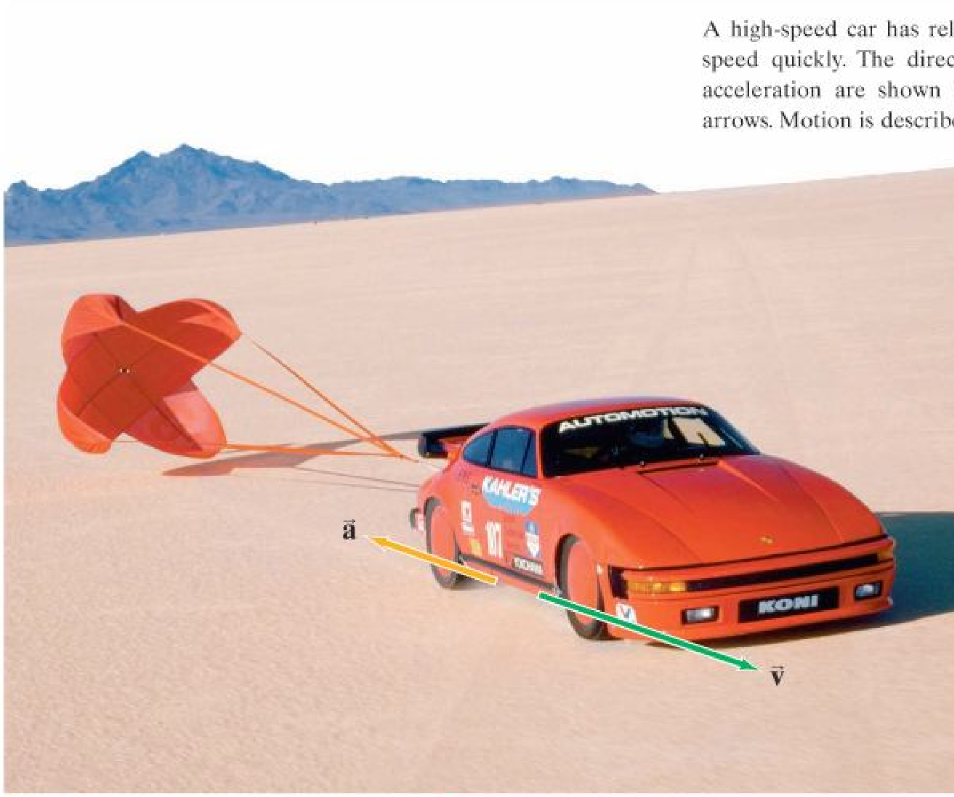


A high-speed car has released a parachute to reduce its speed quickly. The directions of the car's velocity and acceleration are shown by the green ($\vec{v}$) and gold ($\vec{a}$) arrows. Motion is described using the concepts of velocity and acceleration. We see here that the acceleration $\vec{a}$ can sometimes be in the opposite direction from the velocity $\vec{v}$. We will also examine in detail motion with constant acceleration, including the vertical motion of objects falling under gravity.

CHAPTER 2

Describing Motion: Kinematics in One Dimension

The motion of objects—baseballs, automobiles, joggers, and even the Sun and Moon—is an obvious part of everyday life. It was not until the sixteenth and seventeenth centuries that our modern understanding of motion was established. Many individuals contributed to this understanding, particularly Galileo Galilei (1564–1642) and Isaac Newton (1642–1727).

The study of the motion of objects, and the related concepts of force and energy, form the field called **mechanics**. Mechanics is customarily divided into two parts: **kinematics**, which is the description of how objects move, and **dynamics**, which deals with force and why objects move as they do. This Chapter and the next deal with kinematics.

For now we only discuss objects that move without rotating (Fig. 2–1a). Such motion is called **translational motion**. In this Chapter we will be concerned with describing an object that moves along a straight-line path, which is one-dimensional translational motion. In Chapter 3 we will describe translational motion in two (or three) dimensions along paths that are not straight. (We discuss rotation, as in Fig. 2–1b, in Chapter 8.)

We will often use the concept, or *model*, of an idealized **particle** which is considered to be a mathematical point and to have no spatial extent (no size). A particle can undergo only translational motion. The particle model is useful in many real situations where we are interested only in translational motion and the object's size is not so significant. For example, we might consider a billiard ball, or even a spacecraft traveling toward the Moon, as a particle for many purposes.

(a) (b)

FIGURE 2–1 The pinecone in (a) undergoes pure translation as it falls, whereas in (b) it is rotating as well as translating.

2–1 Reference Frames and Displacement

All measurements are made relative to a frame of reference

Any measurement of position, distance, or speed must be made with respect to a **reference frame**, or **frame of reference**. For example, while you are on a train traveling at 80 km/h, suppose a person walks past you toward the front of the train at a speed of, say, 5 km/h (Fig. 2–2). This 5 km/h is the person's speed with respect to the train as frame of reference. With respect to the ground, that person is moving at a speed of $80\,\text{km/h} + 5\,\text{km/h} = 85\,\text{km/h}$. It is always important to specify the frame of reference when stating a speed. In everyday life, we usually mean "with respect to the Earth" without even thinking about it, but the reference frame must be specified whenever there might be confusion.

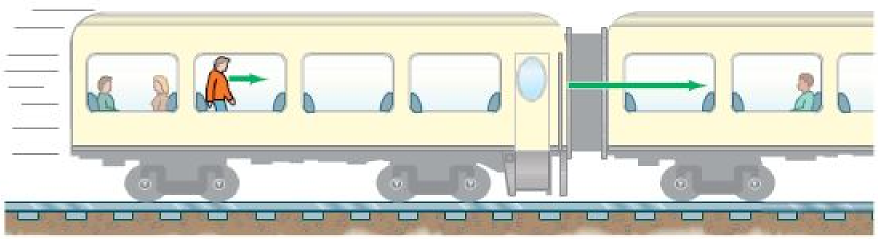


FIGURE 2–2 A person walks toward the front of a train at 5 km/h. The train is moving 80 km/h with respect to the ground, so the walking person's speed, relative to the ground, is 85 km/h.

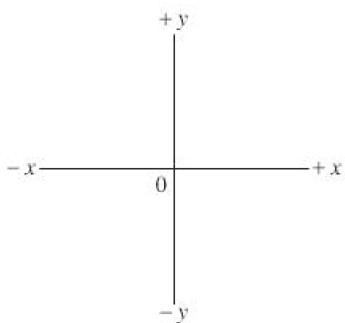


FIGURE 2–3 Standard set of *xy* coordinate axes.

When specifying the motion of an object, it is important to specify not only the speed but also the direction of motion. Often we can specify a direction by using north, east, south, and west, and by "up" and "down." In physics, we often draw a set of **coordinate axes**, as shown in Fig. 2–3, to represent a frame of reference. We can always place the origin 0, and the directions of the x and y axes, as we like for convenience. The x and y axes are always perpendicular to each other. Objects positioned to the right of the origin of coordinates (0) on the x axis have an x coordinate which we usually choose to be positive; objects to the left of 0 then have a negative x coordinate. The position along the y axis is usually considered positive when above 0, and negative when below 0, although the reverse convention can be used if convenient. Any point on the plane can be specified by giving its x and y coordinates. In three dimensions, a z axis perpendicular to the x and y axes is added.

For one-dimensional motion, we often choose the x axis as the line along which the motion takes place. Then the **position** of an object at any moment is given by its x coordinate. If the motion is vertical, as for a dropped object, we usually use the y axis.

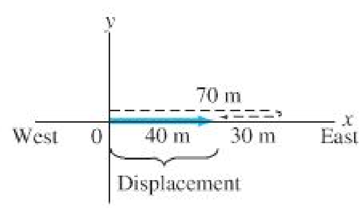
Displacement

We need to make a distinction between the *distance* an object has traveled and its **displacement**, which is defined as the *change in position* of the object. That is, *displacement is how far the object is from its starting point.* To see the distinction between total distance and displacement, imagine a person walking 70 m to the east and then turning around and walking back (west) a distance of 30 m (see Fig. 2–4). The total *distance* traveled is 100 m, but the *displacement* is only 40 m since the person is now only 40 m from the starting point.

CAUTION

The displacement may not equal the total distance traveled

FIGURE 2–4 A person walks 70 m east, then 30 m west. The total distance traveled is 100 m (path is shown dashed in black); but the displacement, shown as a blue arrow, is 40 m to the east.



Displacement is a quantity that has both magnitude and direction. Such quantities are called **vectors**, and are represented by arrows in diagrams. For example, in Fig. 2–4, the blue arrow represents the displacement whose magnitude is 40 m and whose direction is to the right (east).

We will deal with vectors more fully in Chapter 3. For now, we deal only with motion in one dimension, along a line. In this case, vectors which point in one direction will have a positive sign, whereas vectors that point in the opposite direction will have a negative sign, along with their magnitude.

Consider the motion of an object over a particular time interval. Suppose that at some initial time, call it t_1, the object is on the x axis at the position x_1 in the coordinate system shown in Fig. 2–5. At some later time, t_2, suppose the object has moved to position x_2. The displacement of our object is $x_2 - x_1$, and is represented by the arrow pointing to the right in Fig. 2–5. It is convenient to write

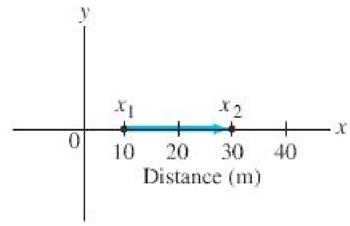


FIGURE 2–5 The arrow represents the displacement $x_2 - x_1$. Distances are in meters.

$$\Delta x = x_2 - x_1,$$

where the symbol Δ (Greek letter delta) means "change in." Then Δx means "the change in x," or "change in position," which is the displacement. Note that the "change in" any quantity means the final value of that quantity, minus the initial value.

Δ means final value minus initial value

Suppose $x_1 = 10.0\text{ m}$ and $x_2 = 30.0\text{ m}$. Then

$$\Delta x = x_2 - x_1 = 30.0\text{ m} - 10.0\text{ m} = 20.0\text{ m},$$

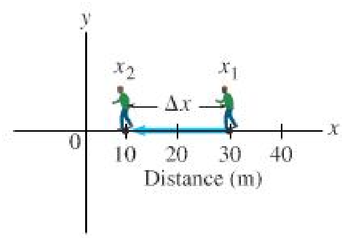
so the displacement is 20.0 m in the positive direction, as in Fig. 2–5.

Now consider an object moving to the left as shown in Fig. 2–6. Here the object, say, a person, starts at $x_1 = 30.0\text{ m}$ and walks to the left to the point $x_2 = 10.0\text{ m}$. In this case

$$\Delta x = x_2 - x_1 = 10.0\text{ m} - 30.0\text{ m} = -20.0\text{ m},$$

and the blue arrow representing the vector displacement points to the left. The displacement is 20.0 m in the negative direction. This example illustrates that for one-dimensional motion along the x axis, a vector pointing to the right has a positive sign, whereas a vector pointing to the left has a negative sign.

FIGURE 2–6 For the displacement $\Delta x = x_2 - x_1 = 10.0\text{ m} - 30.0\text{ m}$, the displacement vector points to the left.



2–2 Average Velocity

Consider a racing sprinter, a galloping horse, a speeding Ferrari, or a rocket shot off into space. The most obvious aspect of their motion is how fast they are moving, which brings us to the idea of speed and velocity.

The term "speed" refers to how far an object travels in a given time interval, regardless of direction. If a car travels 240 kilometers (km) in 3 hours (h), we say its average speed was 80 km/h. In general, the **average speed** of an object is defined as *the total distance traveled along its path divided by the time it takes to travel this distance*:

$$\text{average speed} = \frac{\text{distance traveled}}{\text{time elapsed}}. \qquad \textbf{(2–1)}$$

Average speed

The terms "velocity" and "speed" are often used interchangeably in ordinary language. But in physics we make a distinction between the two. Speed is simply a positive number, with units. **Velocity**, on the other hand, is used to signify both the *magnitude* (numerical value) of how fast an object is moving and also the *direction* in which it is moving. (Velocity is therefore a vector.) There is a second difference between speed and velocity: namely, the **average velocity** is defined in terms of *displacement*, rather than total distance traveled:

Velocity

$$\text{average velocity} = \frac{\text{displacement}}{\text{time elapsed}} = \frac{\text{final position} - \text{initial position}}{\text{time elapsed}}.$$

Average velocity

CAUTION

Average speed is not necessarily equal to the magnitude of the average velocity

Average speed and average velocity have the same magnitude when the motion is all in one direction. In other cases, they may differ: recall the walk we described earlier, in Fig. 2–4, where a person walked 70 m east and then 30 m west. The total distance traveled was 70 m + 30 m = 100 m, but the displacement was 40 m. Suppose this walk took 70 s to complete. Then the average speed was:

$$\frac{\text{distance}}{\text{time elapsed}} = \frac{100\text{ m}}{70\text{ s}} = 1.4\text{ m/s}.$$

The magnitude of the average velocity, on the other hand, was:

$$\frac{\text{displacement}}{\text{time elapsed}} = \frac{40\text{ m}}{70\text{ s}} = 0.57\text{ m/s}.$$

This difference between the speed and the magnitude of the velocity can occur when we calculate *average* values.

To discuss one-dimensional motion of an object in general, suppose that at some moment in time, call it t_1, the object is on the x axis at position x_1 in a coordinate system, and at some later time, t_2, suppose it is at position x_2. The elapsed time is $t_2 - t_1$; during this time interval the displacement of our object is $\Delta x = x_2 - x_1$. Then the average velocity, defined as *the displacement divided by the elapsed time*, can be written

Average velocity

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}, \qquad \textbf{(2–2)}$$

where v stands for velocity and the bar ($^{-}$) over the v is a standard symbol meaning "average."

The **elapsed time**, or **time interval**, $t_2 - t_1$, is the time that has passed during our chosen period of observation.

PROBLEM SOLVING

+ or − sign can signify the direction for linear motion

For the usual case of the $+x$ axis to the right, note that if x_2 is less than x_1, the object is moving to the left, and then $\Delta x = x_2 - x_1$ is less than zero. The sign of the displacement, and thus of the average velocity, indicates the direction: the average velocity is positive for an object moving to the right along the $+x$ axis and negative when the object moves to the left. The direction of the average velocity is always the same as the direction of the displacement.

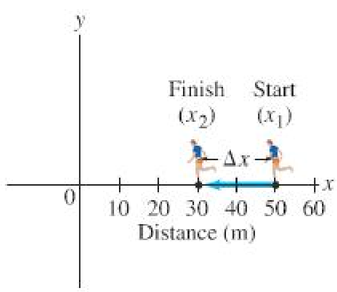


FIGURE 2–7 Example 2–1. A person runs from $x_1 = 50.0$ m to $x_2 = 30.5$ m. The displacement is -19.5 m.

EXAMPLE 2–1 Runner's average velocity. The position of a runner as a function of time is plotted as moving along the x axis of a coordinate system. During a 3.00-s time interval, the runner's position changes from $x_1 = 50.0$ m to $x_2 = 30.5$ m, as shown in Fig. 2–7. What was the runner's average velocity?

APPROACH We want to find the average velocity, which is the displacement divided by the elapsed time.

SOLUTION The displacement is $\Delta x = x_2 - x_1 = 30.5\text{ m} - 50.0\text{ m} = -19.5\text{ m}$. The elapsed time, or time interval, is $\Delta t = 3.00$ s. The average velocity is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{-19.5\text{ m}}{3.00\text{ s}} = -6.50\text{ m/s}.$$

The displacement and average velocity are negative, which tells us that the runner is moving to the left along the x axis, as indicated by the arrow in Fig. 2–7. Thus we can say that the runner's average velocity is 6.50 m/s to the left.

EXAMPLE 2–2 Distance a cyclist travels. How far can a cyclist travel in 2.5 h along a straight road if her average velocity is 18 km/h?

APPROACH We are given the average velocity and the time interval (= 2.5 h). We want to find the distance traveled, so we solve Eq. 2–2 for Δx.

SOLUTION We rewrite Eq. 2–2 as $\Delta x = \bar{v}\,\Delta t$, and find

$$\Delta x = \bar{v}\,\Delta t = (18\text{ km/h})(2.5\text{ h}) = 45\text{ km}.$$

2–3 Instantaneous Velocity

If you drive a car 150 km along a straight road in one direction for 2.0 h, the magnitude of your average velocity is 75 km/h. It is unlikely, though, that you were moving at precisely 75 km/h at every instant. To deal with this situation we need the concept of *instantaneous velocity*, which is the velocity at any instant of time. (Its magnitude is the number, with units, indicated by a speedometer; Fig. 2–8.) More precisely, the **instantaneous velocity** at any moment is defined as *the average velocity during an infinitesimally short time interval*. That is, starting with Eq. 2–2,

$$\bar{v} = \frac{\Delta x}{\Delta t},$$

we define instantaneous velocity as the average velocity as we let Δt become extremely small, approaching zero. We can write the definition of instantaneous velocity, v, for one-dimensional motion as

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}. \qquad \textbf{(2–3)}$$

Instantaneous velocity

FIGURE 2–8 Car speedometer showing mi/h in white, and km/h in orange.

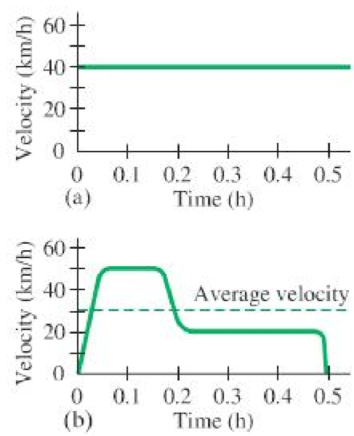
The notation $\lim_{\Delta t \to 0}$ means the ratio $\Delta x/\Delta t$ is to be evaluated in the limit of Δt approaching zero.

For instantaneous velocity we use the symbol v, whereas for average velocity we use $\bar{v}$, with a bar. In the rest of this book, when we use the term "velocity," it will refer to instantaneous velocity. When we want to speak of the average velocity, we will make this clear by including the word "average."

Note that the *instantaneous* speed always equals the magnitude of the instantaneous velocity. Why? Because the distance and the magnitude of the displacement become the same when they become infinitesimally small.

If an object moves at a uniform (that is, constant) velocity during a particular time interval, then its instantaneous velocity at any instant is the same as its average velocity (see Fig. 2–9a). But in many situations this is not the case. For example, a car may start from rest, speed up to 50 km/h, remain at that velocity for a time, then slow down to 20 km/h in a traffic jam, and finally stop at its destination after traveling a total of 15 km in 30 min. This trip is plotted on the graph of Fig. 2–9b. Also shown on the graph is the average velocity (dashed line), which is $\bar{v} = \Delta x/\Delta t = 15\text{ km}/0.50\text{ h} = 30\text{ km/h}$.

FIGURE 2–9 Velocity of a car as a function of time: (a) at constant velocity; (b) with varying velocity.



2–4 Acceleration

An object whose velocity is changing is said to be accelerating. For instance, a car whose velocity increases in magnitude from zero to 80 km/h is accelerating. Acceleration specifies how rapidly the velocity of an object is changing.

Average acceleration is defined as the change in velocity divided by the time taken to make this change:

$$\text{average acceleration} = \frac{\text{change of velocity}}{\text{time elapsed}}.$$

In symbols, the average acceleration, $\bar{a}$, during a time interval $\Delta t = t_2 - t_1$ over which the velocity changes by $\Delta v = v_2 - v_1$, is defined as

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}. \qquad \textbf{(2–4)}$$

Average acceleration

Acceleration is also a vector, but for one-dimensional motion, we need only use a plus or minus sign to indicate direction relative to a chosen coordinate system.

The **instantaneous acceleration**, a, can be defined in analogy to instantaneous velocity, for any specific instant:

Instantaneous acceleration

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}. \qquad \textbf{(2–5)}$$

Here Δv is the very small change in velocity during the very short time interval Δt.

EXAMPLE 2–3 **Average acceleration.** A car accelerates along a straight road from rest to 75 km/h in 5.0 s, Fig. 2–10. What is the magnitude of its average acceleration?

At rest means $v = 0$

APPROACH Average acceleration is the change in velocity divided by elapsed time, 5.0 s. The car starts from rest, so $v_1 = 0$. The final velocity is $v_2 = 75\ \text{km/h}$.

SOLUTION From Eq. 2–4, the average acceleration is

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{75\ \text{km/h} - 0\ \text{km/h}}{5.0\ \text{s}} = 15\,\frac{\text{km/h}}{\text{s}}.$$

This is read as "fifteen kilometers per hour per second" and means that, on average, the velocity changed by 15 km/h during each second. That is, assuming the acceleration was constant, during the first second the car's velocity increased from zero to 15 km/h. During the next second its velocity increased by another 15 km/h, reaching a velocity of 30 km/h at $t = 2.0\ \text{s}$, and so on. See Fig. 2–10.

NOTE Our result contains two different time units: hours and seconds. We usually prefer to use only seconds. To do so we can change km/h to m/s (see Section 1–6, and Example 1–5):

$$75\ \text{km/h} = \left(75\,\frac{\cancel{\text{km}}}{\cancel{\text{h}}}\right)\left(\frac{1000\ \text{m}}{1\ \cancel{\text{km}}}\right)\left(\frac{1\ \cancel{\text{h}}}{3600\ \text{s}}\right) = 21\ \text{m/s}.$$

Then

$$\bar{a} = \frac{21\ \text{m/s} - 0.0\ \text{m/s}}{5.0\ \text{s}} = 4.2\,\frac{\text{m/s}}{\text{s}} = 4.2\,\frac{\text{m}}{\text{s}^2}.$$

We almost always write the units for acceleration as m/s^2 (meters per second squared), as we just did, instead of m/s/s. This is possible because:

$$\frac{\text{m/s}}{\text{s}} = \frac{\text{m}}{\text{s}\cdot\text{s}} = \frac{\text{m}}{\text{s}^2}.$$

According to the calculation in Example 2–3, the velocity changed on the average by 4.2 m/s during each second, for a total change of 21 m/s over the 5.0 s.

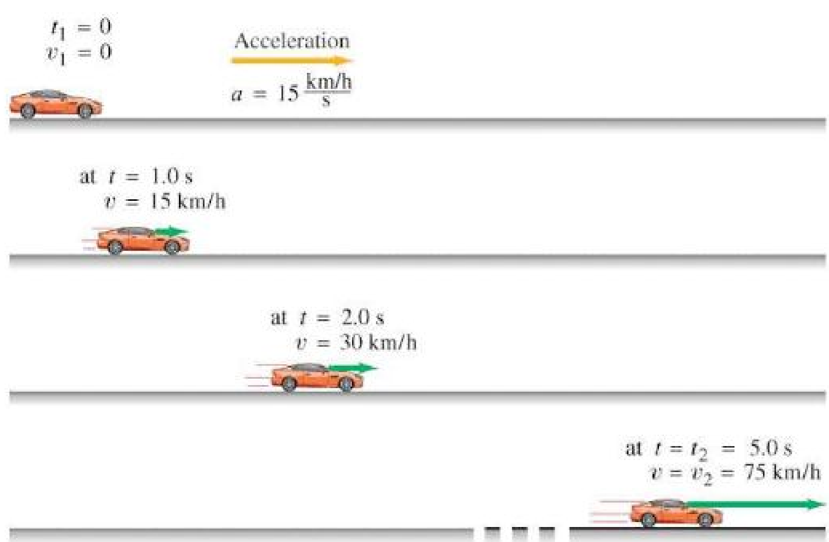


FIGURE 2–10 Example 2–3. The car is shown at the start with $v_1 = 0$ at $t_1 = 0$. The car is shown three more times, at $t = 1.0\ \text{s}$, $t = 2.0\ \text{s}$, and at the end of our time interval, $t_2 = 5.0\ \text{s}$. We assume the acceleration is constant and equals 15 km/h/s. The green arrows represent the velocity vectors; the length of each arrow represents the magnitude of the velocity at that moment. The acceleration vector is the orange arrow. Distances are not to scale.

Note that *acceleration tells us how quickly the velocity changes*, whereas *velocity tells us how quickly the position changes.*

CAUTION
Distinguish velocity from acceleration

CONCEPTUAL EXAMPLE 2–4 **Velocity and acceleration.** (*a*) If the velocity of an object is zero, does it mean that the acceleration is zero? (*b*) If the acceleration is zero, does it mean that the velocity is zero? Think of some examples.

CAUTION
If v or a is zero, is the other zero too?

RESPONSE A zero velocity does not necessarily mean that the acceleration is zero, nor does a zero acceleration mean that the velocity is zero. (*a*) For example, when you put your foot on the gas pedal of your car which is at rest, the velocity starts from zero but the acceleration is not zero since the velocity of the car changes. (How else could your car start forward if its velocity weren't changing—that is, accelerating?) (*b*) As you cruise along a straight highway at a constant velocity of 100 km/h, your acceleration is zero: $a = 0$, $v \neq 0$.

EXERCISE A A car is advertised to go from zero to 60 mi/h in 6.0 s. What does this say about the car: (*a*) it is fast (high speed); or (*b*) it accelerates well?

EXAMPLE 2–5 **Car slowing down.** An automobile is moving to the right along a straight highway, which we choose to be the positive *x* axis (Fig. 2–11). Then the driver puts on the brakes. If the initial velocity (when the driver hits the brakes) is $v_1 = 15.0\,\mathrm{m/s}$, and it takes 5.0 s to slow down to $v_2 = 5.0\,\mathrm{m/s}$, what was the car's average acceleration?

APPROACH We are given the initial and final velocities and the elapsed time, so we can calculate $\bar{a}$ using Eq. 2–4.

SOLUTION We use Eq. 2–4 and call the initial time $t_1 = 0$; then $t_2 = 5.0\,\mathrm{s}$. (Note that our choice of $t_1 = 0$ doesn't affect the calculation of $\bar{a}$ because only $\Delta t = t_2 - t_1$ appears in Eq. 2–4.) Then

$$\bar{a} = \frac{5.0\,\mathrm{m/s} - 15.0\,\mathrm{m/s}}{5.0\,\mathrm{s}} = -2.0\,\mathrm{m/s^2}.$$

The negative sign appears because the final velocity is less than the initial velocity. In this case the direction of the acceleration is to the left (in the negative *x* direction)—even though the velocity is always pointing to the right. We say that the acceleration is $2.0\,\mathrm{m/s^2}$ to the left, and it is shown in Fig. 2–11 as an orange arrow.

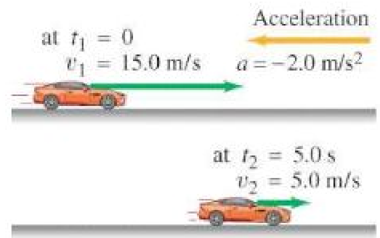


FIGURE 2–11 Example 2–5, showing the position of the car at times t_1 and t_2, as well as the car's velocity represented by the green arrows. The acceleration vector (orange) points to the left as the car slows down while moving to the right.

Deceleration

When an object is slowing down, we sometimes say it is **decelerating**. But be careful: deceleration does *not* mean that the acceleration is necessarily negative. For an object moving to the right along the positive *x* axis and slowing down (as in Fig. 2–11), the acceleration *is* negative. But the same car moving to the left (decreasing *x*), and slowing down, has positive acceleration that points to the right, as shown in Fig. 2–12. We have a deceleration whenever the magnitude of the velocity is decreasing, and then the velocity and acceleration point in opposite directions.

CAUTION
Deceleration means the magnitude of the velocity is decreasing; it does not *necessarily mean a is negative*

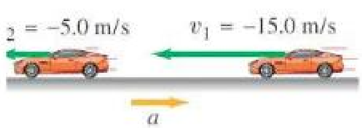


FIGURE 2–12 The car of Example 2–5, now moving to the *left* and decelerating. The acceleration is

$$a = \frac{v_2 - v_1}{\Delta t} = \frac{-5.0\,\mathrm{m/s} - (-15.0\,\mathrm{m/s})}{5.0\,\mathrm{s}} = \frac{-5.0\,\mathrm{m/s} + 15.0\,\mathrm{m/s}}{5.0\,\mathrm{s}} = +2.0\,\mathrm{m/s}.$$

EXERCISE B A car moves along the *x* axis. What is the sign of the car's acceleration if it is moving in the positive *x* direction with (*a*) increasing speed or (*b*) decreasing speed? What is the sign of the acceleration if the car moves in the negative direction with (*c*) increasing speed or (*d*) decreasing speed?

2–5 Motion at Constant Acceleration

Let a = constant

Many practical situations occur in which the acceleration is constant or nearly constant. We now examine this situation when the magnitude of the acceleration is constant and the motion is in a straight line. In this case, the instantaneous and average accelerations are equal.

We now use our definitions of velocity and acceleration to derive a set of extremely useful equations that relate x, v, a, and t when a is constant, allowing us to determine any one of these variables if we know the others.

x (at $t = 0$) = x_0
v (at $t = 0$) = v_0
t = elapsed time

To simplify our notation, let us take the initial time in any discussion to be zero, and we call it t_0: $t_1 = t_0 = 0$. (This is effectively starting a stopwatch at t_0.) We can then let $t_2 = t$ be the elapsed time. The initial position (x_1) and the initial velocity (v_1) of an object will now be represented by x_0 and v_0, since they represent x and v at $t = 0$. At time t the position and velocity will be called x and v (rather than x_2 and v_2). The average velocity during the time interval $t - t_0$ will be (Eq. 2–2)

$$\bar{v} = \frac{x - x_0}{t - t_0} = \frac{x - x_0}{t}$$

since we chose $t_0 = 0$. The acceleration, assumed constant in time, is (Eq. 2–4)

$$a = \frac{v - v_0}{t}.$$

A common problem is to determine the velocity of an object after any elapsed time t, when we are given the object's constant acceleration. We can solve such problems by solving for v in the last equation to obtain:

v related to a and t (a = constant, t = elapsed time)

$$v = v_0 + at. \qquad \text{[constant acceleration]} \quad \textbf{(2–6)}$$

For example, it may be known that the acceleration of a particular motorcycle is $4.0\,\text{m/s}^2$, and we wish to determine how fast it will be going after an elapsed time $t = 6.0\,\text{s}$ when it starts from rest ($v_0 = 0$ at $t_0 = 0$). At $t = 6.0\,\text{s}$, the velocity will be $v = at = (4.0\,\text{m/s}^2)(6.0\,\text{s}) = 24\,\text{m/s}$.

Next, let us see how to calculate the position of an object after a time t when it is undergoing constant acceleration. The definition of average velocity (Eq. 2–2) is $\bar{v} = (x - x_0)/t$, which we can rewrite as

$$x = x_0 + \bar{v}t. \qquad \textbf{(2–7)}$$

Because the velocity increases at a uniform rate, the average velocity, $\bar{v}$, will be midway between the initial and final velocities:

CAUTION
Average velocity, but only if a = constant

$$\bar{v} = \frac{v_0 + v}{2}. \qquad \text{[constant acceleration]} \quad \textbf{(2–8)}$$

(Careful: Eq. 2–8 is not necessarily valid if the acceleration is not constant.) We combine the last two Equations with Eq. 2–6 and find

$$x = x_0 + \bar{v}t = x_0 + \left(\frac{v_0 + v}{2}\right)t$$
$$= x_0 + \left(\frac{v_0 + v_0 + at}{2}\right)t$$

or

x related to a and t (a = constant)

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2. \qquad \text{[constant acceleration]} \quad \textbf{(2–9)}$$

Equations 2–6, 2–8, and 2–9 are three of the four most useful equations for motion at constant acceleration. We now derive the fourth equation, which is useful in situations where the time t is not known. We begin with Eq. 2–7 and substitute in Eq. 2–8:

$$x = x_0 + \bar{v}t = x_0 + \left(\frac{v + v_0}{2}\right)t.$$

Next we solve Eq. 2–6 for t, obtaining

$$t = \frac{v - v_0}{a},$$

and substituting this into the previous equation we have

$$x = x_0 + \left(\frac{v + v_0}{2}\right)\left(\frac{v - v_0}{a}\right) = x_0 + \frac{v^2 - v_0^2}{2a}.$$

We solve this for v^2 and obtain

$$v^2 = v_0^2 + 2a(x - x_0), \qquad \text{[constant acceleration]} \quad \textbf{(2–10)}$$

v related to a and x (a = constant)

which is the useful equation we sought.

We now have four equations relating position, velocity, acceleration, and time, when the acceleration a is constant. We collect these kinematic equations here in one place for future reference (the tan background screen emphasizes their usefulness):

$$v = v_0 + at \qquad [a = \text{constant}] \quad \textbf{(2–11a)}$$

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 \qquad [a = \text{constant}] \quad \textbf{(2–11b)}$$

$$v^2 = v_0^2 + 2a(x - x_0) \qquad [a = \text{constant}] \quad \textbf{(2–11c)}$$

$$\overline{v} = \frac{v + v_0}{2}. \qquad [a = \text{constant}] \quad \textbf{(2–11d)}$$

Kinematic equations for constant acceleration (we'll use them a lot)

These useful equations are not valid unless a is a constant. In many cases we can set $x_0 = 0$, and this simplifies the above equations a bit. Note that x represents position, not distance, that $x - x_0$ is the displacement, and that t is the elapsed time.

EXAMPLE 2–6 **Runway design.** You are designing an airport for small planes. One kind of airplane that might use this airfield must reach a speed before takeoff of at least 27.8 m/s (100 km/h), and can accelerate at $2.00\ \text{m/s}^2$. (*a*) If the runway is 150 m long, can this airplane reach the required speed for take off? (*b*) If not, what minimum length must the runway have?

PHYSICS APPLIED
Airport design

APPROACH The plane's acceleration is given as constant $(a = 2.00\ \text{m/s}^2)$, so we can use the kinematic equations for constant acceleration. In (*a*), we are given that the plane can travel a distance of 150 m. The plane starts from rest, so $v_0 = 0$ and we take $x_0 = 0$. We want to find its velocity, to determine if it will be at least 27.8 m/s. We want to find v when we are given:

Known	Wanted
$x_0 = 0$	v
$v_0 = 0$	
$x = 150\ \text{m}$	
$a = 2.00\ \text{m/s}^2$	

SOLUTION (*a*) Of the above four equations, Eq. 2–11c will give us v when we know v_0, a, x, and x_0:

$$v^2 = v_0^2 + 2a(x - x_0)$$
$$= 0 + 2(2.0\ \text{m/s}^2)(150\ \text{m}) = 600\ \text{m}^2/\text{s}^2$$
$$v = \sqrt{600\ \text{m}^2/\text{s}^2} = 24.5\ \text{m/s}.$$

PROBLEM SOLVING
Equations 2–11 are valid only when the acceleration is constant, which we assume in this Example

This runway length is *not* sufficient.

(*b*) Now we want to find the minimum length of runway, $x - x_0$, given $v = 27.8\ \text{m/s}$ and $a = 2.00\ \text{m/s}^2$. So we again use Eq. 2–11c, but rewritten as

$$(x - x_0) = \frac{v^2 - v_0^2}{2a} = \frac{(27.8\ \text{m/s})^2 - 0}{2(2.0\ \text{m/s}^2)} = 193\ \text{m}.$$

A 200-m runway is more appropriate for this plane.

2–6 Solving Problems

Before doing more worked-out Examples, let us look at how to approach problem solving. First, it is important to note that physics is *not* a collection of equations to be memorized. (In fact, rather than memorizing the very useful Eqs. 2–11, it is better to understand how to derive them from the definitions of velocity and acceleration as we did above.) Simply searching for an equation that might work can lead you to a wrong result and will surely not help you understand physics. A better approach is to use the following (rough) procedure, which we put in a special "Box." (Other such Problem Solving Boxes, as an aid, will be found throughout the book.)

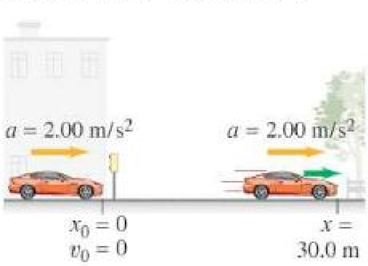
PROBLEM SOLVING

1. Read and **reread** the whole problem carefully before trying to solve it.
2. Decide what **object** (or objects) you are going to study, and for what **time interval**. You can often choose the initial time to be $t = 0$.
3. **Draw** a **diagram** or picture of the situation, with coordinate axes wherever applicable. [You can place the origin of coordinates and the axes wherever you like to make your calculations easier. You also choose which direction is positive and which is negative. Usually we choose the x axis to the right as positive.]
4. Write down what quantities are "**known**" or "given," and then what you *want* to know. Consider quantities both at the beginning and at the end of the chosen time interval. You may need to "translate" stated language into physical terms, such as "starts from rest" means $v_0 = 0$.
5. Think about which **principles of physics** apply in this problem. Use common sense and your own experiences. Then plan an approach.
6. Consider which **equations** (and/or definitions) relate the quantities involved. Before using them, be sure their **range of validity** includes your problem (for example, Eqs. 2–11 are valid only when the acceleration is constant). If you find an applicable equation that involves only known quantities and one desired unknown, **solve** the equation algebraically for the unknown. In many instances several sequential calculations, or a combination of equations, may be needed. It is often preferable to solve algebraically for the desired unknown before putting in numerical values.
7. Carry out the **calculation** if it is a numerical problem. Keep one or two extra digits during the calculations, but round off the final answer(s) to the correct number of significant figures (Section 1–4).
8. Think carefully about the result you obtain: Is it **reasonable**? Does it make sense according to your own intuition and experience? A good check is to do a rough **estimate** using only powers of ten, as discussed in Section 1–7. Often it is preferable to do a rough estimate at the *start* of a numerical problem because it can help you focus your attention on finding a path toward a solution.
9. A very important aspect of doing problems is keeping track of **units**. An equals sign implies the units on each side must be the same, just as the numbers must. If the units do not balance, a mistake has no doubt been made. This can serve as a **check** on your solution (but it only tells you if you're wrong, not if you're right). And: always use a consistent set of units.

PROBLEM SOLVING

"Starting from rest" means $v = 0$ at $t = 0$ [i.e., $v_0 = 0$]

FIGURE 2–13 Example 2–7.



EXAMPLE 2–7 **Acceleration of a car.** How long does it take a car to cross a 30.0-m-wide intersection after the light turns green, if the car accelerates from rest at a constant $2.00\ m/s^2$?

APPROACH We follow the Problem Solving Box, step by step.

SOLUTION

1. **Reread** the problem. Be sure you understand what it asks for (here, a time period).
2. The **object** under study is the car. We need to choose the **time interval** during which we look at the car's motion: we choose $t = 0$, the initial time, to be the moment the car starts to accelerate from rest ($v_0 = 0$); the time t is the instant the car has traveled the full 30.0-m width of the intersection.
3. **Draw** a **diagram**: the situation is shown in Fig. 2–13, where the car is shown moving along the positive x axis. We choose $x_0 = 0$ at the front bumper of the car before it starts to move.

4. The "**knowns**" and the "wanted" are shown in the Table in the margin, and we choose $x_0 = 0$. Note that "starting from rest" means $v = 0$ at $t = 0$; that is, $v_0 = 0$.
5. The **physics**: the motion takes place at constant acceleration, so we can use the kinematic equations, Eqs. 2–11.
6. **Equations**: we want to find the time, given the distance and acceleration; Eq. 2–11b is perfect since the only unknown quantity is t. Setting $v_0 = 0$ and $x_0 = 0$ in Eq. 2–11b $\left(x = x_0 + v_0 t + \frac{1}{2}at^2\right)$, we can solve for t:

$$x = \tfrac{1}{2}at^2,$$

$$t^2 = \frac{2x}{a},$$

so

$$t = \sqrt{\frac{2x}{a}}.$$

7. The **calculation**:

$$t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(30.0\,\text{m})}{2.00\,\text{m/s}^2}} = 5.48\,\text{s}.$$

This is our answer. Note that the units come out correctly.

8. We can check the **reasonableness** of the answer by calculating the final velocity $v = at = (2.00\,\text{m/s}^2)(5.48\,\text{s}) = 10.96\,\text{m/s}$, and then finding $x = x_0 + \bar{v}t = 0 + \frac{1}{2}(10.96\,\text{m/s} + 0)(5.48\,\text{s}) = 30.0\,\text{m}$, which is our given distance.
9. We checked the **units**, and they came out perfectly (seconds).

NOTE In steps 6 and 7, when we took the square root, we should have written $t = \pm\sqrt{2x/a} = \pm 5.48\,\text{s}$. Mathematically there are two solutions. But the second solution, $t = -5.48\,\text{s}$, is a time *before* our chosen time interval and makes no sense physically. We say it is "unphysical" and ignore it.

Known	Wanted
$x_0 = 0$	t
$x = 30.0\,\text{m}$	
$a = 2.00\,\text{m/s}^2$	
$v_0 = 0$	

PROBLEM SOLVING
Check your answer

We explicitly followed the steps of the Problem Solving Box in Example 2–7. In upcoming Examples, we will use our usual "approach" and "solution" to avoid being wordy.

EXAMPLE 2–8 ESTIMATE **Air bags.** Suppose you want to design an air-bag system that can protect the driver in a head-on collision at a speed of 100 km/h (60 mph). Estimate how fast the air bag must inflate (Fig. 2–14) to effectively protect the driver. How does the use of a seat belt help the driver?

PHYSICS APPLIED
Car safety—air bags

APPROACH We assume the acceleration is roughly constant, so we can use Eqs. 2–11. Both Eqs. 2–11a and 2–11b contain t, our desired unknown. They both contain a, so we must first find a, which we can do using Eq. 2–11c if we know the distance x over which the car crumples. A rough estimate might be about 1 meter. We choose the time interval to start at the instant of impact with the car moving at $v_0 = 100\,\text{km/h}$, and to end when the car comes to rest $(v = 0)$ after traveling 1 m.

SOLUTION We convert the given initial speed to SI units: $100\,\text{km/h} = 100 \times 10^3\,\text{m}/3600\,\text{s} = 28\,\text{m/s}$. We then find the acceleration from Eq. 2–11c:

$$a = -\frac{v_0^2}{2x} = -\frac{(28\,\text{m/s})^2}{2.0\,\text{m}} = -390\,\text{m/s}^2.$$

This enormous acceleration takes place in a time given by (Eq. 2–11a):

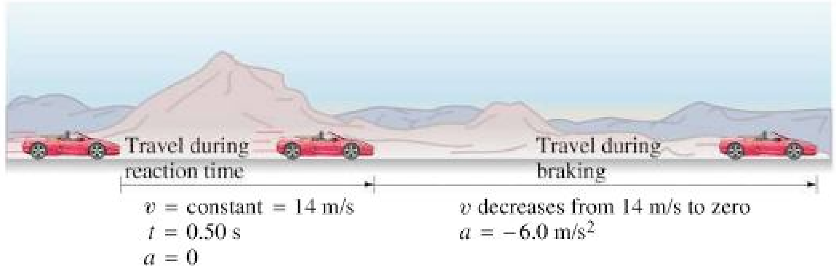
$$t = \frac{v - v_0}{a} = \frac{0 - 28\,\text{m/s}}{-390\,\text{m/s}^2} = 0.07\,\text{s}.$$

To be effective, the air bag would need to inflate faster than this.

What does the air bag do? It spreads the force over a large area of the chest (to avoid puncture of the chest by the steering wheel). The seat belt keeps the person in a stable position against the expanding air bag.

FIGURE 2–14 An air bag deploying on impact. Example 2–8.

FIGURE 2–15 Example 2–9: stopping distance for a braking car.



PHYSICS APPLIED
Braking distances

EXAMPLE 2–9 ESTIMATE Braking distances. Estimate the minimum stopping distance for a car, which is important for traffic safety and traffic design. The problem is best dealt with in two parts, two separate time intervals. (1) The first time interval begins when the driver decides to hit the brakes, and ends when the foot touches the brake pedal. This is the "reaction time" during which the speed is constant, so $a = 0$. (2) The second time interval is the actual braking period when the vehicle slows down ($a \neq 0$) and comes to a stop. The stopping distance depends on the reaction time of the driver, the initial speed of the car (the final speed is zero), and the acceleration of the car. For a dry road and good tires, good brakes can decelerate a car at a rate of about $5\,\mathrm{m/s^2}$ to $8\,\mathrm{m/s^2}$. Calculate the total stopping distance for an initial velocity of 50 km/h (14 m/s ≈ 31 mi/h) and assume the acceleration of the car is $-6.0\,\mathrm{m/s^2}$ (the minus sign appears because the velocity is taken to be in the positive x direction and its magnitude is decreasing). Reaction time for normal drivers varies from perhaps 0.3 s to about 1.0 s; take it to be 0.50 s.

APPROACH During the "reaction time," part (1), the car moves at constant speed of 14 m/s, so $a = 0$. Once the brakes are applied, part (2), the acceleration is $a = -6.0\,\mathrm{m/s^2}$ and is constant over this time interval. For both parts a is constant, so we can use Eqs. 2–11.

Part 1: Reaction time

Known	Wanted
$t = 0.50\,\mathrm{s}$	x
$v_0 = 14\,\mathrm{m/s}$	
$v = 14\,\mathrm{m/s}$	
$a = 0$	
$x_0 = 0$	

SOLUTION Part (1). We take $x_0 = 0$ for the first part of the problem, in which the car travels at a constant speed of 14 m/s during the time interval when the driver is reacting (0.50 s). See Fig. 2–15 and the Table in the margin. To find x, the position of the car at $t = 0.50\,\mathrm{s}$ (when the brakes are applied), we cannot use Eq. 2–11c because x is multiplied by a, which is zero. But Eq. 2–11b works:

$$x = v_0 t + 0 = (14\,\mathrm{m/s})(0.50\,\mathrm{s}) = 7.0\,\mathrm{m}.$$

Thus the car travels 7.0 m during the driver's reaction time, until the moment the brakes are applied. We will use this result as input to part (2).

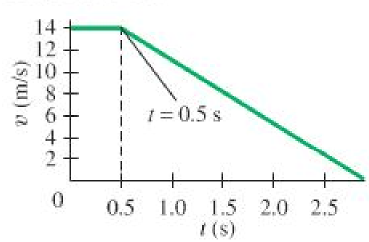
Part 2: Braking

Known	Wanted
$x_0 = 7.0\,\mathrm{m}$	x
$v_0 = 14\,\mathrm{m/s}$	
$v = 0$	
$a = -6.0\,\mathrm{m/s^2}$	

Part (2). Now we consider the second time interval, during which the brakes are applied and the car is brought to rest. We have an initial position $x_0 = 7.0\,\mathrm{m}$ (result of part (1)), and other variables are shown in the Table in the margin. Equation 2–11a doesn't contain x; Eq. 2–11b contains x but also the unknown t. Equation 2–11c, $v^2 - v_0^2 = 2a(x - x_0)$, is what we want; after setting $x_0 = 7.0\,\mathrm{m}$, we solve for x, the final position of the car (when it stops):

$$\begin{aligned} x &= x_0 + \frac{v^2 - v_0^2}{2a} \\ &= 7.0\,\mathrm{m} + \frac{0 - (14\,\mathrm{m/s})^2}{2(-6.0\,\mathrm{m/s^2})} = 7.0\,\mathrm{m} + \frac{-196\,\mathrm{m^2/s^2}}{-12\,\mathrm{m/s^2}} \\ &= 7.0\,\mathrm{m} + 16\,\mathrm{m} = 23\,\mathrm{m}. \end{aligned}$$

FIGURE 2–16 Example 2–9. Graph of v vs. t.



The car traveled 7.0 m while the driver was reacting and another 16 m during the braking period before coming to a stop. The total distance traveled was then 23 m. Figure 2–16 shows a graph of v vs. t: v is constant from $t = 0$ to $t = 0.50\,\mathrm{s}$ and decreases linearly, to zero, after $t = 0.50\,\mathrm{s}$.

NOTE From the equation above for x, we see that the stopping distance after you hit the brakes ($= x - x_0$) increases with the *square* of the initial speed, not just linearly with speed. If you are traveling twice as fast, it takes four times the distance to stop.

The analysis of motion we have been discussing in this Chapter is basically algebraic. It is sometimes helpful to use a graphical interpretation as well; see the optional Section 2–8.

2–7 Falling Objects

One of the most common examples of uniformly accelerated motion is that of an object allowed to fall freely near the Earth's surface. That a falling object is accelerating may not be obvious at first. And beware of thinking, as was widely believed until the time of Galileo (Fig. 2–17), that heavier objects fall faster than lighter objects and that the speed of fall is proportional to how heavy the object is.

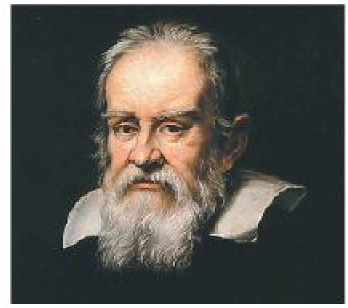


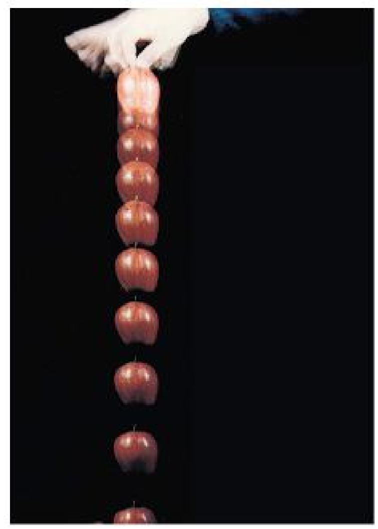
FIGURE 2–17 Galileo Galilei (1564–1642).

CAUTION

The speed of a falling object is NOT proportional to its mass or weight

Galileo's analysis of falling objects made use of his new and creative technique of imagining what would happen in idealized (simplified) cases. For free fall, he postulated that all objects would fall with the *same constant acceleration* in the absence of air or other resistance. He showed that this postulate predicts that for an object falling from rest, the distance traveled will be proportional to the square of the time (Fig. 2–18); that is, $d \propto t^2$. We can see this from Eq. 2–11b, but Galileo was the first to derive this mathematical relation. [Among Galileo's great contributions to science was to establish such mathematical relations, and to insist on specific experimental consequences that could be quantitatively checked, such as $d \propto t^2$.]

FIGURE 2–18 Multiflash photograph of a falling apple, at equal time intervals. The apple falls farther during each successive interval, which means it is accelerating.



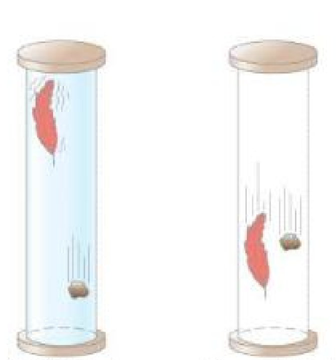
To support his claim that falling objects increase in speed as they fall, Galileo made use of a clever argument: a heavy stone dropped from a height of 2 m will drive a stake into the ground much further than will the same stone dropped from a height of only 0.2 m. Clearly, the stone must be moving faster in the former case.

As we saw, Galileo also claimed that *all* objects, light or heavy, fall with the *same* acceleration, at least in the absence of air. If you hold a piece of paper horizontally in one hand and a heavier object—say, a baseball—in the other, and release them at the same time as in Fig. 2–19a, the heavier object will reach the ground first. But if you repeat the experiment, this time crumpling the paper into a small wad (see Fig. 2–19b), you will find that the two objects reach the floor at nearly the same time.

Galileo was sure that air acts as a resistance to very light objects that have a large surface area. But in many circumstances this air resistance is negligible. In a chamber from which the air has been removed, even light objects like a feather or a horizontally held piece of paper will fall with the same acceleration as any other object (see Fig. 2–20). Such a demonstration in vacuum was not possible in Galileo's time, which makes Galileo's achievement all the greater. Galileo is often called the "father of modern science," not only for the content of his science (astronomical discoveries, inertia, free fall), but also for his style or approach to science (idealization and simplification, mathematization of theory, theories that have testable consequences, experiments to test theoretical predictions).

(a) (b)

FIGURE 2–19 (a) A ball and a light piece of paper are dropped at the same time. (b) Repeated, with the paper wadded up.



Air-filled tube (a)

Evacuated tube (b)

FIGURE 2–20 A rock and a feather are dropped simultaneously (a) in air, (b) in a vacuum.

Galileo's specific contribution to our understanding of the motion of falling objects can be summarized as follows:

Galileo's hypothesis: free fall is at constant acceleration g

at a given location on the Earth and in the absence of air resistance, all objects fall with the same constant acceleration.

We call this acceleration the **acceleration due to gravity** on the Earth, and we give it the symbol g. Its magnitude is approximately

Acceleration due to gravity

$$g = 9.80\ \mathrm{m/s^2}. \qquad \text{[at surface of Earth]}$$

In British units g is about $32\ \mathrm{ft/s^2}$. Actually, g varies slightly according to latitude and elevation, but these variations are so small that we will ignore them for most purposes. The effects of air resistance are often small, and we will neglect them for the most part. However, air resistance will be noticeable even on a reasonably heavy object if the velocity becomes large.† Acceleration due to gravity is a vector, as is any acceleration, and its direction is toward the center of the Earth.

When dealing with freely falling objects we can make use of Eqs. 2–11, where for a we use the value of g given above. Also, since the motion is vertical we will substitute y in place of x, and y_0 in place of x_0. We take $y_0 = 0$ unless otherwise specified. *It is arbitrary whether we choose y to be positive in the upward direction or in the downward direction; but we must be consistent about it throughout a problem's solution.*

PROBLEM SOLVING
You choose y to be positive either up or down

EXAMPLE 2–10 **Falling from a tower.** Suppose that a ball is dropped ($v_0 = 0$) from a tower 70.0 m high. How far will the ball have fallen after a time $t_1 = 1.00\ \mathrm{s}$, $t_2 = 2.00\ \mathrm{s}$, and $t_3 = 3.00\ \mathrm{s}$?

"Drop" means $v_0 = 0$

APPROACH Let us take y as positive downward. We neglect any air resistance. Thus the acceleration is $a = g = +9.80\ \mathrm{m/s^2}$, which is positive because we have chosen downward as positive. We set $v_0 = 0$ and $y_0 = 0$. We want to find the position y of the ball after three different time intervals. Equation 2–11b, with x replaced by y, relates the given quantities (t, a, and v_0) to the unknown y.

SOLUTION We set $t = t_1 = 1.00\ \mathrm{s}$ in Eq. 2–11b:

$$\begin{aligned} y_1 &= v_0 t_1 + \tfrac{1}{2} a t_1^2 \\ &= 0 + \tfrac{1}{2} a t_1^2 = \tfrac{1}{2}(9.80\ \mathrm{m/s^2})(1.00\ \mathrm{s})^2 = 4.90\ \mathrm{m}. \end{aligned}$$

The ball has fallen a distance of 4.90 m during the time interval $t = 0$ to $t_1 = 1.00\ \mathrm{s}$. Similarly, after 2.00 s ($= t_2$), the ball's position is

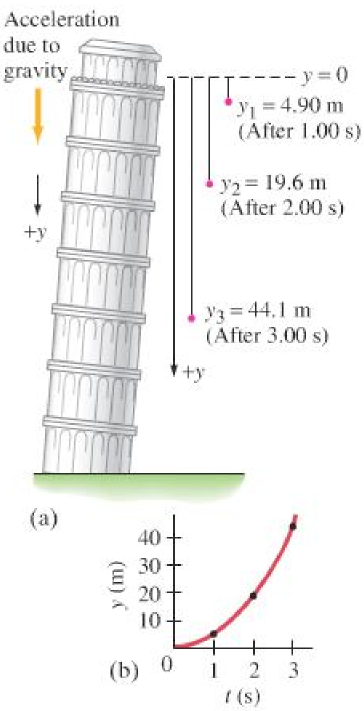
$$y_2 = \tfrac{1}{2} a t_2^2 = \tfrac{1}{2}(9.80\ \mathrm{m/s^2})(2.00\ \mathrm{s})^2 = 19.6\ \mathrm{m}.$$

Finally, after 3.00 s ($= t_3$), the ball's position is (see Fig. 2–21)

$$y_3 = \tfrac{1}{2} a t_3^2 = \tfrac{1}{2}(9.80\ \mathrm{m/s^2})(3.00\ \mathrm{s})^2 = 44.1\ \mathrm{m}.$$

NOTE Whenever we say "dropped," we mean $v_0 = 0$.

FIGURE 2–21 Example 2–10. (a) An object dropped from a tower falls with progressively greater speed and covers greater distance with each successive second. (See also Fig. 2–18.) (b) Graph of y vs. t.



EXAMPLE 2–11 **Thrown down from a tower.** Suppose the ball in Example 2–10 is *thrown* downward with an initial velocity of 3.00 m/s, instead of being dropped. (*a*) What then would be its position after 1.00 s and 2.00 s? (*b*) What would its speed be after 1.00 s and 2.00 s? Compare with the speeds of a dropped ball.

APPROACH We can approach this in the same way as in Example 2–10. Again we use Eq. 2–11b, but now v_0 is not zero, it is $v_0 = 3.00\ \mathrm{m/s}$.

SOLUTION (*a*) At $t = 1.00\ \mathrm{s}$, the position of the ball as given by Eq. 2–11b is

$$y = v_0 t + \tfrac{1}{2} a t^2 = (3.00\ \mathrm{m/s})(1.00\ \mathrm{s}) + \tfrac{1}{2}(9.80\ \mathrm{m/s^2})(1.00\ \mathrm{s})^2 = 7.90\ \mathrm{m}.$$

At $t = 2.00\ \mathrm{s}$, (time interval $t = 0$ to $t = 2.00\ \mathrm{s}$), the position is

$$y = v_0 t + \tfrac{1}{2} a t^2 = (3.00\ \mathrm{m/s})(2.00\ \mathrm{s}) + \tfrac{1}{2}(9.80\ \mathrm{m/s^2})(2.00\ \mathrm{s})^2 = 25.6\ \mathrm{m}.$$

As expected, the ball falls farther each second than if it were dropped with $v_0 = 0$.

†The speed of an object falling in air (or other fluid) does not increase indefinitely. If the object falls far enough, it will reach a maximum velocity called the **terminal velocity** due to air resistance.

(b) The velocity is obtained from Eq. 2–11a:

$$v = v_0 + at$$
$$= 3.00\,\text{m/s} + (9.80\,\text{m/s}^2)(1.00\,\text{s}) = 12.8\,\text{m/s} \qquad [\text{at } t_1 = 1.00\,\text{s}]$$
$$= 3.00\,\text{m/s} + (9.80\,\text{m/s}^2)(2.00\,\text{s}) = 22.6\,\text{m/s}. \qquad [\text{at } t_2 = 2.00\,\text{s}]$$

In Example 2–10, when the ball was dropped ($v_0 = 0$), the first term (v_0) in these equations was zero, so

$$v = 0 + at$$
$$= (9.80\,\text{m/s}^2)(1.00\,\text{s}) = 9.80\,\text{m/s} \qquad [\text{at } t_1 = 1.00\,\text{s}]$$
$$= (9.80\,\text{m/s}^2)(2.00\,\text{s}) = 19.6\,\text{m/s}. \qquad [\text{at } t_2 = 2.00\,\text{s}]$$

NOTE For both Examples 2–10 and 2–11, the speed increases linearly in time by 9.80 m/s during each second. But the speed of the downwardly thrown ball at any moment is always 3.00 m/s (its initial speed) higher than that of a dropped ball.

EXAMPLE 2–12 **Ball thrown upward, I.** A person throws a ball *upward* into the air with an initial velocity of 15.0 m/s. Calculate (a) how high it goes, and (b) how long the ball is in the air before it comes back to his hand.

APPROACH We are not concerned here with the throwing action, but only with the motion of the ball *after* it leaves the thrower's hand (Fig. 2–22) and until it comes back to his hand again. Let us choose y to be positive in the upward direction and negative in the downward direction. (This is a different convention from that used in Examples 2–10 and 2–11, and so illustrates our options.) The acceleration due to gravity will have a negative sign, $a = -g = -9.80\,\text{m/s}^2$. As the ball rises, its speed decreases until it reaches the highest point (B in Fig. 2–22), where its speed is zero for an instant; then it descends, with increasing speed.

SOLUTION (a) We consider the time interval from when the ball leaves the thrower's hand until the ball reaches the highest point. To determine the maximum height, we calculate the position of the ball when its velocity equals zero ($v = 0$ at the highest point). At $t = 0$ (point A in Fig. 2–22) we have $y_0 = 0$, $v_0 = 15.0\,\text{m/s}$, and $a = -9.80\,\text{m/s}^2$. At time t (maximum height), $v = 0$, $a = -9.80\,\text{m/s}^2$, and we wish to find y. We use Eq. 2–11c, replacing x with y: $v^2 = v_0^2 + 2ay$. We solve this equation for y:

$$y = \frac{v^2 - v_0^2}{2a} = \frac{0 - (15.0\,\text{m/s})^2}{2(-9.80\,\text{m/s}^2)} = 11.5\,\text{m}.$$

The ball reaches a height of 11.5 m above the hand.

(b) Now we need to choose a different time interval to calculate how long the ball is in the air before it returns to his hand. We could do this calculation in two parts by first determining the time required for the ball to reach its highest point, and then determining the time it takes to fall back down. However, it is simpler to consider the time interval for the entire motion from A to B to C (Fig. 2–22) in one step and use Eq. 2–11b. We can do this because y (or x) represents position or displacement, and not the total distance traveled. Thus, at both points A and C, $y = 0$. We use Eq. 2–11b with $a = -9.80\,\text{m/s}^2$ and find

$$y = v_0 t + \tfrac{1}{2}at^2$$
$$0 = (15.0\,\text{m/s})t + \tfrac{1}{2}(-9.80\,\text{m/s}^2)t^2.$$

This equation is readily factored (we factor out one t):

$$(15.0\,\text{m/s} - 4.90\,\text{m/s}^2\,t)\,t = 0.$$

There are two solutions:

$$t = 0 \qquad \text{and} \qquad t = \frac{15.0\,\text{m/s}}{4.90\,\text{m/s}^2} = 3.06\,\text{s}.$$

The first solution ($t = 0$) corresponds to the initial point (A) in Fig. 2–22, when the ball was first thrown from $y = 0$. The second solution, $t = 3.06\,\text{s}$, corresponds to point C, when the ball has returned to $y = 0$. Thus the ball is in the air 3.06 s.

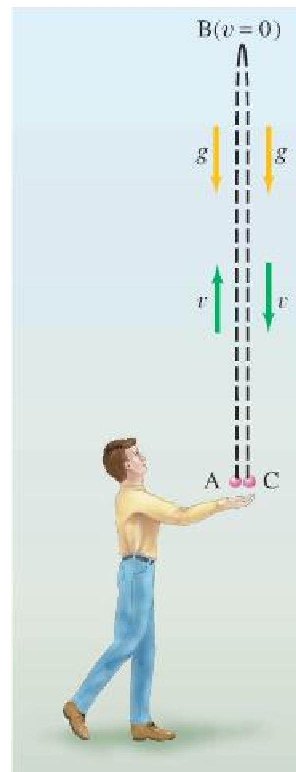


FIGURE 2–22 An object thrown into the air leaves the thrower's hand at A, reaches its maximum height at B, and returns to the original position at C. Examples 2–12, 2–13, 2–14, and 2–15.

We did not consider the throwing action in this Example. Why? Because during the throw, the thrower's hand is touching the ball and accelerating the ball at a rate unknown to us—the acceleration is *not* g. We consider only the time when the ball is in the air and the acceleration is equal to g.

CAUTION

Quadratic equations have two solutions. Sometimes only one corresponds to reality, sometimes both

Every quadratic equation (where the variable is squared) mathematically produces two solutions. In physics, sometimes only one solution corresponds to the real situation, as in Example 2–7, in which case we ignore the "unphysical" solution. But in Example 2–12, both solutions to our equation in t^2 are physically meaningful: $t = 0$ and $t = 3.06\,\mathrm{s}$.

CONCEPTUAL EXAMPLE 2–13 **Two possible misconceptions.** Give examples to show the error in these two common misconceptions: (1) that acceleration and velocity are always in the same direction, and (2) that an object thrown upward has zero acceleration at the highest point (B in Fig. 2–22).

CAUTION

(1) Velocity and acceleration are not always in the same direction; the acceleration (of gravity) always points down

(2) $a \neq 0$ even at the highest point of a trajectory

RESPONSE Both are wrong. (1) Velocity and acceleration are *not* necessarily in the same direction. When the ball in Example 2–12 is moving upward, its velocity is positive (upward), whereas the acceleration is negative (downward). (2) At the highest point (B in Fig. 2–22), the ball has zero velocity for an instant. Is the acceleration also zero at this point? No. The velocity near the top of the arc points upward, then becomes zero (for zero time) at the highest point, and then points downward. Gravity does not stop acting, so $a = -g = -9.80\,\mathrm{m/s^2}$ even there. Thinking that $a = 0$ at point B would lead to the conclusion that upon reaching point B, the ball would stay there: if the acceleration (= rate of change of velocity) were zero, the velocity would stay zero at the highest point, and the ball would stay up there without falling. In sum, the acceleration of gravity always points down toward the Earth, even when the object is moving up.

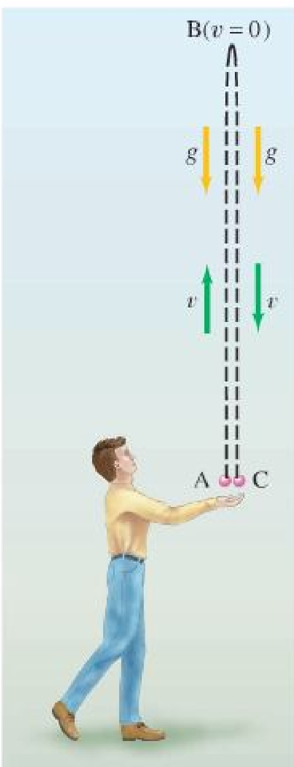


FIGURE 2–22 (Repeated for Examples 2–13, 2–14, and 2–15.)

EXAMPLE 2–14 **Ball thrown upward, II.** Let us consider again the ball thrown upward of Example 2–12, and make more calculations. Calculate (*a*) how much time it takes for the ball to reach the maximum height (point B in Fig. 2–22), and (*b*) the velocity of the ball when it returns to the thrower's hand (point C).

APPROACH Again we assume the acceleration is constant, so Eqs. 2–11 are valid. We have the height of 11.5 m from Example 2–12. Again we take y as positive upward.

SOLUTION (*a*) We consider the time interval between the throw $(t = 0,$ $v_0 = 15.0\,\mathrm{m/s})$ and the top of the path $(y = +11.5\,\mathrm{m})$, $v = 0$, and we want to find t. The acceleration is constant at $a = -g = -9.80\,\mathrm{m/s^2}$. Both Eqs. 2–11a and 2–11b contain the time t with other quantities known. Let us use Eq. 2–11a with $a = -9.80\,\mathrm{m/s^2}$, $v_0 = 15.0\,\mathrm{m/s}$, and $v = 0$:

$$v = v_0 + at;$$

setting $v = 0$ and solving for t gives

$$t = -\frac{v_0}{a} = -\frac{15.0\,\mathrm{m/s}}{-9.80\,\mathrm{m/s^2}} = 1.53\,\mathrm{s}.$$

This is just half the time it takes the ball to go up and fall back to its original position [3.06 s, calculated in part (*b*) of Example 2–12]. Thus it takes the same time to reach the maximum height as to fall back to the starting point.

(*b*) Now we consider the time interval from the throw $(t = 0,\ v_0 = 15.0\,\mathrm{m/s})$ until the ball's return to the hand, which occurs at $t = 3.06\,\mathrm{s}$ (as calculated in Example 2–12), and we want to find v when $t = 3.06\,\mathrm{s}$:

$$v = v_0 + at = 15.0\,\mathrm{m/s} - \left(9.80\,\mathrm{m/s^2}\right)(3.06\,\mathrm{s}) = -15.0\,\mathrm{m/s}.$$

Note the symmetry: the speed at any height is the same when going up as when coming down (but the direction is opposite)

NOTE The ball has the same magnitude of velocity when it returns to the starting point as it did initially, but in the opposite direction (this is the meaning of the negative sign). Thus, as we gathered from part (*a*), the motion is symmetrical about the maximum height.

> **EXERCISE C** Two balls are thrown from a cliff. One is thrown directly up, the other directly down. Both balls have the same initial speed, and both hit the ground below the cliff. Which ball hits the ground at the greater speed: (*a*) the ball thrown upward, (*b*) the ball thrown downward, or (*c*) both the same? Ignore air resistance. [*Hint*: See the result of Example 2–14, part (*b*).]

The acceleration of objects such as rockets and fast airplanes is often given as a multiple of $g = 9.80\,\mathrm{m/s^2}$. For example, a plane pulling out of a dive and undergoing 3.00 g's would have an acceleration of $(3.00)(9.80\,\mathrm{m/s^2}) = 29.4\,\mathrm{m/s^2}$.

Acceleration expressed in g's

> **EXERCISE D** If a car is said to accelerate at $0.50\,g$, what is its acceleration in $\mathrm{m/s^2}$?

Additional Example—Using the Quadratic Formula

EXAMPLE 2–15 **Ball thrown upward, III.** For the ball in Example 2–14, calculate at what time t the ball passes a point 8.00 m above the person's hand.

APPROACH We choose the time interval from the throw ($t = 0$, $v_0 = 15.0\,\mathrm{m/s}$) until the time t (to be determined) when the ball is at position $y = 8.00\,\mathrm{m}$, using Eq. 2–11b.

SOLUTION We want t, given $y = 8.00\,\mathrm{m}$, $y_0 = 0$, $v_0 = 15.0\,\mathrm{m/s}$, and $a = -9.80\,\mathrm{m/s^2}$. We use Eq. 2–11b:

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2$$

$$8.00\,\mathrm{m} = 0 + (15.0\,\mathrm{m/s})\,t + \tfrac{1}{2}(-9.80\,\mathrm{m/s^2})t^2.$$

To solve any quadratic equation of the form $at^2 + bt + c = 0$, where a, b, and c are constants (a is *not* acceleration here), we use the **quadratic formula** (see Appendix A–4):

PROBLEM SOLVING
Using the quadratic formula

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

We rewrite our y equation just above in standard form, $at^2 + bt + c = 0$:

$$(4.90\,\mathrm{m/s^2})t^2 - (15.0\,\mathrm{m/s})\,t + (8.00\,\mathrm{m}) = 0.$$

So the coefficient a is $4.90\,\mathrm{m/s^2}$, b is $-15.0\,\mathrm{m/s}$, and c is 8.00 m. Putting these into the quadratic formula, we obtain

$$t = \frac{15.0\,\mathrm{m/s} \pm \sqrt{(15.0\,\mathrm{m/s})^2 - 4(4.90\,\mathrm{m/s^2})(8.00\,\mathrm{m})}}{2(4.90\,\mathrm{m/s^2})},$$

which gives us $t = 0.69\,\mathrm{s}$ and $t = 2.37\,\mathrm{s}$. Are both solutions valid? Yes, because the ball passes $y = 8.00\,\mathrm{m}$ when it goes up ($t = 0.69\,\mathrm{s}$) and again when it comes down ($t = 2.37\,\mathrm{s}$).

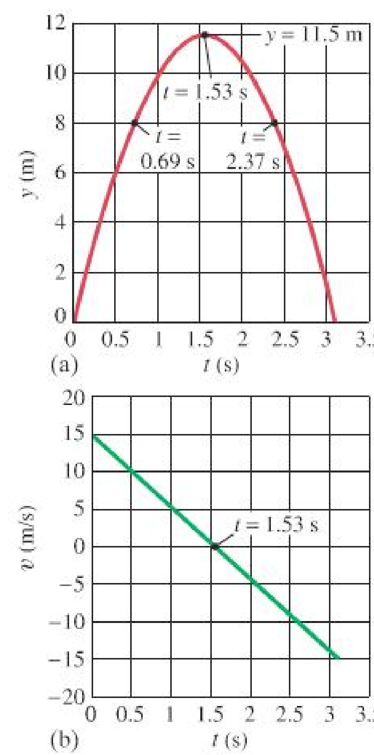


FIGURE 2–23 Graphs of (a) y vs. t, (b) v vs. t for a ball thrown upward, Examples 2–12, 2–14, and 2–15.

For some people, graphs can be a help in understanding. Figure 2–23 shows graphs of y vs. t and v vs. t for the ball thrown upward in Fig. 2–22, incorporating the results of Examples 2–12, 2–14, and 2–15. We shall discuss some useful properties of graphs in the next Section.

We will use the word "vertical" a lot in this book. What does it mean? (Try to respond before reading on.) Vertical is defined as the line along which an object falls. Or, if you put a small sphere on the end of a string and let it hang, the string represents a vertical line (sometimes called a *plumb line*).

> **EXERCISE E** What does *horizontal* mean?

* 2–8 Graphical Analysis of Linear Motion†

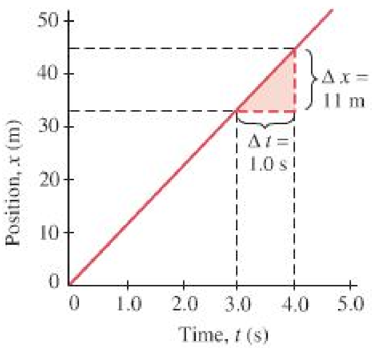


FIGURE 2–24 Graph of position vs. time for an object moving at a uniform velocity of 11 m/s.

Figure 2–9 showed the graph of the velocity of a car versus time for two cases of linear motion: (a) constant velocity, and (b) a particular case in which the magnitude of the velocity varied. It is also useful to graph, or "plot," the position x (or y) as a function of time, as we did in Fig. 2–23a. The time t is considered the independent variable and is measured along the horizontal axis. The position, x, the dependent variable, is measured along the vertical axis.

Let us make a graph of x vs. t, and make the choice that at $t = 0$, the position is $x_0 = 0$. First we consider a car moving at a constant velocity of 40 km/h, which is equivalent to 11 m/s. Equation 2–11b tells us $x = vt$, and we see that x increases by 11 m every second. Thus, the position increases linearly in time, so the graph of x vs. t is a straight line, as shown in Fig. 2–24. Each point on this straight line tells us the car's position at a particular time. For example, at $t = 3.0\,\text{s}$, the position is 33 m, and at $t = 4.0\,\text{s}$, $x = 44\,\text{m}$, as indicated by the dashed lines. The small (shaded) triangle on the graph indicates the **slope** of the straight line, which is defined as the change in the dependent variable (Δx) divided by the corresponding change in the independent variable (Δt):

$$\text{slope} = \frac{\Delta x}{\Delta t}.$$

Velocity = slope of x vs. t graph

We see, using the definition of average velocity (Eq. 2–2), that the *slope of the x vs. t graph is equal to the velocity*. And, as can be seen from the small triangle on the graph, $\Delta x/\Delta t = (11\,\text{m})/(1.0\,\text{s}) = 11\,\text{m/s}$, which is the given velocity.

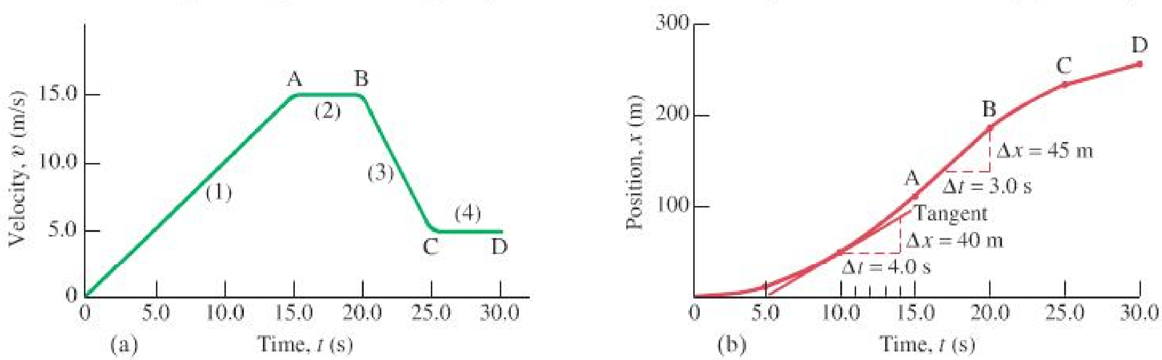
The slope of the x vs. t graph is everywhere the same if the velocity is constant, as in Fig. 2–24. But if the velocity changes, as in Fig. 2–25a, the slope of the x vs. t graph also varies. Consider, for example, a car that (1) accelerates uniformly from rest to 15 m/s in 15 s, after which (2) it remains at a constant velocity of 15 m/s for the next 5.0 s; (3) during the following 5.0 s, the car slows down uniformly to 5.0 m/s, and then (4) remains at this constant velocity. This velocity as a function of time is shown in the graph of Fig. 2–25a. To construct the x vs. t graph, we can use Eq. 2–11b ($x = x_0 + v_0 t + \frac{1}{2}at^2$) with constant acceleration for the interval $t = 0$ to $t = 15\,\text{s}$ and for $t = 20\,\text{s}$ to $t = 25\,\text{s}$; for the constant velocity period $t = 15\,\text{s}$ to $t = 20\,\text{s}$, and after $t = 25\,\text{s}$, we set $a = 0$. The result is the x vs. t graph of Fig. 2–25b.

Slope of a curve

From the origin to point A, the x vs. t graph (Fig. 2–25b) is not a straight line, but is curved. The **slope** of a curve at any point is defined as the *slope of the tangent to the curve at that point*. (The *tangent* is a straight line drawn so it touches the curve only at that one point, but does not pass across or through the curve.) For example, the tangent to the x vs. t curve at the time $t = 10.0\,\text{s}$ is drawn on the graph of Fig. 2–25b. A triangle is drawn with Δt chosen to be 4.0 s;

†Some Sections of this book, such as this one, may be considered *optional* at the discretion of the instructor. See the Preface for more details.

FIGURE 2–25 (a) Velocity vs. time and (b) displacement vs. time for an object with variable velocity. (See text.)



Δx can be measured off the graph for this chosen Δt and is found to be 40 m. Thus, the slope of the curve at $t = 10.0$ s, which equals the instantaneous velocity at that instant, is $v = \Delta x/\Delta t = 40\,\text{m}/4.0\,\text{s} = 10\,\text{m/s}$.

In the region between A and B (Fig. 2–25b) the x vs. t graph is a straight line because the slope (equal to the velocity) is constant. The slope can be measured using the triangle shown for the time interval between $t = 17$ s and $t = 20$ s, where the increase in x is 45 m: $\Delta x/\Delta t = 45\,\text{m}/3.0\,\text{s} = 15\,\text{m/s}$.

The slope of an x vs. t graph at any point is $\Delta x/\Delta t$ and thus equals the velocity of the object being described at that moment. Similarly, the slope at any point of a v vs. t graph is $\Delta v/\Delta t$ and so (by Eq. 2–4) equals the acceleration at that moment.

Suppose we were given the x vs. t graph of Fig. 2–25b. We could measure the slopes at a number of points and plot these slopes as a function of time. Since the slope equals the velocity, we could thus reconstruct the v vs. t graph! In other words, given the graph of x vs. t, we can determine the velocity as a function of time using graphical methods, instead of using equations. This technique is particularly useful when the acceleration is not constant, for then Eqs. 2–11 cannot be used.

If, instead, we are given the v vs. t graph, as in Fig. 2–25a, we can determine the position, x, as a function of time using a graphical procedure, which we illustrate by applying it to the v vs. t graph of Fig. 2–25a. We divide the total time interval into subintervals, as shown in Fig. 2–26a, where only six are shown (by dashed vertical lines). In each interval, a *horizontal* dashed line is drawn to indicate the average velocity during that time interval. For example, in the first interval, the velocity increases at a constant rate from zero to 5.0 m/s, so $\bar{v} = 2.5$ m/s; and in the fourth interval the velocity is a constant 15 m/s, so $\bar{v} = 15$ m/s (no horizontal dashed line is shown in Fig. 2–26a since it coincides with the curve itself). The displacement (change in position) during any subinterval is $\Delta x = \bar{v}\,\Delta t$. Thus the displacement during each subinterval equals the product of $\bar{v}$ and Δt, which is just the *area of the rectangle* (height × base $= \bar{v} \times \Delta t$), shown shaded in rose, for that interval. The total displacement after 25 s, say, will be the sum of the areas of the first five rectangles.

If the velocity varies a great deal, it may be difficult to estimate $\bar{v}$ from the graph. To reduce this difficulty, we can choose to divide the time interval into many more—but narrower—subintervals of time, making each Δt smaller as shown in Fig. 2–26b. More intervals give a better approximation. Ideally, we could let Δt approach zero; this leads to the techniques of integral calculus, which we don't discuss here. The result, in any case, is that *the total displacement between any two times is equal to the area under the v vs. t graph between these two times.*

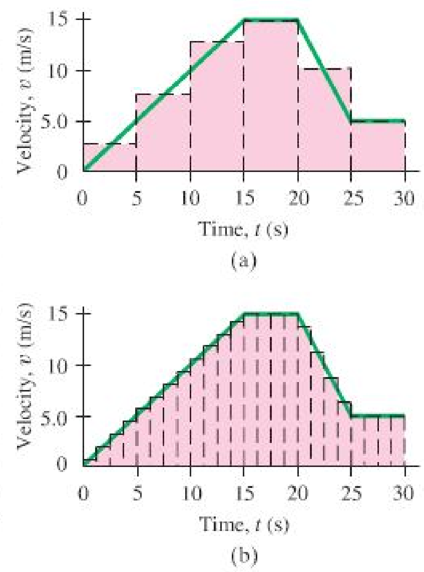


FIGURE 2–26 Determining the displacement from the graph of v vs. t is done by calculating areas.

Displacement = area under v vs. t graph

EXAMPLE 2–16 **Displacement using v vs. t graph.** A space probe accelerates uniformly from 50 m/s at $t = 0$ to 150 m/s at $t = 10$ s. How far did it move between $t = 2.0$ s and $t = 6.0$ s?

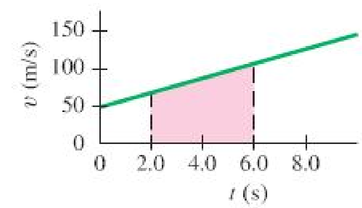
APPROACH A graph of v vs. t can be drawn as shown in Fig. 2–27. We need to calculate the area of the shaded region, which is a trapezoid. The area will be the average of the heights (in units of velocity) times the width (which is 4.0 s).

SOLUTION The acceleration is $a = (150\,\text{m/s} - 50\,\text{m/s})/10\,\text{s} = 10\,\text{m/s}^2$. Using Eq. 2–11a, or Fig. 2–27, at $t = 2.0$ s, $v = 70$ m/s; and at $t = 6.0$ s, $v = 110$ m/s. Thus the area, $(\bar{v} \times \Delta t)$, which equals Δx, is

$$\Delta x = \left(\frac{70\,\text{m/s} + 110\,\text{m/s}}{2}\right)(4.0\,\text{s}) = 360\,\text{m}.$$

NOTE For this case of constant acceleration, we could use Eqs. 2–11 and we would get the same result.

FIGURE 2–27 Example 2–16. The shaded area represents the displacement during the time interval $t = 2.0$ s to $t = 6.0$ s.



In cases where the acceleration is not constant, the area can be obtained by counting squares on graph paper.

Summary

[The Summary that appears at the end of each Chapter in this book gives a brief overview of the main ideas of the Chapter. The Summary *cannot* serve to give an understanding of the material, which can be accomplished only by a detailed reading of the Chapter.]

Kinematics deals with the description of how objects move. The description of the motion of any object must always be given relative to some particular **reference frame**.

The **displacement** of an object is the change in position of the object.

Average speed is the distance traveled divided by the elapsed time or time interval, Δt, the time period over which we choose to make our observations. An object's **average velocity** over a particular time interval Δt is its displacement Δx during that time interval, divided by Δt:

$$\bar{v} = \frac{\Delta x}{\Delta t}. \qquad \textbf{(2–2)}$$

The **instantaneous velocity**, whose magnitude is the same as the *instantaneous speed*, is defined as the average velocity taken over an infinitesimally short time interval.

Acceleration is the change of velocity per unit time. An object's **average acceleration** over a time interval Δt is

$$\bar{a} = \frac{\Delta v}{\Delta t}, \qquad \textbf{(2–4)}$$

where Δv is the change of velocity during the time interval Δt. **Instantaneous acceleration** is the average acceleration taken over an infinitesimally short time interval.

If an object has position x_0 and velocity v_0 at time $t = 0$ and moves in a straight line with constant acceleration, the velocity v and position x at a later time t are related to the acceleration a, the initial position x_0, and the initial velocity v_0 by Eqs. 2–11:

$$v = v_0 + at, \qquad x = x_0 + v_0 t + \tfrac{1}{2}at^2,$$
$$v^2 = v_0^2 + 2a(x - x_0), \qquad \bar{v} = \frac{v + v_0}{2}. \qquad \textbf{(2–11)}$$

Objects that move vertically near the surface of the Earth, either falling or having been projected vertically up or down, move with the constant downward **acceleration due to gravity**, whose magnitude is $g = 9.80\,\mathrm{m/s^2}$ if air resistance can be ignored. We can apply Eqs. 2–11 for constant acceleration to objects that move up or down freely near the Earth's surface.

[*The slope of a curve at any point on a graph is the slope of the tangent to the curve at that point. If the graph is x vs. t, the slope is $\Delta x/\Delta t$ and equals the velocity at that point. The area under a v vs. t graph equals the displacement between any two chosen times.]

Questions

1. Does a car speedometer measure speed, velocity, or both?

2. Can an object have a varying speed if its velocity is constant? If yes, give examples.

3. When an object moves with constant velocity, does its average velocity during any time interval differ from its instantaneous velocity at any instant?

4. In drag racing, is it possible for the car with the greatest speed crossing the finish line to lose the race? Explain.

5. If one object has a greater speed than a second object, does the first necessarily have a greater acceleration? Explain, using examples.

6. Compare the acceleration of a motorcycle that accelerates from 80 km/h to 90 km/h with the acceleration of a bicycle that accelerates from rest to 10 km/h in the same time.

7. Can an object have a northward velocity and a southward acceleration? Explain.

8. Can the velocity of an object be negative when its acceleration is positive? What about vice versa?

9. Give an example where both the velocity and acceleration are negative.

10. Two cars emerge side by side from a tunnel. Car A is traveling with a speed of 60 km/h and has an acceleration of 40 km/h/min. Car B has a speed of 40 km/h and has an acceleration of 60 km/h/min. Which car is passing the other as they come out of the tunnel? Explain your reasoning.

11. Can an object be increasing in speed as its acceleration decreases? If so, give an example. If not, explain.

12. A baseball player hits a foul ball straight up into the air. It leaves the bat with a speed of 120 km/h. In the absence of air resistance, how fast will the ball be traveling when the catcher catches it?

13. As a freely falling object speeds up, what is happening to its acceleration due to gravity—does it increase, decrease, or stay the same?

14. How would you estimate the maximum height you could throw a ball vertically upward? How would you estimate the maximum speed you could give it?

15. You travel from point A to point B in a car moving at a constant speed of 70 km/h. Then you travel the same distance from point B to another point C, moving at a constant speed of 90 km/h. Is your average speed for the entire trip from A to C 80 km/h? Explain why or why not.

16. In a lecture demonstration, a 3.0-m-long vertical string with ten bolts tied to it at equal intervals is dropped from the ceiling of the lecture hall. The string falls on a tin plate, and the class hears the clink of each bolt as it hits the plate. The sounds will not occur at equal time intervals. Why? Will the time between clinks increase or decrease near the end of the fall? How could the bolts be tied so that the clinks occur at equal intervals?

17. Which one of these motions is *not* at constant acceleration: a rock falling from a cliff, an elevator moving from the second floor to the fifth floor making stops along the way, a dish resting on a table?

18. An object that is thrown vertically upward will return to its original position with the same speed as it had initially if air resistance is negligible. If air resistance is appreciable, will this result be altered, and if so, how? [*Hint*: The acceleration due to air resistance is always in a direction opposite to the motion.]

19. Can an object have zero velocity and nonzero acceleration at the same time? Give examples.

20. Can an object have zero acceleration and nonzero velocity at the same time? Give examples.

* **21.** Describe in words the motion plotted in Fig. 2–28 in terms of v, a, etc. [*Hint*: First try to duplicate the motion plotted by walking or moving your hand.]

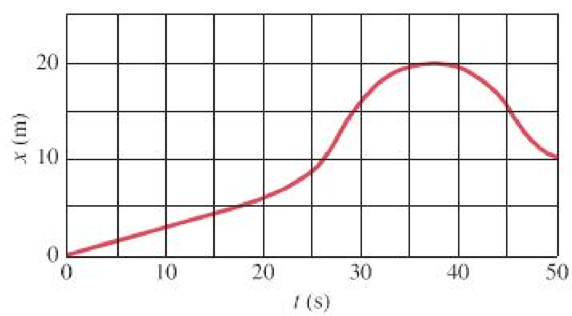


FIGURE 2–28 Question 21, Problems 50, 51, and 55.

* **22.** Describe in words the motion of the object graphed in Fig. 2–29.

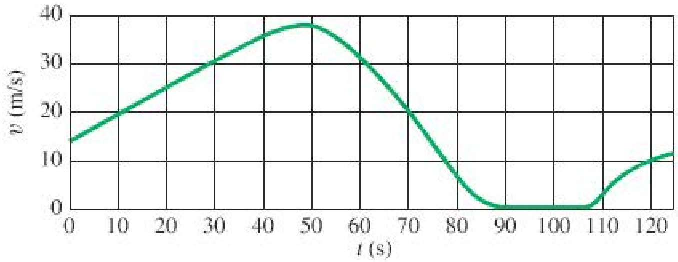


FIGURE 2–29 Question 22, Problems 49 and 54.

Problems

[The Problems at the end of each Chapter are ranked I, II, or III according to estimated difficulty, with (I) Problems being easiest. Level III are meant as challenges for the best students. The Problems are arranged by Section, meaning that the reader should have read up to and including that Section, but not only that Section—Problems often depend on earlier material. Finally, there is a set of unranked "General Problems" not arranged by Section number.]

2–1 to 2–3 Speed and Velocity

1. (I) What must be your car's average speed in order to travel 235 km in 3.25 h?

2. (I) A bird can fly 25 km/h. How long does it take to fly 15 km?

3. (I) If you are driving 110 km/h along a straight road and you look to the side for 2.0 s, how far do you travel during this inattentive period?

4. (I) Convert 35 mi/h to (*a*) km/h, (*b*) m/s, and (*c*) ft/s.

5. (I) A rolling ball moves from $x_1 = 3.4$ cm to $x_2 = -4.2$ cm during the time from $t_1 = 3.0$ s to $t_2 = 6.1$ s. What is its average velocity?

6. (II) A particle at $t_1 = -2.0$ s is at $x_1 = 3.4$ cm and at $t_2 = 4.5$ s is at $x_2 = 8.5$ cm. What is its average velocity? Can you calculate its average speed from these data?

7. (II) You are driving home from school steadily at 95 km/h for 130 km. It then begins to rain and you slow to 65 km/h. You arrive home after driving 3 hours and 20 minutes. (*a*) How far is your hometown from school? (*b*) What was your average speed?

8. (II) According to a rule-of-thumb, every five seconds between a lightning flash and the following thunder gives the distance to the flash in miles. Assuming that the flash of light arrives in essentially no time at all, estimate the speed of sound in m/s from this rule.

9. (II) A person jogs eight complete laps around a quarter-mile track in a total time of 12.5 min. Calculate (*a*) the average speed and (*b*) the average velocity, in m/s.

10. (II) A horse canters away from its trainer in a straight line, moving 116 m away in 14.0 s. It then turns abruptly and gallops halfway back in 4.8 s. Calculate (*a*) its average speed and (*b*) its average velocity for the entire trip, using "away from the trainer" as the positive direction.

11. (II) Two locomotives approach each other on parallel tracks. Each has a speed of 95 km/h with respect to the ground. If they are initially 8.5 km apart, how long will it be before they reach each other? (See Fig. 2–30).

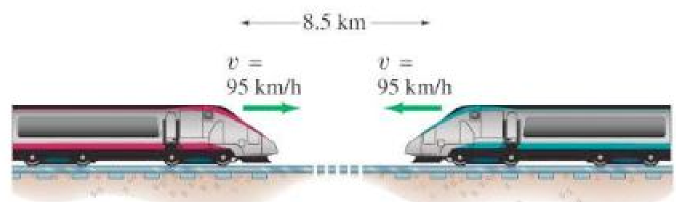


FIGURE 2–30 Problem 11.

12. (II) A car traveling 88 km/h is 110 m behind a truck traveling 75 km/h. How long will it take the car to reach the truck?

13. (II) An airplane travels 3100 km at a speed of 790 km/h, and then encounters a tailwind that boosts its speed to 990 km/h for the next 2800 km. What was the total time for the trip? What was the average speed of the plane for this trip? [*Hint*: Think carefully before using Eq. 2–11d.]

14. (II) Calculate the average speed and average velocity of a complete round-trip in which the outgoing 250 km is covered at 95 km/h, followed by a 1.0-hour lunch break, and the return 250 km is covered at 55 km/h.

15. (III) A bowling ball traveling with constant speed hits the pins at the end of a bowling lane 16.5 m long. The bowler hears the sound of the ball hitting the pins 2.50 s after the ball is released from his hands. What is the speed of the ball? The speed of sound is 340 m/s.

2–4 Acceleration

16. (I) A sports car accelerates from rest to 95 km/h in 6.2 s. What is its average acceleration in m/s^2?

17. (I) A sprinter accelerates from rest to 10.0 m/s in 1.35 s. What is her acceleration (*a*) in m/s^2, and (*b*) in km/h^2?

18. (II) At highway speeds, a particular automobile is capable of an acceleration of about $1.6\,\mathrm{m/s^2}$. At this rate, how long does it take to accelerate from 80 km/h to 110 km/h?

19. (II) A sports car moving at constant speed travels 110 m in 5.0 s. If it then brakes and comes to a stop in 4.0 s, what is its acceleration in $\mathrm{m/s^2}$? Express the answer in terms of "g's," where $1.00\,g = 9.80\,\mathrm{m/s^2}$.

20. (III) The position of a racing car, which starts from rest at $t = 0$ and moves in a straight line, is given as a function of time in the following Table. Estimate (*a*) its velocity and (*b*) its acceleration as a function of time. Display each in a Table and on a graph.

t (s)	0	0.25	0.50	0.75	1.00	1.50	2.00	2.50
x (m)	0	0.11	0.46	1.06	1.94	4.62	8.55	13.79

t (s)	3.00	3.50	4.00	4.50	5.00	5.50	6.00
x (m)	20.36	28.31	37.65	48.37	60.30	73.26	87.16

2–5 and 2–6 Motion at Constant Acceleration

21. (I) A car accelerates from 13 m/s to 25 m/s in 6.0 s. What was its acceleration? How far did it travel in this time? Assume constant acceleration.

22. (I) A car slows down from 23 m/s to rest in a distance of 85 m. What was its acceleration, assumed constant?

23. (I) A light plane must reach a speed of 33 m/s for takeoff. How long a runway is needed if the (constant) acceleration is $3.0\,\mathrm{m/s^2}$?

24. (II) A world-class sprinter can burst out of the blocks to essentially top speed (of about 11.5 m/s) in the first 15.0 m of the race. What is the average acceleration of this sprinter, and how long does it take her to reach that speed?

25. (II) A car slows down uniformly from a speed of 21.0 m/s to rest in 6.00 s. How far did it travel in that time?

26. (II) In coming to a stop, a car leaves skid marks 92 m long on the highway. Assuming a deceleration of $7.00\,\mathrm{m/s^2}$, estimate the speed of the car just before braking.

27. (II) A car traveling 85 km/h strikes a tree. The front end of the car compresses and the driver comes to rest after traveling 0.80 m. What was the average acceleration of the driver during the collision? Express the answer in terms of "g's," where $1.00\,g = 9.80\,\mathrm{m/s^2}$.

28. (II) Determine the stopping distances for a car with an initial speed of 95 km/h and human reaction time of 1.0 s, for an acceleration (*a*) $a = -4.0\,\mathrm{m/s^2}$; (*b*) $a = -8.0\,\mathrm{m/s^2}$.

29. (III) Show that the equation for the stopping distance of a car is $d_S = v_0 t_R - v_0^2/(2a)$, where v_0 is the initial speed of the car, t_R is the driver's reaction time, and a is the constant acceleration (and is negative).

30. (III) A car is behind a truck going 25 m/s on the highway. The car's driver looks for an opportunity to pass, guessing that his car can accelerate at $1.0\,\mathrm{m/s^2}$. He gauges that he has to cover the 20-m length of the truck, plus 10 m clear room at the rear of the truck and 10 m more at the front of it. In the oncoming lane, he sees a car approaching, probably also traveling at 25 m/s. He estimates that the car is about 400 m away. Should he attempt the pass? Give details.

31. (III) A runner hopes to complete the 10,000-m run in less than 30.0 min. After exactly 27.0 min, there are still 1100 m to go. The runner must then accelerate at $0.20\,\mathrm{m/s^2}$ for how many seconds in order to achieve the desired time?

32. (III) A person driving her car at 45 km/h approaches an intersection just as the traffic light turns yellow. She knows that the yellow light lasts only 2.0 s before turning red, and she is 28 m away from the near side of the intersection (Fig. 2–31). Should she try to stop, or should she speed up to cross the intersection before the light turns red? The intersection is 15 m wide. Her car's maximum deceleration is $-5.8\,\mathrm{m/s^2}$, whereas it can accelerate from 45 km/h to 65 km/h in 6.0 s. Ignore the length of her car and her reaction time.

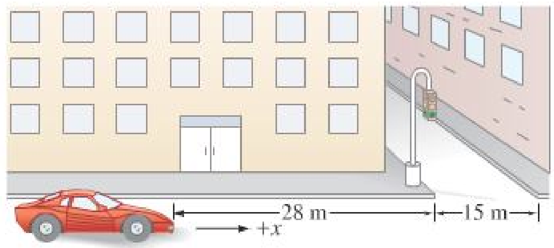


FIGURE 2–31 Problem 32.

2–7 Falling Objects [neglect air resistance]

33. (I) A stone is dropped from the top of a cliff. It hits the ground below after 3.25 s. How high is the cliff?

34. (I) If a car rolls gently ($v_0 = 0$) off a vertical cliff, how long does it take it to reach 85 km/h?

35. (I) Estimate (*a*) how long it took King Kong to fall straight down from the top of the Empire State Building (380 m high), and (*b*) his velocity just before "landing"?

36. (II) A baseball is hit nearly straight up into the air with a speed of 22 m/s. (*a*) How high does it go? (*b*) How long is it in the air?

37. (II) A ballplayer catches a ball 3.0 s after throwing it vertically upward. With what speed did he throw it, and what height did it reach?

38. (II) An object starts from rest and falls under the influence of gravity. Draw graphs of (*a*) its speed and (*b*) the distance it has fallen, as a function of time from $t = 0$ to $t = 5.00\,\mathrm{s}$. Ignore air resistance.

39. (II) A helicopter is ascending vertically with a speed of 5.20 m/s. At a height of 125 m above the Earth, a package is dropped from a window. How much time does it take for the package to reach the ground? [*Hint*: The package's initial speed equals the helicopter's.]

40. (II) For an object falling freely from rest, show that the distance traveled during each successive second increases in the ratio of successive odd integers (1, 3, 5, etc.). This was first shown by Galileo. See Figs. 2–18 and 2–21.

41. (II) If air resistance is neglected, show (algebraically) that a ball thrown vertically upward with a speed v_0 will have the same speed, v_0, when it comes back down to the starting point.

42. (II) A stone is thrown vertically upward with a speed of 18.0 m/s. (*a*) How fast is it moving when it reaches a height of 11.0 m? (*b*) How long is required to reach this height? (*c*) Why are there two answers to (*b*)?

43. (III) Estimate the time between each photoflash of the apple in Fig. 2–18 (or number of photoflashes per second). Assume the apple is about 10 cm in diameter. [*Hint*: Use two apple positions, but not the unclear ones at the top.]

44. (III) A falling stone takes 0.28 s to travel past a window 2.2 m tall (Fig. 2–32). From what height above the top of the window did the stone fall?

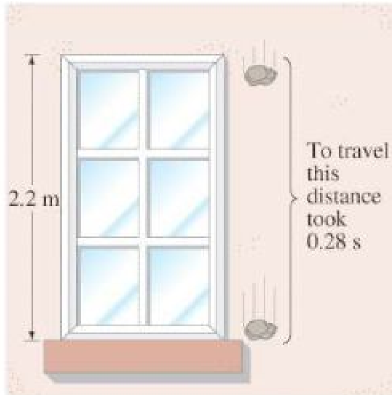


FIGURE 2–32 Problem 44.

45. (III) A rock is dropped from a sea cliff, and the sound of it striking the ocean is heard 3.2 s later. If the speed of sound is 340 m/s, how high is the cliff?

46. (III) Suppose you adjust your garden hose nozzle for a hard stream of water. You point the nozzle vertically upward at a height of 1.5 m above the ground (Fig. 2–33). When you quickly move the nozzle away from the vertical, you hear the water striking the ground next to you for another 2.0 s. What is the water speed as it leaves the nozzle?

FIGURE 2–33 Problem 46.

47. (III) A stone is thrown vertically upward with a speed of 12.0 m/s from the edge of a cliff 70.0 m high (Fig. 2–34). (*a*) How much later does it reach the bottom of the cliff? (*b*) What is its speed just before hitting? (*c*) What total distance did it travel?

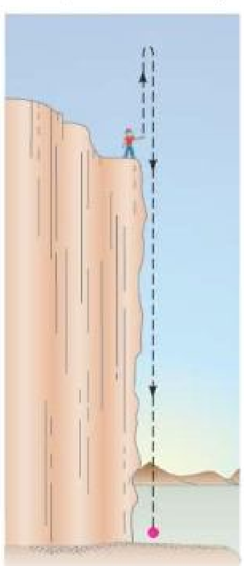


FIGURE 2–34 Problem 47.

48. (III) A baseball is seen to pass upward by a window 28 m above the street with a vertical speed of 13 m/s. If the ball was thrown from the street, (*a*) what was its initial speed, (*b*) what altitude does it reach, (*c*) when was it thrown, and (*d*) when does it reach the street again?

* 2–8 Graphical Analysis

* **49.** (I) Figure 2–29 shows the velocity of a train as a function of time. (*a*) At what time was its velocity greatest? (*b*) During what periods, if any, was the velocity constant? (*c*) During what periods, if any, was the acceleration constant? (*d*) When was the magnitude of the acceleration greatest?

* **50.** (II) The position of a rabbit along a straight tunnel as a function of time is plotted in Fig. 2–28. What is its instantaneous velocity (*a*) at $t = 10.0$ s and (*b*) at $t = 30.0$ s? What is its average velocity (*c*) between $t = 0$ and $t = 5.0$ s, (*d*) between $t = 25.0$ s and $t = 30.0$ s, and (*e*) between $t = 40.0$ s and $t = 50.0$ s?

* **51.** (II) In Fig. 2–28, (*a*) during what time periods, if any, is the velocity constant? (*b*) At what time is the velocity greatest? (*c*) At what time, if any, is the velocity zero? (*d*) Does the object move in one direction or in both directions during the time shown?

* **52.** (II) A certain type of automobile can accelerate approximately as shown in the velocity–time graph of Fig. 2–35. (The short flat spots in the curve represent shifting of the gears.) (*a*) Estimate the average acceleration of the car in second gear and in fourth gear. (*b*) Estimate how far the car traveled while in fourth gear.

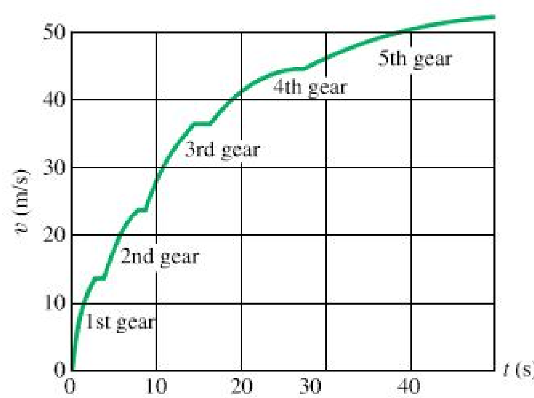


FIGURE 2–35 Problems 52 and 53. The velocity of an automobile as a function of time, starting from a dead stop. The jumps in the curve represent gear shifts.

* **53.** (II) Estimate the average acceleration of the car in the previous Problem (Fig. 2–35) when it is in (*a*) first, (*b*) third, and (*c*) fifth gear. (*d*) What is its average acceleration through the first four gears?

* **54.** (II) In Fig. 2–29, estimate the distance the object traveled during (*a*) the first minute, and (*b*) the second minute.

* **55.** (II) Construct the v vs. t graph for the object whose displacement as a function of time is given by Fig. 2–28.

* **56.** (II) Figure 2–36 is a position versus time graph for the motion of an object along the x axis. Consider the time interval from A to B. (*a*) Is the object moving in the positive or negative direction? (*b*) Is the object speeding up or slowing down? (*c*) Is the acceleration of the object positive or negative? Now consider the time interval from D to E. (*d*) Is the object moving in the positive or negative direction? (*e*) Is the object speeding up or slowing down? (*f*) Is the acceleration of the object positive or negative? (*g*) Finally, answer these same three questions for the time interval from C to D.

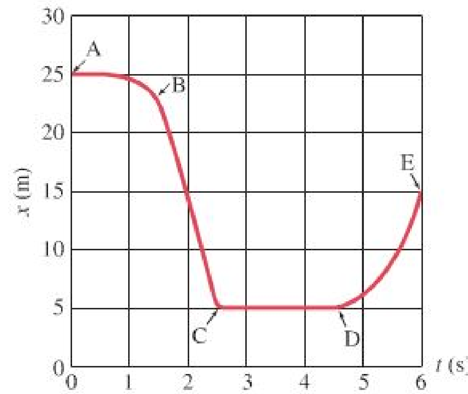


FIGURE 2–36 Problem 56.

General Problems

57. A person jumps from a fourth-story window 15.0 m above a firefighter's safety net. The survivor stretches the net 1.0 m before coming to rest, Fig. 2–37. (*a*) What was the average deceleration experienced by the survivor when she was slowed to rest by the net? (*b*) What would you do to make it "safer" (that is, to generate a smaller deceleration): would you stiffen or loosen the net? Explain.

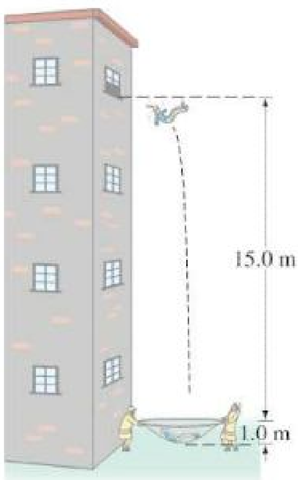


FIGURE 2–37 Problem 57.

58. The acceleration due to gravity on the Moon is about one-sixth what it is on Earth. If an object is thrown vertically upward on the Moon, how many times higher will it go than it would on Earth, assuming the same initial velocity?

59. A person who is properly constrained by an over-the-shoulder seat belt has a good chance of surviving a car collision if the deceleration does not exceed about 30 "g's" $(1.0\ g = 9.8\ \mathrm{m/s^2})$. Assuming uniform deceleration of this value, calculate the distance over which the front end of the car must be designed to collapse if a crash brings the car to rest from 100 km/h.

60. Agent Bond is standing on a bridge, 12 m above the road below, and his pursuers are getting too close for comfort. He spots a flatbed truck approaching at 25 m/s, which he measures by knowing that the telephone poles the truck is passing are 25 m apart in this country. The bed of the truck is 1.5 m above the road, and Bond quickly calculates how many poles away the truck should be when he jumps down from the bridge onto the truck to make his getaway. How many poles is it?

61. Suppose a car manufacturer tested its cars for front-end collisions by hauling them up on a crane and dropping them from a certain height. (*a*) Show that the speed just before a car hits the ground, after falling from rest a vertical distance H, is given by $\sqrt{2gH}$. What height corresponds to a collision at (*b*) 60 km/h? (*c*) 100 km/h?

62. Every year the Earth travels about 10^9 km as it orbits the Sun. What is Earth's average speed in km/h?

63. A 95-m-long train begins uniform acceleration from rest. The front of the train has a speed of 25 m/s when it passes a railway worker who is standing 180 m from where the front of the train started. What will be the speed of the last car as it passes the worker? (See Fig. 2–38.)

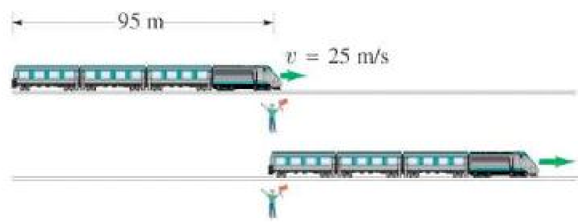


FIGURE 2–38 Problem 63.

64. A person jumps off a diving board 4.0 m above the water's surface into a deep pool. The person's downward motion stops 2.0 m below the surface of the water. Estimate the average deceleration of the person while under the water.

65. In the design of a rapid transit system, it is necessary to balance the average speed of a train against the distance between stops. The more stops there are, the slower the train's average speed. To get an idea of this problem, calculate the time it takes a train to make a 9.0-km trip in two situations: (*a*) the stations at which the trains must stop are 1.8 km apart (a total of 6 stations, including those at the ends); and (*b*) the stations are 3.0 km apart (4 stations total). Assume that at each station the train accelerates at a rate of $1.1\ \mathrm{m/s^2}$ until it reaches 90 km/h, then stays at this speed until its brakes are applied for arrival at the next station, at which time it decelerates at $-2.0\ \mathrm{m/s^2}$. Assume it stops at each intermediate station for 20 s.

66. Pelicans tuck their wings and free fall straight down when diving for fish. Suppose a pelican starts its dive from a height of 16.0 m and cannot change its path once committed. If it takes a fish 0.20 s to perform evasive action, at what minimum height must it spot the pelican to escape? Assume the fish is at the surface of the water.

67. In putting, the force with which a golfer strikes a ball is planned so that the ball will stop within some small distance of the cup, say, 1.0 m long or short, in case the putt is missed. Accomplishing this from an uphill lie (that is, putting downhill, see Fig. 2–39) is more difficult than from a downhill lie. To see why, assume that on a particular green the ball decelerates constantly at $2.0\ \mathrm{m/s^2}$ going downhill, and constantly at $3.0\ \mathrm{m/s^2}$ going uphill. Suppose we have an uphill lie 7.0 m from the cup. Calculate the allowable range of initial velocities we may impart to the ball so that it stops in the range 1.0 m short to 1.0 m long of the cup. Do the same for a downhill lie 7.0 m from the cup. What in your results suggests that the downhill putt is more difficult?

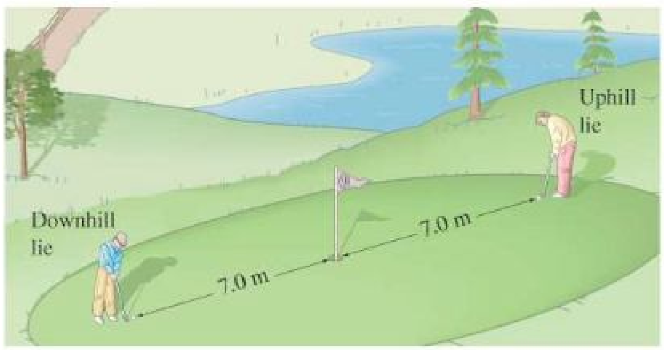


FIGURE 2–39 Problem 67. Golf on Wednesday morning.

68. A fugitive tries to hop on a freight train traveling at a constant speed of 6.0 m/s. Just as an empty box car passes him, the fugitive starts from rest and accelerates at $a = 4.0\ \mathrm{m/s^2}$ to his maximum speed of 8.0 m/s. (*a*) How long does it take him to catch up to the empty box car? (*b*) What is the distance traveled to reach the box car?

69. A stone is dropped from the roof of a high building. A second stone is dropped 1.50 s later. How far apart are the stones when the second one has reached a speed of 12.0 m/s?

70. A race car driver must average 200.0 km/h over the course of a time trial lasting ten laps. If the first nine laps were done at 198.0 km/h, what average speed must be maintained for the last lap?

71. A bicyclist in the Tour de France crests a mountain pass as he moves at 18 km/h. At the bottom, 4.0 km farther, his speed is 75 km/h. What was his average acceleration (in $\mathrm{m/s^2}$) while riding down the mountain?

72. Two children are playing on two trampolines. The first child can bounce up one-and-a-half times higher than the second child. The initial speed up of the second child is 5.0 m/s. (*a*) Find the maximum height the second child reaches. (*b*) What is the initial speed of the first child? (*c*) How long was the first child in the air?

73. An automobile traveling 95 km/h overtakes a 1.10-km-long train traveling in the same direction on a track parallel to the road. If the train's speed is 75 km/h, how long does it take the car to pass it, and how far will the car have traveled in this time? See Fig. 2–40. What are the results if the car and train are traveling in opposite directions?

FIGURE 2–40 Problem 73.

74. A baseball pitcher throws a baseball with a speed of 44 m/s. In throwing the baseball, the pitcher accelerates the ball through a displacement of about 3.5 m, from behind the body to the point where it is released (Fig. 2–41). Estimate the average acceleration of the ball during the throwing motion.

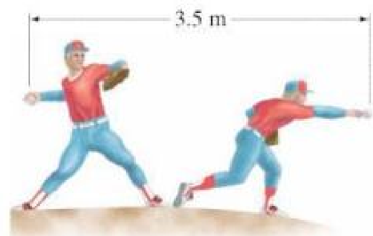


FIGURE 2–41 Problem 74.

75. A rocket rises vertically, from rest, with an acceleration of $3.2\ \mathrm{m/s^2}$ until it runs out of fuel at an altitude of 1200 m. After this point, its acceleration is that of gravity, downward. (*a*) What is the velocity of the rocket when it runs out of fuel? (*b*) How long does it take to reach this point? (*c*) What maximum altitude does the rocket reach? (*d*) How much time (total) does it take to reach maximum altitude? (*e*) With what velocity does the rocket strike the Earth? (*f*) How long (total) is it in the air?

76. Consider the street pattern shown in Fig. 2–42. Each intersection has a traffic signal, and the speed limit is 50 km/h. Suppose you are driving from the west at the speed limit. When you are 10 m from the first intersection, all the lights turn green. The lights are green for 13 s each. (*a*) Calculate the time needed to reach the third stoplight. Can you make it through all three lights without stopping? (*b*) Another car was stopped at the first light when all the lights turned green. It can accelerate at the rate of $2.0\ \mathrm{m/s^2}$ to the speed limit. Can the second car make it through all three lights without stopping?

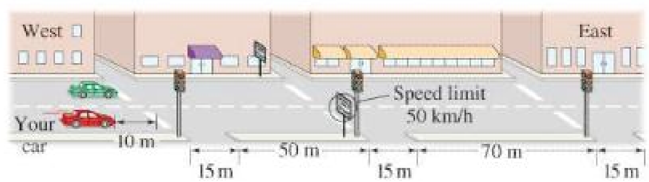


FIGURE 2–42 Problem 76.

77. A police car at rest, passed by a speeder traveling at a constant 120 km/h, takes off in hot pursuit. The police officer catches up to the speeder in 750 m, maintaining a constant acceleration. (*a*) Qualitatively plot the position vs. time graph for both cars from the police car's start to the catch-up point. Calculate (*b*) how long it took the police officer to overtake the speeder, (*c*) the required police car acceleration, and (*d*) the speed of the police car at the overtaking point.

78. A stone is dropped from the roof of a building; 2.00 s after that, a second stone is thrown straight down with an initial speed of 25.0 m/s, and the two stones land at the same time. (*a*) How long did it take the first stone to reach the ground? (*b*) How high is the building? (*c*) What are the speeds of the two stones just before they hit the ground?

79. Two stones are thrown vertically up at the same time. The first stone is thrown with an initial velocity of 11.0 m/s from a 12th-floor balcony of a building and hits the ground after 4.5 s. With what initial velocity should the second stone be thrown from a 4^{th}-floor balcony so that it hits the ground at the same time as the first stone? Make simple assumptions, like equal-height floors.

80. If there were no air resistance, how long would it take a free-falling parachutist to fall from a plane at 3200 m to an altitude of 350 m, where she will pull her ripcord? What would her speed be at 350 m? (In reality, the air resistance will restrict her speed to perhaps 150 km/h.)

81. A fast-food restaurant uses a conveyor belt to send the burgers through a grilling machine. If the grilling machine is 1.1 m long and the burgers require 2.5 min to cook, how fast must the conveyor belt travel? If the burgers are spaced 15 cm apart, what is the rate of burger production (in burgers/min)?

82. Bill can throw a ball vertically at a speed 1.5 times faster than Joe can. How many times higher will Bill's ball go than Joe's?

83. You stand at the top of a cliff while your friend stands on the ground below you. You drop a ball from rest and see that it takes 1.2 s for the ball to hit the ground below. Your friend then picks up the ball and throws it up to you, such that it just comes to rest in your hand. What is the speed with which your friend threw the ball?

84. Two students are asked to find the height of a particular building using a barometer. Instead of using the barometer as an altitude-measuring device, they take it to the roof of the building and drop it off, timing its fall. One student reports a fall time of 2.0 s, and the other, 2.3 s. How much difference does the 0.3 s make for the estimates of the building's height?

* 85. Figure 2–43 shows the position vs. time graph for two bicycles, A and B. (*a*) Is there any instant at which the two bicycles have the same velocity? (*b*) Which bicycle has the larger acceleration? (*c*) At which instant(s) are the bicycles passing each other? Which bicycle is passing the other? (*d*) Which bicycle has the highest instantaneous velocity? (*e*) Which bicycle has the higher average velocity?

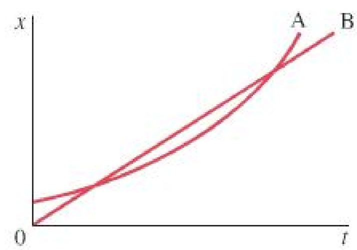


FIGURE 2–43 Problem 85.

Answers to Exercises

A: (*b*).
B: (*a*) +; (*b*) −; (*c*) −; (*d*) +.
C: (*c*).
D: $4.9\ m/s^2$.
E: That plane on which a smooth ball will not roll; or perpendicular to vertical.