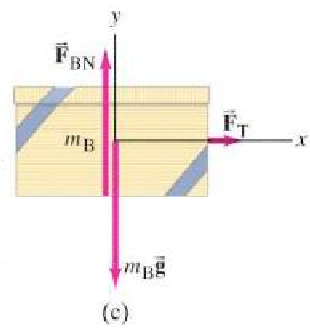
PROBLEM SOLVING
Cords can pull but can't push; tension exists throughout a cord

When a flexible cord pulls on an object, the cord is said to be under **tension**, and the force it exerts on the object is the tension F_T. If the cord has negligible mass, the force exerted at one end is transmitted undiminished to each adjacent piece of cord along the entire length to the other end. Why? Because $\Sigma\vec{F} = m\vec{a} = 0$ for the cord if the cord's mass m is zero (or negligible) no matter what $\vec{a}$ is. Hence the forces pulling on the cord at its two ends must add up to zero (F_T and $-F_T$). Note that flexible cords and strings can only pull. They can't push because they bend.

FIGURE 4–22 Example 4–12. (a) Two boxes, A and B, are connected by a cord. A person pulls horizontally on box A with force $F_P = 40.0\text{ N}$. (b) Free-body diagram for box A. (c) Free-body diagram for box B.







Our next Example involves two boxes connected by a cord. We can refer to this group of objects as a system. A *system* is any group of one or more objects we choose to consider and study.

EXAMPLE 4–12 **Two boxes connected by a cord.** Two boxes, A and B, are connected by a lightweight cord and are resting on a smooth (frictionless) table. The boxes have masses of 12.0 kg and 10.0 kg. A horizontal force F_P of 40.0 N is applied to the 10.0-kg box, as shown in Fig. 4–22a. Find (*a*) the acceleration of each box, and (*b*) the tension in the cord connecting the boxes.

APPROACH We streamline our approach by not listing each step. We have two boxes so we need to draw a free-body diagram for each box. To draw them correctly, we must consider the forces on *each* box by itself, so that Newton's second law can be applied to each. The person exerts a force F_P on box A. Box A exerts a force F_T on the connecting cord, and the cord exerts an opposite but equal magnitude force F_T back on box A (Newton's third law). These two horizontal forces on box A are shown in Fig. 4–22b, along with the force of gravity $m_A\vec{g}$ downward and the normal force $\vec{F}_{AN}$ exerted upward by the table. The cord is light, so we neglect its mass. The tension at each end of the cord is thus the same. Hence the cord exerts a force F_T on the second box. Figure 4–22c shows the forces on box B, which are $\vec{F}_T$, $m_B\vec{g}$, and the normal force $\vec{F}_{BN}$. There will be only horizontal motion. We take the positive x axis to the right.

SOLUTION (*a*) We apply $\Sigma F_x = ma_x$ to box A:

$$\Sigma F_x = F_P - F_T = m_A a_A. \qquad \text{[box A]}$$

For box B, the only horizontal force is F_T, so

$$\Sigma F_x = F_T = m_B a_B. \qquad \text{[box B]}$$

The boxes are connected, and if the cord remains taut and doesn't stretch, then the two boxes will have the same acceleration a. Thus $a_A = a_B = a$. We are given $m_A = 10.0\text{ kg}$ and $m_B = 12.0\text{ kg}$. We can add the two equations above to eliminate an unknown (F_T) and obtain

$$(m_A + m_B)a = F_P - F_T + F_T = F_P$$

or

$$a = \frac{F_P}{m_A + m_B} = \frac{40.0\text{ N}}{22.0\text{ kg}} = 1.82\text{ m/s}^2.$$

This is what we sought.

Alternate Solution We would have obtained the same result had we considered a single system, of mass $m_A + m_B$, acted on by a net horizontal force equal to F_P. (The tension forces F_T would then be considered internal to the system as a whole, and summed together would make zero contribution to the net force on the *whole* system.)

PROBLEM SOLVING
An alternate analysis

(*b*) From the equation above for box B $(F_T = m_B a_B)$, the tension in the cord is

$$F_T = m_B a = (12.0\text{ kg})(1.82\text{ m/s}^2) = 21.8\text{ N}.$$

Thus, F_T is less than $F_P(= 40.0\text{ N})$, as we expect, since F_T acts to accelerate only m_B.

NOTE It might be tempting to say that the force the person exerts, F_P, acts not only on box A but also on box B. It doesn't. F_P acts only on box A. It affects box B via the tension in the cord, F_T, which acts on box B and accelerates it.

CAUTION
For any object, use only the forces on that object in calculating $\Sigma F = ma$

PHYSICS APPLIED

Elevator (as Atwood's machine)

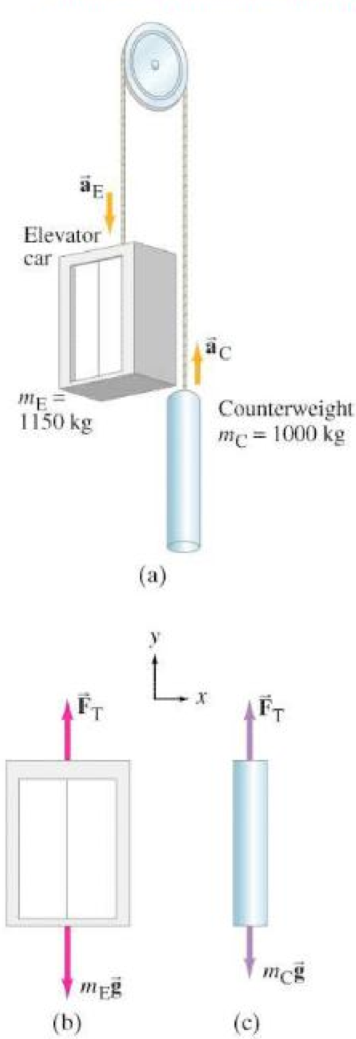


FIGURE 4–23 Example 4–13. (a) Atwood's machine in the form of an elevator–counterweight system. (b) and (c) Free-body diagrams for the two objects.

Additional Examples

Here are some more worked-out Examples to give you practice in solving a wide range of Problems.

EXAMPLE 4–13 Elevator and counterweight (Atwood's machine). A system of two objects suspended over a pulley by a flexible cable, as shown in Fig. 4–23a, is sometimes referred to as an *Atwood's machine*. Consider the real-life application of an elevator (m_E) and its counterweight (m_C). To minimize the work done by the motor to raise and lower the elevator safely, m_E and m_C are similar in mass. We leave the motor out of the system for this calculation, and assume that the cable's mass is negligible and that the mass of the pulley, as well as any friction, is small and ignorable. These assumptions ensure that the tension F_T in the cable has the same magnitude on both sides of the pulley. Let the mass of the counterweight be $m_C = 1000\,\text{kg}$. Assume the mass of the empty elevator is 850 kg, and its mass when carrying four passengers is $m_E = 1150\,\text{kg}$. For the latter case $(m_E = 1150\,\text{kg})$, calculate (*a*) the acceleration of the elevator and (*b*) the tension in the cable.

APPROACH Again we have two objects, and we will need to apply Newton's second law to each of them separately. Each mass has two forces acting on it: gravity downward and the cable tension pulling upward, $\vec{F}_T$. Figures 4–23b and c show the free-body diagrams for the elevator (m_E) and for the counterweight (m_C). The elevator, being the heavier, will accelerate downward, whereas the counterweight will accelerate upward. The magnitudes of their accelerations will be equal (we assume the cable doesn't stretch). For the counterweight, $m_C g = (1000\,\text{kg})(9.80\,\text{m/s}^2) = 9800\,\text{N}$, so F_T must be greater than 9800 N (in order that m_C will accelerate upward). For the elevator, $m_E g = (1150\,\text{kg})(9.80\,\text{m/s}^2) = 11{,}300\,\text{N}$, which must have greater magnitude than F_T so that m_E accelerates downward. Thus our calculation must give F_T between 9800 N and 11,300 N.

SOLUTION (*a*) To find F_T as well as the acceleration a, we apply Newton's second law, $\Sigma F = ma$, to each object. We take upward as the positive y direction for both objects. With this choice of axes, $a_C = a$ because m_C accelerates upward, and $a_E = -a$ because m_E accelerates downward. Thus

$$F_T - m_E g = m_E a_E = -m_E a$$
$$F_T - m_C g = m_C a_C = +m_C a.$$

We can subtract the first equation from the second to get

$$(m_E - m_C)g = (m_E + m_C)a,$$

where a is now the only unknown. We solve this for a:

$$a = \frac{m_E - m_C}{m_E + m_C}\,g = \frac{1150\,\text{kg} - 1000\,\text{kg}}{1150\,\text{kg} + 1000\,\text{kg}}\,g = 0.070\,g = 0.68\,\text{m/s}^2.$$

The elevator (m_E) accelerates downward (and the counterweight m_C upward) at $a = 0.070g = 0.68\,\text{m/s}^2$.

(*b*) The tension in the cable F_T can be obtained from either of the two $\Sigma F = ma$ equations, setting $a = 0.070g = 0.68\,\text{m/s}^2$:

$$F_T = m_E g - m_E a = m_E(g - a)$$
$$= 1150\,\text{kg}\,(9.80\,\text{m/s}^2 - 0.68\,\text{m/s}^2) = 10{,}500\,\text{N},$$
$$F_T = m_C g + m_C a = m_C(g + a)$$
$$= 1000\,\text{kg}\,(9.80\,\text{m/s}^2 + 0.68\,\text{m/s}^2) = 10{,}500\,\text{N},$$

which are consistent. As predicted, our result lies between 9800 N and 11,300 N.

NOTE We can check our equation for the acceleration a in this Example by noting that if the masses were equal $(m_E = m_C)$, then our equation above for a would give $a = 0$, as we should expect. Also, if one of the masses is zero (say, $m_C = 0$), then the other mass $(m_E \neq 0)$ would be predicted by our equation to accelerate at $a = g$, again as expected.

PROBLEM SOLVING

Check your result by seeing if it works in situations where the answer is easily guessed

CONCEPTUAL EXAMPLE 4–14 **The advantage of a pulley.** A mover is trying to lift a piano (slowly) up to a second-story apartment (Fig. 4–24). He is using a rope looped over two pulleys as shown. What force must he exert on the rope to slowly lift the piano's 2000-N weight?

RESPONSE The magnitude of the tension force F_T within the rope is the same at any point along the rope if we assume we can ignore its mass. First notice the forces acting on the lower pulley at the piano. The weight of the piano pulls down on the pulley via a short cable. The tension in the rope, looped through this pulley, pulls up *twice*, once on each side of the pulley. Let us apply Newton's second law to the pulley–piano combination (of mass m):

$$2F_T - mg = ma.$$

To move the piano with constant speed (set $a = 0$ in this equation) thus requires a tension in the rope, and hence a pull on the rope, of $F_T = mg/2$. The mover can exert a force equal to half the piano's weight. We say the pulley has given a **mechanical advantage** of 2, since without the pulley the mover would have to exert twice the force.

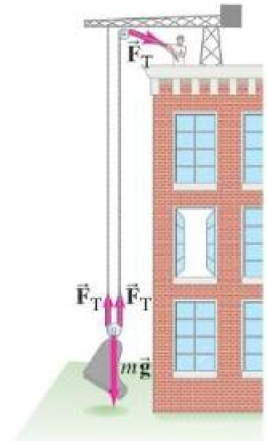


FIGURE 4–24 Example 4–14.

EXAMPLE 4–15 **Getting the car out of the mud.** Finding her car stuck in the mud, a bright graduate of a good physics course ties a strong rope to the back bumper of the car, and the other end to a boulder, as shown in Fig. 4–25a. She pushes at the midpoint of the rope with her maximum effort, which she estimates to be a force $F_P \approx 300\,\text{N}$. The car just begins to budge with the rope at an angle θ (see the Figure), which she estimates to be 5°. With what force is the rope pulling on the car? Neglect the mass of the rope.

How to get out of the mud

APPROACH First, note that the tension in a rope is always along the rope. Any component perpendicular to the rope would cause the rope to bend or buckle (as it does here where $\vec{\mathbf{F}}_P$ acts)—in other words, a rope can support a tension force only along its length. Let $\vec{\mathbf{F}}_{BR}$ and $\vec{\mathbf{F}}_{CR}$ be the forces on the boulder and on the car, exerted via the tension in the rope, as shown in Fig. 4–25a. Let us choose to look at the forces on the tiny section of rope where she pushes. The free-body diagram is shown in Fig. 4–25b, which shows $\vec{\mathbf{F}}_P$ as well as the tensions in the rope (note that we have used Newton's third law: $\vec{\mathbf{F}}_{RB} = -\vec{\mathbf{F}}_{BR}$, $\vec{\mathbf{F}}_{RC} = -\vec{\mathbf{F}}_{CR}$). At the moment the car budges, the acceleration is still essentially zero, so $\vec{\mathbf{a}} = 0$.

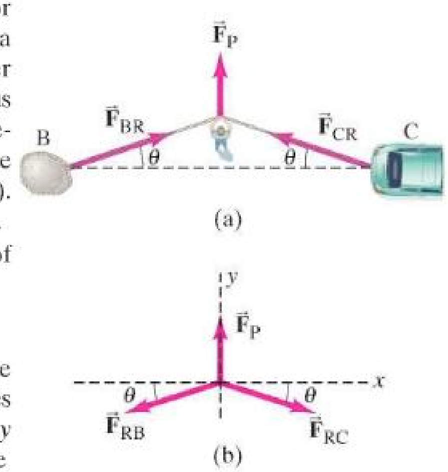


FIGURE 4–25 Example 4–15. (a) Getting a car out of the mud, showing the forces on the boulder, on the car, and exerted by the person. (b) The free-body diagram: forces on a small segment of rope.

SOLUTION For the x component of $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}} = 0$ on that small section of rope (Fig. 4–25b), we have

$$\Sigma F_x = F_{RB}\cos\theta - F_{RC}\cos\theta = 0.$$

Hence $F_{RB} = F_{RC}$, and these forces represent the magnitude of the tension in the rope, call it F_T; then we can write $F_T = F_{RB} = F_{RC}$. In the y direction, the forces acting are F_P, and the components of F_{RB} and F_{RC} that point in the negative y direction (each equal to $F_T \sin\theta$). So for the y component of $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$, we have

$$\Sigma F_y = F_P - 2F_T \sin\theta = 0.$$

We solve this for F_T, and insert $\theta = 5°$ and $F_P \approx 300\,\text{N}$, which were given:

$$F_T = \frac{F_P}{2\sin\theta} \approx \frac{300\,\text{N}}{2\sin 5°} \approx 1700\,\text{N}.$$

When our physics graduate exerted a force of 300 N on the rope, the force produced on the car was 1700 N. She was able to magnify her effort almost six times using this technique!

NOTE Notice the symmetry of the problem, which ensures that $F_{RB} = F_{RC}$.

PROBLEM SOLVING
Use any symmetry present to simplify a problem

NOTE Compare Figs. 4–25a and b. Notice that we cannot write down Newton's second law using Fig. 4–25a because the force vectors are not acting on the same object. It is only by choosing a tiny section of rope as our object, and using Newton's third law (in this case, the boulder and the car pulling back on the rope with forces F_{RB} and F_{RC}), that all forces apply to the same object.

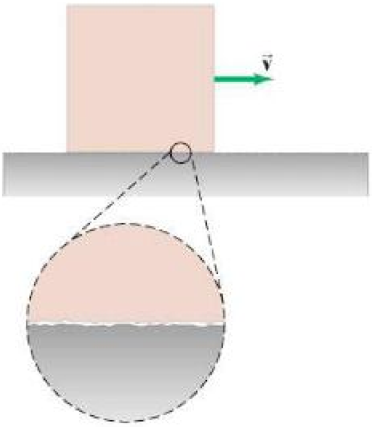


FIGURE 4–26 An object moving to the right on a table or floor. The two surfaces in contact are rough, at least on a microscopic scale.

4–8 Problems Involving Friction, Inclines

Friction

Until now we have ignored friction, but it must be taken into account in most practical situations. Friction exists between two solid surfaces because even the smoothest looking surface is quite rough on a microscopic scale, Fig. 4–26. When we try to slide an object across another surface, these microscopic bumps impede the motion. Exactly what is happening at the microscopic level is not yet fully understood. It is thought that the atoms on a bump of one surface may come so close to the atoms of the other surface that attractive electric forces between the atoms can "bond" as a tiny weld between the two surfaces. Sliding an object across a surface is often jerky, perhaps due to the making and breaking of these bonds. Even when a round object rolls across a surface, there is still some friction, called *rolling friction*, although it is generally much less than when an object slides across a surface. We focus now on sliding friction, which is usually called **kinetic friction** (*kinetic* is from the Greek for "moving").

When an object slides along a rough surface, the force of kinetic friction acts opposite to the direction of the object's velocity. The magnitude of the force of kinetic friction depends on the nature of the two sliding surfaces. For given surfaces, experiment shows that the friction force is approximately proportional to the *normal force* between the two surfaces, which is the force that either object exerts on the other, perpendicular to their common surface of contact (see Fig. 4–27). The force of friction between hard surfaces in many cases depends very little on the total surface area of contact; that is, the friction force on this book is roughly the same whether it is being slid on its wide face or on its spine, assuming the surfaces have the same smoothness. We consider a simple model of friction in which we make this assumption that the friction force is independent of area. Then we write the proportionality between the friction force F_{fr} and the normal force F_N as an equation by inserting a constant of proportionality, μ_k:

Kinetic friction

$$F_{fr} = \mu_k F_N .$$

This relation is not a fundamental law; it is an experimental relation between the magnitude of the friction force F_{fr}, which acts parallel to the two surfaces, and the magnitude of the normal force F_N, which acts perpendicular to the surfaces. It is *not* a vector equation since the two forces have directions perpendicular to one another. The term μ_k is called the *coefficient of kinetic friction*, and its value depends on the nature of the two surfaces. Measured values for a variety of surfaces are given in Table 4–2. These are only approximate, however, since μ depends on whether the surfaces are wet or dry, on how much they have been sanded or rubbed, if any burrs remain, and other such factors. But μ_k is roughly independent of the sliding speed, as well as the area in contact.

$\vec{F}_{fr} \perp \vec{F}_N$

FIGURE 4–27 When an object is pulled by an applied force ($\vec{F}_A$) along a surface, the force of friction $\vec{F}_{fr}$ opposes the motion. The magnitude of $\vec{F}_{fr}$ is proportional to the magnitude of the normal force (F_N).

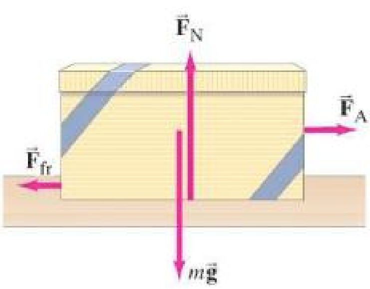


TABLE 4–2 Coefficients of Friction[†]

Surfaces	Coefficient of Static Friction, μ_s	Coefficient of Kinetic Friction, μ_k
Wood on wood	0.4	0.2
Ice on ice	0.1	0.03
Metal on metal (lubricated)	0.15	0.07
Steel on steel (unlubricated)	0.7	0.6
Rubber on dry concrete	1.0	0.8
Rubber on wet concrete	0.7	0.5
Rubber on other solid surfaces	1–4	1
Teflon® on Teflon in air	0.04	0.04
Teflon on steel in air	0.04	0.04
Lubricated ball bearings	<0.01	<0.01
Synovial joints (in human limbs)	0.01	0.01

[†] Values are approximate and intended only as a guide.

What we have been discussing up to now is *kinetic friction*, when one object slides over another. There is also **static friction**, which refers to a force parallel to the two surfaces that can arise even when they are not sliding. Suppose an object such as a desk is resting on a horizontal floor. If no horizontal force is exerted on the desk, there also is no friction force. But now suppose you try to push the desk, and it doesn't move. You are exerting a horizontal force, but the desk isn't moving, so there must be another force on the desk keeping it from moving (the net force is zero on an object that doesn't move). This is the force of *static friction* exerted by the floor on the desk. If you push with a greater force without moving the desk, the force of static friction also has increased. If you push hard enough, the desk will eventually start to move, and kinetic friction takes over. At this point, you have exceeded the maximum force of static friction, which is given by $F_{fr}(\max) = \mu_s F_N$, where μ_s is the *coefficient of static friction* (Table 4–2). Since the force of static friction can vary from zero to this maximum value, we write

$$F_{fr} \leq \mu_s F_N .$$

Static friction

You may have noticed that it is often easier to keep a heavy object sliding than it is to start it sliding in the first place. This is consistent with μ_s generally being greater than μ_k (see Table 4–2).

EXAMPLE 4–16 **Friction: static and kinetic.** Our 10.0-kg mystery box rests on a horizontal floor. The coefficient of static friction is $\mu_s = 0.40$ and the coefficient of kinetic friction is $\mu_k = 0.30$. Determine the force of friction, F_{fr}, acting on the box if a horizontal external applied force F_A is exerted on it of magnitude: (*a*) 0, (*b*) 10 N, (*c*) 20 N, (*d*) 38 N, and (*e*) 40 N.

APPROACH We don't know, right off, if we are dealing with static friction or kinetic friction, nor if the box remains at rest or accelerates. We need to draw a free-body diagram, and then determine in each case whether or not the box will move, by using Newton's second law. The forces on the box are gravity $m\vec{\mathbf{g}}$, the normal force exerted by the floor $\vec{\mathbf{F}}_N$, the horizontal applied force $\vec{\mathbf{F}}_A$, and the friction force $\vec{\mathbf{F}}_{fr}$, as shown in Fig. 4–27.

SOLUTION The free-body diagram of the box is shown in Fig. 4–27. In the vertical direction there is no motion, so Newton's second law in the vertical direction gives $\Sigma F_y = ma_y = 0$, which tells us $F_N - mg = 0$. Hence the normal force is

$$F_N = mg = (10.0\,\text{kg})(9.8\,\text{m/s}^2) = 98\,\text{N}.$$

(*a*) Since no external force F_A is applied in this first case, the box doesn't move, and $F_{fr} = 0$.
(*b*) The force of static friction will oppose any applied force up to a maximum of

$$\mu_s F_N = (0.40)(98\,\text{N}) = 39\,\text{N}.$$

When the applied force is $F_A = 10\,\text{N}$, the box will not move. Since $\Sigma F_x = F_A - F_{fr} = 0$, then $F_{fr} = 10\,\text{N}$.
(*c*) An applied force of 20 N is also not sufficient to move the box. Thus $F_{fr} = 20\,\text{N}$ to balance the applied force.
(*d*) The applied force of 38 N is still not quite large enough to move the box; so the friction force has now increased to 38 N to keep the box at rest.
(*e*) A force of 40 N will start the box moving since it exceeds the maximum force of static friction, $\mu_s F_N = (0.40)(98\,\text{N}) = 39\,\text{N}$. Instead of static friction, we now have kinetic friction, and its magnitude is

$$F_{fr} = \mu_k F_N = (0.30)(98\,\text{N}) = 29\,\text{N}.$$

There is now a net (horizontal) force on the box of magnitude $F = 40\,\text{N} - 29\,\text{N} = 11\,\text{N}$, so the box will accelerate at a rate

$$a_x = \frac{\Sigma F}{m} = \frac{11\,\text{N}}{10\,\text{kg}} = 1.1\,\text{m/s}^2$$

as long as the applied force is 40 N. Figure 4–28 shows a graph that summarizes this Example.

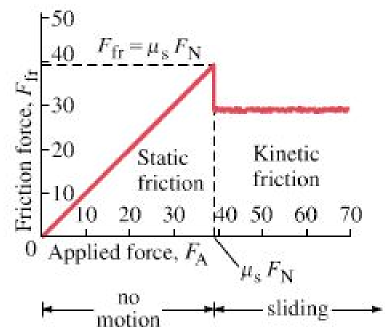


FIGURE 4–28 Example 4–16. Magnitude of the force of friction as a function of the external force applied to an object initially at rest. As the applied force is increased in magnitude, the force of static friction increases linearly to just match it, until the applied force equals $\mu_s F_N$. If the applied force increases further, the object will begin to move, and the friction force drops to a roughly constant value characteristic of kinetic friction.

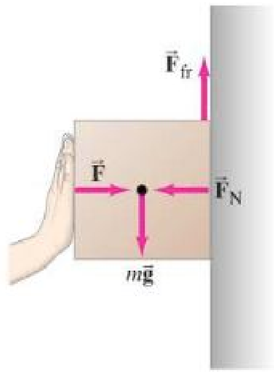


FIGURE 4–29 Example 4–17.

CONCEPTUAL EXAMPLE 4–17 **A box against a wall.** You can hold a box against a rough wall (Fig. 4–29) and prevent it from slipping down by pressing hard horizontally. How does the application of a horizontal force keep an object from moving vertically?

RESPONSE This won't work well if the wall is slippery. You need friction. Even then, if you don't press hard enough, the box will slip. The horizontal force you apply produces a normal force on the box exerted by the wall. The force of gravity mg, acting downward on the box, can now be balanced by an upward friction force whose magnitude is proportional to the normal force. The harder you push, the greater F_N is and the greater F_{fr} can be. If you don't press hard enough, then $mg > \mu_s F_N$ and the box begins to slide down.

Additional Examples

Here are some more worked-out Examples that can help you for solving Problems.

CONCEPTUAL EXAMPLE 4–18 **To push or to pull a sled?** Your little sister wants a ride on her sled. If you are on flat ground, will you exert less force if you push her or pull her? See Figs. 4–30a and b. Assume the same angle θ in each case.

RESPONSE Let us draw free-body diagrams for the sled–sister combination, as shown in Figs. 4–30c and d. They show, for the two cases, the forces exerted by you, $\vec{F}$ (an unknown), by the snow, $\vec{F}_N$ and $\vec{F}_{fr}$, and gravity $m\vec{g}$. (*a*) If you push her, and $\theta > 0$, there is a vertically downward component to your force. Hence the normal force upward exerted by the ground (Fig. 4–30c) will be larger than mg (where m is the mass of sister plus sled). (*b*) If you pull her, your force has a vertically upward component, so the normal force F_N will be less than mg, Fig. 4–30d. Because the friction force is proportional to the normal force, F_{fr} will be less if you pull her. So you exert less force if you pull her.

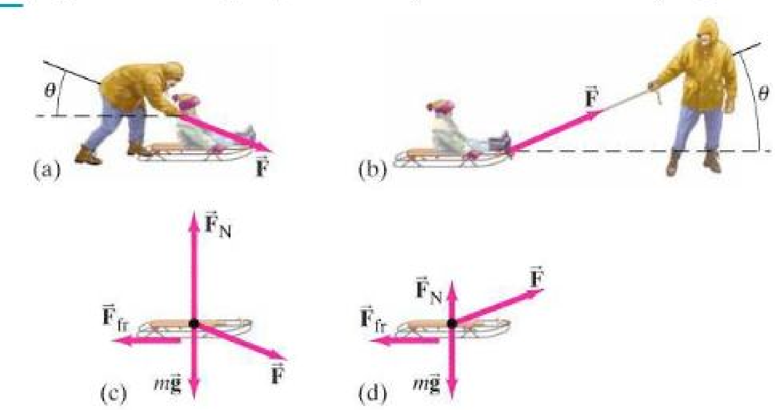
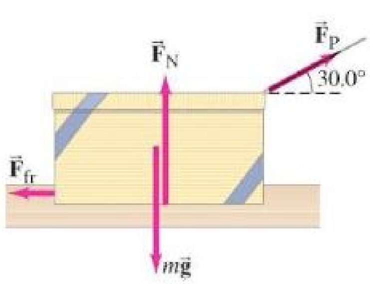


FIGURE 4–30 Example 4–18.

EXAMPLE 4–19 **Pulling against friction.** A 10.0-kg box is pulled along a horizontal surface by a force F_P of 40.0 N applied at a 30.0° angle. This is like Example 4–11 except now there is friction, and we assume a coefficient of kinetic friction of 0.30. Calculate the acceleration.

FIGURE 4–31 Example 4–19.



APPROACH The free-body diagram is like that in Fig. 4–21, but with one more force, that of friction; see Fig. 4–31.

SOLUTION The calculation for the vertical (y) direction is just the same as in Example 4–11, where we saw that $F_{Py} = 20.0\text{ N}$, $F_{Px} = 34.6\text{ N}$, and the normal force is $F_N = 78.0\text{ N}$. Now we apply Newton's second law for the horizontal (x) direction (positive to the right), and include the friction force:

$$F_{Px} - F_{fr} = ma_x.$$

The friction force is kinetic as long as $F_{fr} = \mu_k F_N$ is less than $F_{Px}\,(=34.6\text{ N})$, which it is:

$$F_{fr} = \mu_k F_N = (0.30)(78.0\text{ N}) = 23.4\text{ N}.$$

Hence the box does accelerate:

$$a_x = \frac{F_{Px} - F_{fr}}{m} = \frac{34.6\,\text{N} - 23.4\,\text{N}}{10.0\,\text{kg}} = 1.1\,\text{m/s}^2.$$

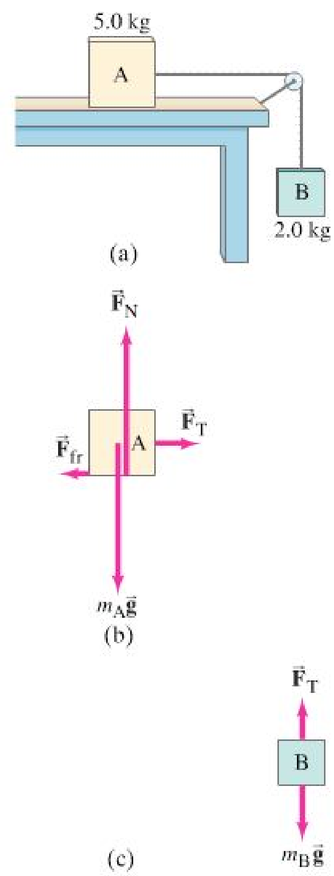
In the absence of friction, as we saw in Example 4–11, the acceleration would be much greater than this.

NOTE Our final answer has only two significant figures because our least significant input value $(\mu_k = 0.30)$ has two.

EXERCISE B If $\mu_k F_N$ were greater than F_{Px}, what would you conclude?

EXAMPLE 4–20 **Two boxes and a pulley.** In Fig. 4–32a, two boxes are connected by a cord running over a pulley. The coefficient of kinetic friction between box A and the table is 0.20. We ignore the mass of the cord and pulley and any friction in the pulley, which means we can assume that a force applied to one end of the cord will have the same magnitude at the other end. We wish to find the acceleration, a, of the system, which will have the same magnitude for both boxes assuming the cord doesn't stretch. As box B moves down, box A moves to the right.

FIGURE 4–32 Example 4–20.



APPROACH We need a free-body diagram for each box, Figs. 4–32b and c, so we can apply Newton's second law to each. The forces on box A are the pulling force of the cord F_T, gravity $m_A g$, the normal force exerted by the table F_N, and a friction force exerted by the table F_{fr}; the forces on box B are gravity $m_B g$, and the cord pulling up, F_T.

SOLUTION Box A does not move vertically, so Newton's second law tells us the normal force just balances the weight,

$$F_N = m_A g = (5.0\,\text{kg})(9.8\,\text{m/s}^2) = 49\,\text{N}.$$

In the horizontal direction, there are two forces on box A (Fig. 4–32b): F_T, the tension in the cord (whose value we don't know), and the force of friction

$$F_{fr} = \mu_k F_N = (0.20)(49\,\text{N}) = 9.8\,\text{N}.$$

The horizontal acceleration is what we wish to find; we use Newton's second law in the x direction, $\Sigma F_{Ax} = m_A a_x$, which becomes (taking the positive direction to the right and setting $a_{Ax} = a$):

$$\Sigma F_{Ax} = F_T - F_{fr} = m_A a. \qquad \text{[box A]}$$

Next consider box B. The force of gravity $m_B g = (2.0\,\text{kg})(9.8\,\text{m/s}^2) = 19.6\,\text{N}$ pulls downward; and the cord pulls upward with a force F_T. So we can write Newton's second law for box B (taking the downward direction as positive):

$$\Sigma F_{By} = m_B g - F_T = m_B a. \qquad \text{[box B]}$$

[Notice that if $a \neq 0$, then F_T is not equal to $m_B g$.]

> **CAUTION**
> *Tension in a cord supporting a falling object may not equal object's weight*

We have two unknowns, a and F_T, and we also have two equations. We solve the box A equation for F_T:

$$F_T = F_{fr} + m_A a,$$

and substitute this into the box B equation:

$$m_B g - F_{fr} - m_A a = m_B a.$$

Now we solve for a and put in numerical values:

$$a = \frac{m_B g - F_{fr}}{m_A + m_B} = \frac{19.6\,\text{N} - 9.8\,\text{N}}{5.0\,\text{kg} + 2.0\,\text{kg}} = 1.4\,\text{m/s}^2,$$

which is the acceleration of box A to the right, and of box B down.

If we wish, we can calculate F_T using the first equation:

$$F_T = F_{fr} + m_A a = 9.8\,\text{N} + (5.0\,\text{kg})(1.4\,\text{m/s}^2) = 17\,\text{N}.$$

NOTE Box B is not in free fall. It does not fall at $a = g$ because an additional force, F_T, is acting upward on it.

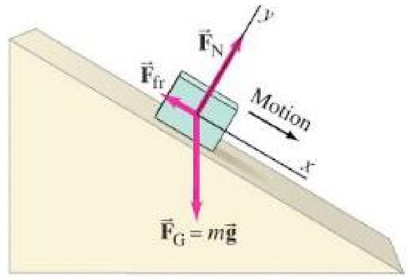


FIGURE 4–33 Forces on an object sliding down an incline.

Inclines

PROBLEM SOLVING
Good choice of coordinate system simplifies the calculation

Now we consider what happens when an object slides down an incline, such as a hill or ramp. Such problems are interesting because gravity is the accelerating force, yet the acceleration is not vertical. Solving problems is usually easier if we choose the xy coordinate system so the x axis points along the incline and the y axis is perpendicular to the incline, as shown in Fig. 4–33. Note also that the normal force is not vertical, but is perpendicular to the sloping surface of the plane in Fig. 4–33.

EXERCISE C Is the gravitational force always perpendicular to an inclined plane? Is it always vertical?

EXERCISE D Is the normal force always perpendicular to an inclined plane? Is it always vertical?

PHYSICS APPLIED
Skiing

EXAMPLE 4–21 **The skier.** The skier in Fig. 4–34 has just begun descending the 30° slope. Assuming the coefficient of kinetic friction is 0.10, calculate (*a*) her acceleration and (*b*) the speed she will reach after 4.0 s.

APPROACH We choose the x axis along the slope, positive pointing downslope in the direction of the skier's motion. The y axis is perpendicular to the surface as shown. The forces acting on the skier are gravity, $\vec{F}_G = m\vec{g}$, which points vertically downward (*not* perpendicular to the slope), and the two forces exerted on her skis by the snow—the normal force perpendicular to the snowy slope (*not* vertical), and the friction force parallel to the surface. These three forces are shown acting at one point in Fig. 4–34b, for convenience, and is our free-body diagram for the skier.

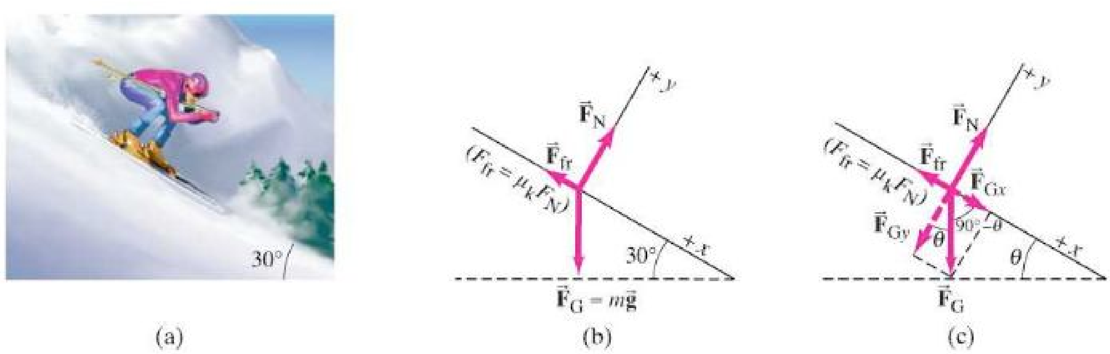


FIGURE 4–34 Example 4–21. A skier descending a slope; $\vec{F}_G = m\vec{g}$ is the force of gravity (weight) on the skier.

SOLUTION We have to resolve only one vector into components, the weight $\vec{\mathbf{F}}_G$, and its components are shown as dashed lines in Fig. 4–34c. To be general, we use θ rather than 30° for now. We use the definitions of sine ("side opposite") and cosine ("side adjacent") to obtain the components:

$$F_{Gx} = mg \sin\theta,$$
$$F_{Gy} = -mg \cos\theta.$$

where F_{Gy} is in the negative y direction.
(*a*) To calculate the skier's acceleration down the hill, a_x, we apply Newton's second law to the x direction:

$$\Sigma F_x = ma_x$$
$$mg \sin\theta - \mu_k F_N = ma_x$$

where the two forces are the x component of the gravity force ($+x$ direction) and the friction force ($-x$ direction). We want to find the value of a_x, but we don't yet know F_N in the last equation. Let's see if we can get F_N from the y component of Newton's second law:

$$\Sigma F_y = ma_y$$
$$F_N - mg\cos\theta = ma_y = 0$$

where we set $a_y = 0$ because there is no motion in the y direction (perpendicular to the slope). Thus we can solve for F_N:

$$F_N = mg\cos\theta$$

and we can substitute this into our equation above for ma_x:

$$mg\sin\theta - \mu_k(mg\cos\theta) = ma_x.$$

There is an m in each term which can be canceled out. Thus (setting $\theta = 30°$ and $\mu_k = 0.10$):

$$a_x = g\sin 30° - \mu_k g\cos 30°$$
$$= 0.50g - (0.10)(0.866)g = 0.41g.$$

The skier's acceleration is 0.41 times the acceleration of gravity, which in numbers is $a = (0.41)(9.8\,\text{m/s}^2) = 4.0\,\text{m/s}^2$. It is interesting that the mass canceled out here, and so we have the useful conclusion that *the acceleration doesn't depend on the mass*. That such a cancellation sometimes occurs, and thus may give a useful conclusion as well as saving calculation, is a big advantage of working with the algebraic equations and putting in the numbers only at the end.

PROBLEM SOLVING
It is often helpful to put in numbers only at the end

(*b*) The speed after 4.0 s is found, since the acceleration is constant, by using Eq. 2–11a:

$$v = v_0 + at$$
$$= 0 + (4.0\,\text{m/s}^2)(4.0\,\text{s}) = 16\,\text{m/s},$$

where we assumed a start from rest.

In problems involving a slope or "inclined plane," it is common to make an error in the direction of the normal force or in the direction of gravity. The normal force is *not* vertical in Example 4–21. It is perpendicular to the slope or plane. And gravity is *not* perpendicular to the slope or plane—gravity acts vertically downward toward the center of the Earth.

CAUTION
Directions of gravity and the normal force

4–9 Problem Solving—A General Approach

A basic part of a physics course is solving problems effectively. The approach discussed here, though emphasizing Newton's laws, can be applied generally for other topics discussed throughout this book.

PROBLEM SOLVING In General

1. **Read** and reread written problems carefully. A common error is to skip a word or two when reading, which can completely change the meaning of a problem.
2. **Draw** an accurate picture or diagram of the situation. (This is probably the most overlooked, yet most crucial, part of solving a problem.) Use arrows to represent vectors such as velocity or force, and label the vectors with appropriate symbols. When dealing with forces and applying Newton's laws, make sure to include all forces on a given object, including unknown ones, and make clear what forces act on what object (otherwise you may make an error in determining the *net force* on a particular object). A separate **free-body diagram** needs to be drawn for each object involved, and it must show *all* the forces acting on a given object (and only on that object). Do not show forces that act on other objects.
3. Choose a convenient xy **coordinate system** (one that makes your calculations easier, such as one axis in the direction of the acceleration). Vectors are to be resolved into components along the coordinate axes. When using Newton's second law, apply $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$ separately to x and y components, remembering that x direction forces are related to a_x, and similarly for y. If more than one object is involved, you can choose different (convenient) coordinate systems for each.
4. List the knowns and the unknowns (what you are trying to determine), and decide what you need in order to find the unknowns. For problems in the present Chapter, we use Newton's laws. More generally, it may help to see if one or more **relationships** (or **equations**) relate the unknowns to the knowns. But be sure each relationship is applicable in the given case. It is very important to know the limitations of each formula or relationship—when it is valid and when not. In this book, the more general equations have been given numbers, but even these can have a limited range of validity (often stated in brackets to the right of the equation).
5. Try to solve the problem approximately, to see if it is doable (to check if enough information has been given) and reasonable. Use your intuition, and make **rough calculations**—see "Order of Magnitude Estimating" in Section 1–7. A rough calculation, or a reasonable guess about what the range of final answers might be, is very useful. And a rough calculation can be checked against the final answer to catch errors in calculation, such as in a decimal point or the powers of 10.
6. **Solve** the problem, which may include algebraic manipulation of equations and/or numerical calculations. Recall the mathematical rule that you need as many independent equations as you have unknowns; if you have three unknowns, for example, then you need three independent equations. It is usually best to work out the algebra symbolically before putting in the numbers. Why? Because (*a*) you can then solve a whole class of similar problems with different numerical values; (*b*) you can check your result for cases already understood (say, $\theta = 0°$ or $90°$); (*c*) there may be cancellations or other simplifications; (*d*) there is usually less chance for numerical error; and (*e*) you may gain better insight into the problem.
7. Be sure to keep track of **units**, for they can serve as a check (they must balance on both sides of any equation).
8. Again consider if your answer is **reasonable**. The use of dimensional analysis, described in Section 1–8, can also serve as a check for many problems.

Summary

Newton's three laws of motion are the basic classical laws describing motion.

Newton's first law (the **law of inertia**) states that if the net force on an object is zero, an object originally at rest remains at rest, and an object in motion remains in motion in a straight line with constant velocity.

Newton's second law states that the acceleration of an object is directly proportional to the net force acting on it, and inversely proportional to its mass:

$$\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}. \qquad (4\text{–}1)$$

Newton's second law is one of the most important and fundamental laws in classical physics.

Newton's third law states that whenever one object exerts a force on a second object, the second object always exerts a force on the first object which is equal in magnitude but opposite in direction:

$$\vec{\mathbf{F}}_{AB} = -\vec{\mathbf{F}}_{BA} \tag{4–2}$$

where $\vec{\mathbf{F}}_{BA}$ is the force on object B exerted by object A.

The tendency of an object to resist a change in its motion is called **inertia**. **Mass** is a measure of the inertia of an object.

Weight refers to the **gravitational force** on an object, and is equal to the product of the object's mass m and the acceleration of gravity $\vec{\mathbf{g}}$:

$$\vec{\mathbf{F}}_G = m\vec{\mathbf{g}}. \tag{4–3}$$

Force, which is a vector, can be considered as a push or pull; or, from Newton's second law, force can be defined as an action capable of giving rise to acceleration. The **net force** on an object is the vector sum of all forces acting on it.

When two objects slide over one another, the force of friction that each object exerts on the other can be written approximately as $F_{fr} = \mu_k F_N$, where F_N is the **normal force** (the force each object exerts on the other perpendicular to their contact surfaces), and μ_k is the coefficient of **kinetic friction**. If the objects are at rest relative to each other, then F_{fr} is just large enough to hold them at rest and satisfies the inequality $F_{fr} < \mu_s F_N$, where μ_s is the coefficient of **static friction**.

For solving problems involving the forces on one or more objects, it is essential to draw a **free-body diagram** for each object, showing all the forces acting on only that object. Newton's second law can be applied to the vector components for each object.

Questions

1. Why does a child in a wagon seem to fall backward when you give the wagon a sharp pull forward?
2. A box rests on the (frictionless) bed of a truck. The truck driver starts the truck and accelerates forward. The box immediately starts to slide toward the rear of the truck bed. Discuss the motion of the box, in terms of Newton's laws, as seen (*a*) by Mary standing on the ground beside the truck, and (*b*) by Chris who is riding on the truck (Fig. 4–35).

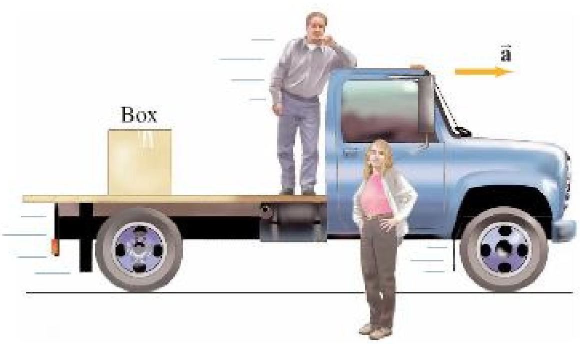


FIGURE 4–35 Question 2.

3. If the acceleration of an object is zero, are no forces acting on it? Explain.
4. Only one force acts on an object. Can the object have zero acceleration? Can it have zero velocity? Explain.
5. When a golf ball is dropped to the pavement, it bounces back up. (*a*) Is a force needed to make it bounce back up? (*b*) If so, what exerts the force?
6. If you walk along a log floating on a lake, why does the log move in the opposite direction?
7. Why might your foot hurt if you kick a heavy desk or a wall?
8. When you are running and want to stop quickly, you must decelerate quickly. (*a*) What is the origin of the force that causes you to stop? (*b*) Estimate (using your own experience) the maximum rate of deceleration of a person running at top speed to come to rest.
9. A stone hangs by a fine thread from the ceiling, and a section of the same thread dangles from the bottom of the stone (Fig. 4–36). If a person gives a sharp pull on the dangling thread, where is the thread likely to break: below the stone or above it? What if the person gives a slow and steady pull? Explain your answers.

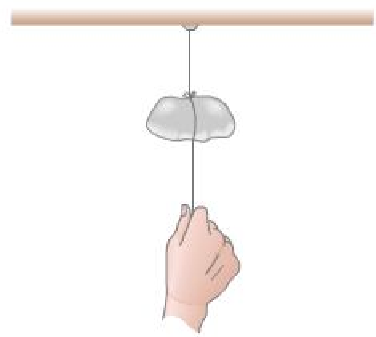


FIGURE 4–36 Question 9.

10. The force of gravity on a 2-kg rock is twice as great as that on a 1-kg rock. Why then doesn't the heavier rock fall faster?
11. Would a spring scale carried to the Moon give accurate results if the scale had been calibrated (*a*) in pounds, or (*b*) in kilograms?
12. You pull a box with a constant force across a frictionless table using an attached rope held horizontally. If you now pull the rope with the same force at an angle to the horizontal (with the box remaining flat on the table), does the acceleration of the box (*a*) remain the same, (*b*) increase, or (*c*) decrease? Explain.
13. When an object falls freely under the influence of gravity there is a net force mg exerted on it by the Earth. Yet by Newton's third law the object exerts an equal and opposite force on the Earth. Why doesn't the Earth move?
14. Compare the effort (or force) needed to lift a 10-kg object when you are on the Moon with the force needed to lift it on Earth. Compare the force needed to throw a 2-kg object horizontally with a given speed on the Moon and on Earth.

15. According to Newton's third law, each team in a tug of war (Fig. 4–37) pulls with equal force on the other team. What, then, determines which team will win?

FIGURE 4–37 Question 15. A tug of war. Describe the forces on each of the teams and on the rope.

16. A person exerts an upward force of 40 N to hold a bag of groceries. Describe the "reaction" force (Newton's third law) by stating (*a*) its magnitude, (*b*) its direction, (*c*) *on* what object it is exerted, and (*d*) *by* what object it is exerted.

17. When you stand still on the ground, how large a force does the ground exert on you? Why doesn't this force make you rise up into the air?

18. Whiplash sometimes results from an automobile accident when the victim's car is struck violently from the rear. Explain why the head of the victim seems to be thrown backward in this situation. Is it really?

19. A heavy crate rests on the bed of a flatbed truck. When the truck accelerates, the crate remains where it is on the truck, so it, too, accelerates. What force causes the crate to accelerate?

20. A block is given a push so that it slides up a ramp. After the block reaches its highest point, it slides back down but the magnitude of its acceleration is less on the descent than on the ascent. Why?

21. What would your bathroom scale read if you weighed yourself on an inclined plane? Assume the mechanism functions properly, even at an angle.

Problems

4–4 to 4–6 Newton's Laws, Gravitational Force, Normal Force

1. (I) What force is needed to accelerate a child on a sled (total mass = 60.0 kg) at $1.25\ \mathrm{m/s^2}$?
2. (I) A net force of 265 N accelerates a bike and rider at $2.30\ \mathrm{m/s^2}$. What is the mass of the bike and rider together?
3. (I) How much tension must a rope withstand if it is used to accelerate a 960-kg car horizontally along a frictionless surface at $1.20\ \mathrm{m/s^2}$?
4. (I) What is the weight of a 76-kg astronaut (*a*) on Earth, (*b*) on the Moon $(g = 1.7\ \mathrm{m/s^2})$, (*c*) on Mars $(g = 3.7\ \mathrm{m/s^2})$, (*d*) in outer space traveling with constant velocity?
5. (II) A 20.0-kg box rests on a table. (*a*) What is the weight of the box and the normal force acting on it? (*b*) A 10.0-kg box is placed on top of the 20.0-kg box, as shown in Fig. 4–38. Determine the normal force that the table exerts on the 20.0-kg box and the normal force that the 20.0-kg box exerts on the 10.0-kg box.

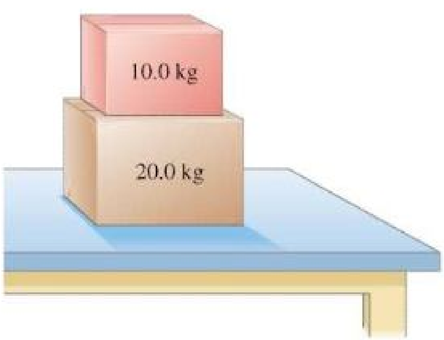


FIGURE 4–38 Problem 5.

6. (II) What average force is required to stop an 1100-kg car in 8.0 s if the car is traveling at 95 km/h?
7. (II) What average force is needed to accelerate a 7.00-gram pellet from rest to 125 m/s over a distance of 0.800 m along the barrel of a rifle?
8. (II) A fisherman yanks a fish vertically out of the water with an acceleration of $2.5\ \mathrm{m/s^2}$ using very light fishing line that has a breaking strength of 22 N. The fisherman unfortunately loses the fish as the line snaps. What can you say about the mass of the fish?
9. (II) A 0.140-kg baseball traveling 35.0 m/s strikes the catcher's mitt, which, in bringing the ball to rest, recoils backward 11.0 cm. What was the average force applied by the ball on the glove?
10. (II) How much tension must a rope withstand if it is used to accelerate a 1200-kg car vertically upward at $0.80\ \mathrm{m/s^2}$?
11. (II) A particular race car can cover a quarter-mile track (402 m) in 6.40 s starting from a standstill. Assuming the acceleration is constant, how many "*g*'s" does the driver experience? If the combined mass of the driver and race car is 485 kg, what horizontal force must the road exert on the tires?
12. (II) A 12.0-kg bucket is lowered vertically by a rope in which there is 163 N of tension at a given instant. What is the acceleration of the bucket? Is it up or down?
13. (II) An elevator (mass 4850 kg) is to be designed so that the maximum acceleration is $0.0680g$. What are the maximum and minimum forces the motor should exert on the supporting cable?
14. (II) A 75-kg petty thief wants to escape from a third-story jail window. Unfortunately, a makeshift rope made of sheets tied together can support a mass of only 58 kg. How might the thief use this "rope" to escape? Give a quantitative answer.
15. (II) A person stands on a bathroom scale in a motionless elevator. When the elevator begins to move, the scale briefly reads only 0.75 of the person's regular weight. Calculate the acceleration of the elevator, and find the direction of acceleration.

16. (II) The cable supporting a 2125-kg elevator has a maximum strength of 21,750 N. What maximum upward acceleration can it give the elevator without breaking?

17. (II) (*a*) What is the acceleration of two falling sky divers (mass 132 kg including parachute) when the upward force of air resistance is equal to one-fourth of their weight? (*b*) After popping open the parachute, the divers descend leisurely to the ground at constant speed. What now is the force of air resistance on the sky divers and their parachute? See Fig. 4–39.

FIGURE 4–39 Problem 17.

18. (III) A person jumps from the roof of a house 3.9-m high. When he strikes the ground below, he bends his knees so that his torso decelerates over an approximate distance of 0.70 m. If the mass of his torso (excluding legs) is 42 kg, find (*a*) his velocity just before his feet strike the ground, and (*b*) the average force exerted on his torso by his legs during deceleration.

4–7 Newton's Laws and Vectors

19. (I) A box weighing 77.0 N rests on a table. A rope tied to the box runs vertically upward over a pulley and a weight is hung from the other end (Fig. 4–40). Determine the force that the table exerts on the box if the weight hanging on the other side of the pulley weighs (*a*) 30.0 N, (*b*) 60.0 N, and (*c*) 90.0 N.

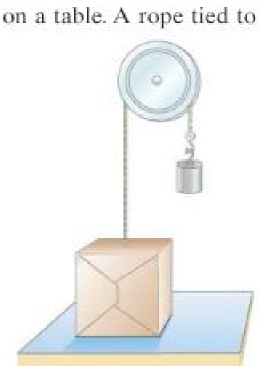


FIGURE 4–40 Problem 19.

20. (I) Draw the free-body diagram for a basketball player (*a*) just before leaving the ground on a jump, and (*b*) while in the air. See Fig. 4–41.

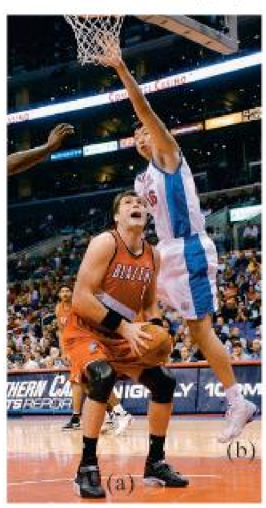


FIGURE 4–41 Problem 20.

21. (I) Sketch the free-body diagram of a baseball (*a*) at the moment it is hit by the bat, and again (*b*) after it has left the bat and is flying toward the outfield.

22. (I) A 650-N force acts in a northwesterly direction. A second 650-N force must be exerted in what direction so that the resultant of the two forces points westward? Illustrate your answer with a vector diagram.

23. (II) Arlene is to walk across a "high wire" strung horizontally between two buildings 10.0 m apart. The sag in the rope when she is at the midpoint is 10.0°, as shown in Fig. 4–42. If her mass is 50.0 kg, what is the tension in the rope at this point?

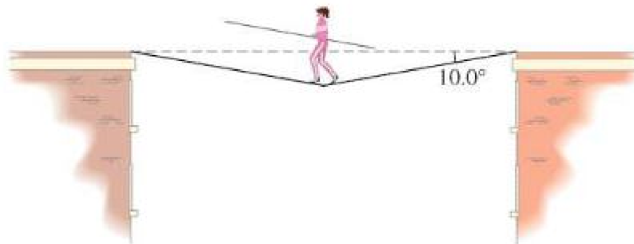


FIGURE 4–42 Problem 23.

24. (II) The two forces $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ shown in Fig. 4–43a and b (looking down) act on a 27.0-kg object on a frictionless tabletop. If $F_1 = 10.2\text{ N}$ and $F_2 = 16.0\text{ N}$, find the net force on the object and its acceleration for (*a*) and (*b*).

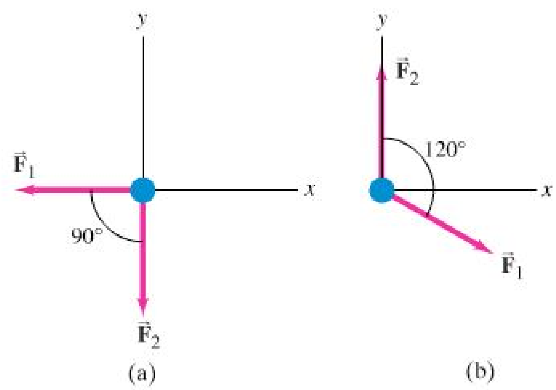


FIGURE 4–43 Problem 24.

25. (II) One 3.2-kg paint bucket is hanging by a massless cord from another 3.2-kg paint bucket, also hanging by a massless cord, as shown in Fig. 4–44. (*a*) If the buckets are at rest, what is the tension in each cord? (*b*) If the two buckets are pulled upward with an acceleration of 1.60 m/s^2 by the upper cord, calculate the tension in each cord.

FIGURE 4–44 Problem 25.

26. (II) A person pushes a 14.0-kg lawn mower at constant speed with a force of $F = 88.0\,\text{N}$ directed along the handle, which is at an angle of 45.0° to the horizontal (Fig. 4–45). (*a*) Draw the free-body diagram showing all forces acting on the mower. Calculate (*b*) the horizontal friction force on the mower, then (*c*) the normal force exerted vertically upward on the mower by the ground. (*d*) What force must the person exert on the lawn mower to accelerate it from rest to 1.5 m/s in 2.5 seconds, assuming the same friction force?

FIGURE 4–45 Problem 26.

27. (II) Two snowcats tow a housing unit to a new location at McMurdo Base, Antarctica, as shown in Fig. 4–46. The sum of the forces $\vec{F}_A$ and $\vec{F}_B$ exerted on the unit by the horizontal cables is parallel to the line L, and $F_A = 4500\,\text{N}$. Determine F_B and the magnitude of $\vec{F}_A + \vec{F}_B$.

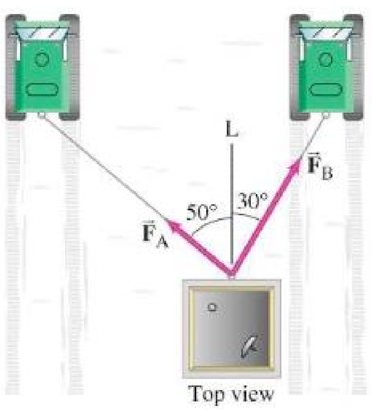


FIGURE 4–46 Problem 27.

28. (II) A train locomotive is pulling two cars of the same mass behind it, Fig. 4–47. Determine the ratio of the tension in the coupling between the locomotive and the first car (F_{T1}), to that between the first car and the second car (F_{T2}), for any nonzero acceleration of the train.

FIGURE 4–47 Problem 28.

29. (II) A window washer pulls herself upward using the bucket–pulley apparatus shown in Fig. 4–48. (*a*) How hard must she pull downward to raise herself slowly at constant speed? (*b*) If she increases this force by 15%, what will her acceleration be? The mass of the person plus the bucket is 65 kg.

FIGURE 4–48 Problem 29.

30. (II) At the instant a race began, a 65-kg sprinter exerted a force of 720 N on the starting block at a 22° angle with respect to the ground. (*a*) What was the horizontal acceleration of the sprinter? (*b*) If the force was exerted for 0.32 s, with what speed did the sprinter leave the starting block?

31. (II) Figure 4–49 shows a block (mass m_A) on a smooth horizontal surface, connected by a thin cord that passes over a pulley to a second block (m_B), which hangs vertically. (*a*) Draw a free-body diagram for each block, showing the force of gravity on each, the force (tension) exerted by the cord, and any normal force. (*b*) Apply Newton's second law to find formulas for the acceleration of the system and for the tension in the cord. Ignore friction and the masses of the pulley and cord.

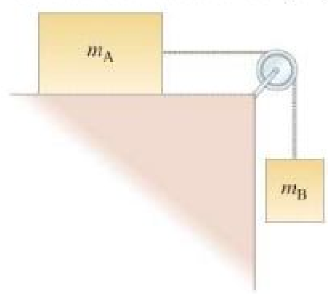


FIGURE 4–49 Problem 31. Mass m_A rests on a smooth horizontal surface, m_B hangs vertically.

32. (II) A pair of fuzzy dice is hanging by a string from your rearview mirror. While you are accelerating from a stoplight to 28 m/s in 6.0 s, what angle θ does the string make with the vertical? See Fig. 4–50.

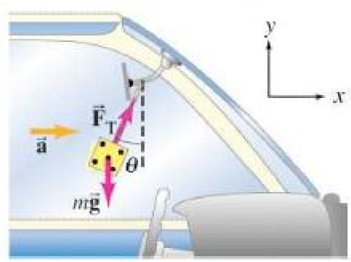


FIGURE 4–50 Problem 32.

33. (III) Three blocks on a frictionless horizontal surface are in contact with each other, as shown in Fig. 4–51. A force $\vec{\mathbf{F}}$ is applied to block A (mass m_A). (*a*) Draw a free-body diagram for each block. Determine (*b*) the acceleration of the system (in terms of m_A, m_B, and m_C), (*c*) the net force on each block, and (*d*) the force of contact that each block exerts on its neighbor. (*e*) If $m_A = m_B = m_C = 12.0$ kg and $F = 96.0$ N, give numerical answers to (*b*), (*c*), and (*d*). Do your answers make sense intuitively?

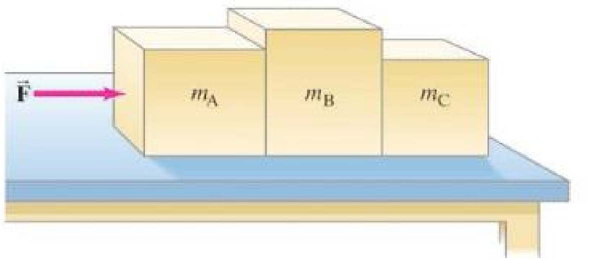


FIGURE 4–51 Problem 33.

34. (III) The two masses shown in Fig. 4–52 are each initially 1.80 m above the ground, and the massless frictionless pulley is 4.8 m above the ground. What maximum height does the lighter object reach after the system is released? [*Hint*: First determine the acceleration of the lighter mass and then its velocity at the moment the heavier one hits the ground. This is its "launch" speed. Assume it doesn't hit the pulley.]

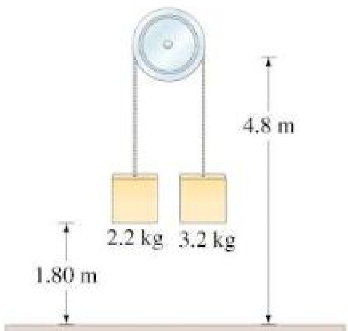


FIGURE 4–52 Problem 34.

35. (III) Suppose two boxes on a frictionless table are connected by a heavy cord of mass 1.0 kg. Calculate the acceleration of each box and the tension at each end of the cord, using the free-body diagrams shown in Fig. 4–53. Assume $F_P = 40.0$ N, and ignore sagging of the cord. Compare your results to Example 4–12 and Fig. 4–22.

4–8 Newton's Laws with Friction; Inclines

36. (I) If the coefficient of kinetic friction between a 35-kg crate and the floor is 0.30, what horizontal force is required to move the crate at a steady speed across the floor? What horizontal force is required if μ_k is zero?

37. (I) A force of 48.0 N is required to start a 5.0-kg box moving across a horizontal concrete floor. (*a*) What is the coefficient of static friction between the box and the floor? (*b*) If the 48.0-N force continues, the box accelerates at $0.70\ \text{m/s}^2$. What is the coefficient of kinetic friction?

38. (I) Suppose that you are standing on a train accelerating at $0.20g$. What minimum coefficient of static friction must exist between your feet and the floor if you are not to slide?

39. (I) What is the maximum acceleration a car can undergo if the coefficient of static friction between the tires and the ground is 0.80?

40. (II) The coefficient of static friction between hard rubber and normal street pavement is about 0.8. On how steep a hill (maximum angle) can you leave a car parked?

41. (II) A 15.0-kg box is released on a 32° incline and accelerates down the incline at $0.30\ \text{m/s}^2$. Find the friction force impeding its motion. What is the coefficient of kinetic friction?

42. (II) A car can decelerate at $-4.80\ \text{m/s}^2$ without skidding when coming to rest on a level road. What would its deceleration be if the road were inclined at 13° uphill? Assume the same static friction coefficient.

43. (II) (*a*) A box sits at rest on a rough 30° inclined plane. Draw the free-body diagram, showing all the forces acting on the box. (*b*) How would the diagram change if the box were sliding down the plane? (*c*) How would it change if the box were sliding up the plane after an initial shove?

44. (II) Drag-race tires in contact with an asphalt surface have a very high coefficient of static friction. Assuming a constant acceleration and no slipping of tires, estimate the coefficient of static friction needed for a drag racer to cover 1.0 km in 12 s, starting from rest.

45. (II) The coefficient of kinetic friction for a 22-kg bobsled on a track is 0.10. What force is required to push it down a 6.0° incline and achieve a speed of 60 km/h at the end of 75 m?

46. (II) For the system of Fig. 4–32 (Example 4–20) how large a mass would box A have to have to prevent any motion from occurring? Assume $\mu_s = 0.30$.

47. (II) A box is given a push so that it slides across the floor. How far will it go, given that the coefficient of kinetic friction is 0.20 and the push imparts an initial speed of 4.0 m/s?

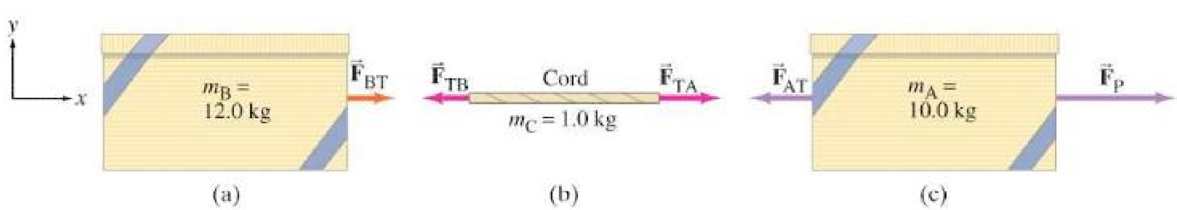


FIGURE 4–53 Problem 35. Free-body diagrams for two boxes on a table connected by a heavy cord, and being pulled to the right as in Fig. 4–22a. Vertical forces, $\vec{\mathbf{F}}_N$ and $\vec{\mathbf{F}}_G$, are not shown.

48. (II) Two crates, of mass 75 kg and 110 kg, are in contact and at rest on a horizontal surface (Fig. 4–54). A 620-N force is exerted on the 75-kg crate. If the coefficient of kinetic friction is 0.15, calculate (*a*) the acceleration of the system, and (*b*) the force that each crate exerts on the other. (*c*) Repeat with the crates reversed.

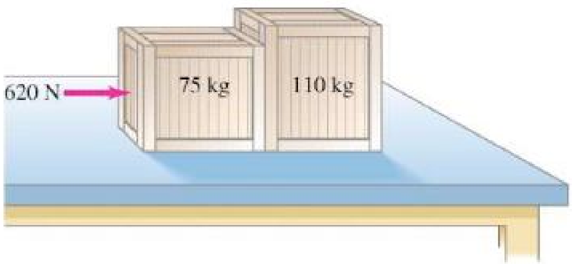


FIGURE 4–54 Problem 48.

49. (II) A flatbed truck is carrying a heavy crate. The coefficient of static friction between the crate and the bed of the truck is 0.75. What is the maximum rate at which the driver can decelerate and still avoid having the crate slide against the cab of the truck?

50. (II) On an icy day, you worry about parking your car in your driveway, which has an incline of 12°. Your neighbor's driveway has an incline of 9.0°, and the driveway across the street is at 6.0°. The coefficient of static friction between tire rubber and ice is 0.15. Which driveway(s) will be safe to park in?

51. (II) A child slides down a slide with a 28° incline, and at the bottom her speed is precisely half what it would have been if the slide had been frictionless. Calculate the coefficient of kinetic friction between the slide and the child.

52. (II) The carton shown in Fig. 4–55 lies on a plane tilted at an angle $\theta = 22.0°$ to the horizontal, with $\mu_k = 0.12$. (*a*) Determine the acceleration of the carton as it slides down the plane. (*b*) If the carton starts from rest 9.30 m up the plane from its base, what will be the carton's speed when it reaches the bottom of the incline?

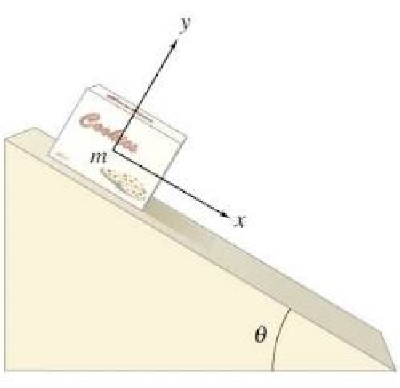


FIGURE 4–55 Carton on inclined plane. Problems 52 and 53.

53. (II) A carton is given an initial speed of 3.0 m/s up the 22.0° plane shown in Fig. 4–55. (*a*) How far up the plane will it go? (*b*) How much time elapses before it returns to its starting point? Ignore friction.

54. (II) A roller coaster reaches the top of the steepest hill with a speed of 6.0 km/h. It then descends the hill, which is at an average angle of 45° and is 45.0 m long. Estimate its speed when it reaches the bottom. Assume $\mu_k = 0.18$.

55. (II) An 18.0-kg box is released on a 37.0° incline and accelerates down the incline at $0.270\ \text{m/s}^2$. Find the friction force impeding its motion. How large is the coefficient of kinetic friction?

56. (II) A small box is held in place against a rough wall by someone pushing on it with a force directed upward at 28° above the horizontal. The coefficients of static and kinetic friction between the box and wall are 0.40 and 0.30, respectively. The box slides down unless the applied force has magnitude 13 N. What is the mass of the box?

57. (II) Piles of snow on slippery roofs can become dangerous projectiles as they melt. Consider a chunk of snow at the ridge of a roof with a pitch of 30°. (*a*) What is the minimum value of the coefficient of static friction that will keep the snow from sliding down? (*b*) As the snow begins to melt, the coefficient of static friction decreases and the snow eventually slips. Assuming that the distance from the chunk to the edge of the roof is 5.0 m and the coefficient of kinetic friction is 0.20, calculate the speed of the snow chunk when it slides off the roof. (*c*) If the edge of the roof is 10.0 m above ground, what is the speed of the snow when it hits the ground?

58. (III) (*a*) Show that the minimum stopping distance for an automobile traveling at speed v is equal to $v^2/2\mu_s g$, where μ_s is the coefficient of static friction between the tires and the road, and g is the acceleration of gravity. (*b*) What is this distance for a 1200-kg car traveling 95 km/h if $\mu_s = 0.75$?

59. (III) A coffee cup on the dashboard of a car slides forward on the dash when the driver decelerates from 45 km/h to rest in 3.5 s or less, but not if he decelerates in a longer time. What is the coefficient of static friction between the cup and the dash?

60. (III) A small block of mass m is given an initial speed v_0 up a ramp inclined at angle θ to the horizontal. It travels a distance d up the ramp and comes to rest. Determine a formula for the coefficient of kinetic friction between block and ramp.

61. (III) The 75-kg climber in Fig. 4–56 is supported in the "chimney" by the friction forces exerted on his shoes and back. The static coefficients of friction between his shoes and the wall, and between his back and the wall, are 0.80 and 0.60, respectively. What is the minimum normal force he must exert? Assume the walls are vertical and that friction forces are both at a maximum. Ignore his grip on the rope.

FIGURE 4–56 Problem 61.

62. (III) Boxes are moved on a conveyor belt from where they are filled to the packing station 11.0 m away. The belt is initially stationary and must finish with zero speed. The most rapid transit is accomplished if the belt accelerates for half the distance, then decelerates for the final half of the trip. If the coefficient of static friction between a box and the belt is 0.60, what is the minimum transit time for each box?

63. (III) A block (mass m_1) lying on a frictionless inclined plane is connected to a mass m_2 by a massless cord passing over a pulley, as shown in Fig. 4–57. (*a*) Determine a formula for the acceleration of the system of the two blocks in terms of m_1, m_2, θ and g. (*b*) What conditions apply to masses m_1 and m_2 for the acceleration to be in one direction (say, m_1 down the plane), or in the opposite direction?

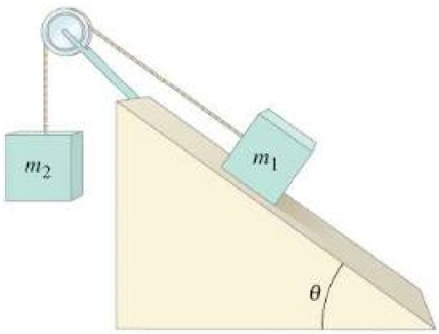


FIGURE 4–57 Problems 63 and 64.

64. (III) (*a*) Suppose the coefficient of kinetic friction between m_1 and the plane in Fig. 4–57 is $\mu_k = 0.15$, and that $m_1 = m_2 = 2.7$ kg. As m_2 moves down, determine the magnitude of the acceleration of m_1 and m_2, given $\theta = 25°$. (*b*) What smallest value of μ_k will keep this system from accelerating?

65. (III) A bicyclist of mass 65 kg (including the bicycle) can coast down a 6.0° hill at a steady speed of 6.0 km/h because of air resistance. How much force must be applied to climb the hill at the same speed and same air resistance?

General Problems

66. According to a simplified model of a mammalian heart, at each pulse approximately 20 g of blood is accelerated from 0.25 m/s to 0.35 m/s during a period of 0.10 s. What is the magnitude of the force exerted by the heart muscle?

67. A person has a reasonable chance of surviving an automobile crash if the deceleration is no more than 30 "g's." Calculate the force on a 70-kg person undergoing this acceleration. What distance is traveled if the person is brought to rest at this rate from 100 km/h?

68. (*a*) If the horizontal acceleration produced by an earthquake is a, and if an object is going to "hold its place" on the ground, show that the coefficient of static friction with the ground must be at least $\mu_s = a/g$. (*b*) The famous Loma Prieta earthquake that stopped the 1989 World Series produced ground accelerations of up to 4.0 m/s^2 in the San Francisco Bay Area. Would a chair have started to slide on a linoleum floor with coefficient of static friction 0.25?

69. An 1150-kg car pulls a 450-kg trailer. The car exerts a horizontal force of 3.8×10^3 N against the ground in order to accelerate. What force does the car exert on the trailer? Assume an effective friction coefficient of 0.15 for the trailer.

70. Police investigators, examining the scene of an accident involving two cars, measure 72-m-long skid marks of one of the cars, which nearly came to a stop before colliding. The coefficient of kinetic friction between rubber and the pavement is about 0.80. Estimate the initial speed of that car assuming a level road.

71. A car starts rolling down a 1-in-4 hill (1-in-4 means that for each 4 m traveled along the road, the elevation change is 1 m). How fast is it going when it reaches the bottom after traveling 55 m? (*a*) Ignore friction. (*b*) Assume an effective coefficient of friction equal to 0.10.

72. A 2.0-kg purse is dropped from the top of the Leaning Tower of Pisa and falls 55 m before reaching the ground with a speed of 29 m/s. What was the average force of air resistance?

73. A cyclist is coasting at a steady speed of 12 m/s but enters a muddy stretch where the effective coefficient of friction is 0.60. Will the cyclist emerge from the muddy stretch without having to pedal if the mud lasts for 11 m? If so, what will be the speed upon emerging?

74. A city planner is working on the redesign of a hilly portion of a city. An important consideration is how steep the roads can be so that even low-powered cars can get up the hills without slowing down. A particular small car, with a mass of 1100 kg, can accelerate on a level road from rest to 21 m/s (75 km/h) in 14.0 s. Using these data, calculate the maximum steepness of a hill.

75. Francesca, who likes physics experiments, dangles her watch from a thin piece of string while the jetliner she is in takes off from JFK Airport (Fig. 4–58). She notices that the string makes an angle of 25° with respect to the vertical as the aircraft accelerates for takeoff, which takes about 18 s. Estimate the takeoff speed of the aircraft.

FIGURE 4–58 Problem 75.

76. A 28.0-kg block is connected to an empty 1.35-kg bucket by a cord running over a frictionless pulley (Fig. 4–59). The coefficient of static friction between the table and the block is 0.450 and the coefficient of kinetic friction between the table and the block is 0.320. Sand is gradually added to the bucket until the system just begins to move. (*a*) Calculate the mass of sand added to the bucket. (*b*) Calculate the acceleration of the system.

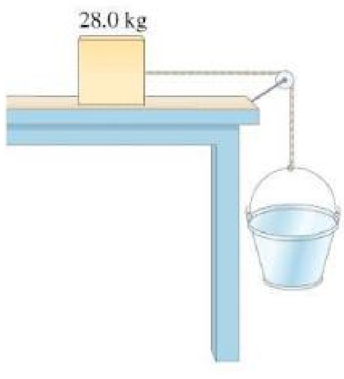


FIGURE 4–59 Problem 76.

77. In the design of a supermarket, there are to be several ramps connecting different parts of the store. Customers will have to push grocery carts up the ramps and it is obviously desirable that this not be too difficult. The engineer has done a survey and found that almost no one complains if the force directed up the ramp is no more than 20 N. Ignoring friction, at what maximum angle θ should the ramps be built, assuming a full 30-kg grocery cart?

78. (*a*) What minimum force F is needed to lift the piano (mass M) using the pulley apparatus shown in Fig. 4–60? (*b*) Determine the tension in each section of rope: F_{T1}, F_{T2}, F_{T3}, and F_{T4}.

FIGURE 4–60 Problem 78.

79. A jet aircraft is accelerating at 3.5 m/s^2 at an angle of 45° above the horizontal. What is the total force that the cockpit seat exerts on the 75-kg pilot?

80. In the design process for a child-restraint chair, an engineer considers the following set of conditions: A 12-kg child is riding in the chair, which is securely fastened to the seat of an automobile (Fig. 4–61). Assume the automobile is involved in a head-on collision with another vehicle. The initial speed v_0 of the car is 45 km/h, and this speed is reduced to zero during the collision time of 0.20 s. Assume a constant car deceleration during the collision and estimate the net horizontal force F that the straps of the restraint chair must exert on the child in order to keep her fixed to the chair. Treat the child as a particle and state any additional assumptions made during your analysis.

FIGURE 4–61 Problem 80.

81. A 7650-kg helicopter accelerates upward at 0.80 m/s^2 while lifting a 1250-kg frame at a construction site, Fig. 4–62. (*a*) What is the lift force exerted by the air on the helicopter rotors? (*b*) What is the tension in the cable (ignore its mass) that connects the frame to the helicopter? (*c*) What force does the cable exert on the helicopter?

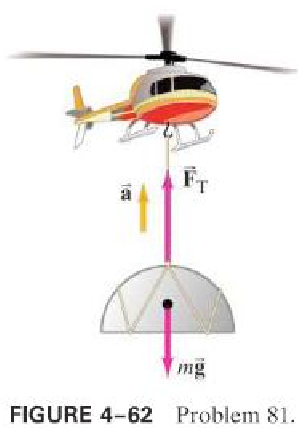


FIGURE 4–62 Problem 81.

82. A super high-speed 12-car Italian train has a mass of 660 metric tons (660,000 kg). It can exert a maximum force of 400 kN horizontally against the tracks, whereas at maximum velocity (300 km/h), it exerts a force of about 150 kN. Calculate (*a*) its maximum acceleration, and (*b*) estimate the force of air resistance at top speed.

83. A 65-kg ice skater coasts with no effort for 75 m until she stops. If the coefficient of kinetic friction between her skates and the ice is $\mu_k = 0.10$, how fast was she moving at the start of her coast?

84. Two rock climbers, Bill and Karen, use safety ropes of similar length. Karen's rope is more elastic, called a *dynamic rope* by climbers. Bill has a *static rope*, not recommended for safety purposes in pro climbing. Karen falls freely about 2.0 m and then the rope stops her over a distance of 1.0 m (Fig. 4–63). (*a*) Estimate, assuming that the force is constant, how large a force she will feel from the rope. (Express the result in multiples of her weight.) (*b*) In a similar fall, Bill's rope stretches by 30 cm only. How many times his weight will the rope pull on him? Which climber is more likely to be hurt?

FIGURE 4–63 Problem 84.

85. A fisherman in a boat is using a "10-lb test" fishing line. This means that the line can exert a force of 45 N without breaking (1 lb = 4.45 N). (*a*) How heavy a fish can the fisherman land if he pulls the fish up vertically at constant speed? (*b*) If he accelerates the fish upward at $2.0\,\mathrm{m/s^2}$, what maximum weight fish can he land? (*c*) Is it possible to land a 15-lb trout on 10-lb test line? Why or why not?

86. An elevator in a tall building is allowed to reach a maximum speed of 3.5 m/s going down. What must the tension be in the cable to stop this elevator over a distance of 2.6 m if the elevator has a mass of 1300 kg including occupants?

87. Two boxes, $m_1 = 1.0\,\mathrm{kg}$ with a coefficient of kinetic friction of 0.10, and $m_2 = 2.0\,\mathrm{kg}$ with a coefficient of 0.20, are placed on a plane inclined at $\theta = 30°$. (*a*) What acceleration does each box experience? (*b*) If a taut string is connected to the boxes (Fig. 4–64), with m_2 initially farther down the slope, what is the acceleration of each box? (*c*) If the initial configuration is reversed with m_1 starting lower with a taut string, what is the acceleration of each box?

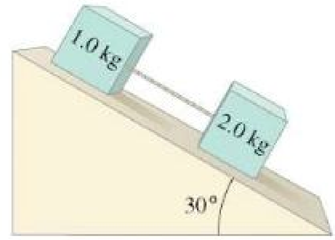


FIGURE 4–64 Problem 87.

88. A 75.0-kg person stands on a scale in an elevator. What does the scale read (in N and in kg) when the elevator is (*a*) at rest, (*b*) ascending at a constant speed of 3.0 m/s, (*c*) falling at 3.0 m/s, (*d*) accelerating upward at $3.0\,\mathrm{m/s^2}$, (*e*) accelerating downward at $3.0\,\mathrm{m/s^2}$?

89. Three mountain climbers who are roped together are ascending an icefield inclined at 21.0° to the horizontal. The last climber slips, pulling the second climber off his feet. The first climber is able to hold them both. If each climber has a mass of 75 kg, calculate the tension in each of the two sections of rope between the three climbers. Ignore friction between the ice and the fallen climbers.

Answers to Exercises

A: (*a*) The same; (*b*) the sports car; (*c*) third law for part (a), second law for part (b).

B: The force applied by the person is insufficient to keep the box moving.

C: No; yes.

D: Yes; no.