The pendulum of a clock is an example of oscillatory motion. Many kinds of oscillatory motion are sinusoidal in time, or nearly so, and are referred to as being simple harmonic motion. Real systems generally have at least some friction, causing the motion to be "damped." When an external sinusoidal force is exerted on a system able to oscillate, resonance occurs if the driving force is at or near the natural frequency of vibration.

Vibrations can give rise to waves—such as water waves or waves traveling along a cord—which travel outward from their source.

CHAPTER 11

Vibrations and Waves

Many objects vibrate or oscillate—an object on the end of a spring, a tuning fork, the balance wheel of an old watch, a pendulum, a plastic ruler held firmly over the edge of a table and gently struck, the strings of a guitar or piano. Spiders detect prey by the vibrations of their webs; cars oscillate up and down when they hit a bump; buildings and bridges vibrate when heavy trucks pass or the wind is fierce. Indeed, because most solids are elastic (see Section 9–5), they vibrate (at least briefly) when given an impulse. Electrical oscillations occur in radio and television sets. At the atomic level, atoms vibrate within a molecule, and the atoms of a solid vibrate about their relatively fixed positions. Because it is so common in everyday life and occurs in so many areas of physics, oscillatory (or vibrational) motion is of great importance. Mechanical vibrations are fully described on the basis of Newtonian mechanics.

Vibrations and wave motion are intimately related subjects. Waves—whether ocean waves, waves on a string, earthquake waves, or sound waves in air—have as their source a vibration. In the case of sound, not only is the source a vibrating object, but so is the detector—the eardrum or the membrane of a microphone. Indeed, when a wave travels through a medium, the medium vibrates (such as air for sound waves). In the second half of this Chapter, after we discuss vibrations, we will discuss simple waves such as those on water or on a string. In Chapter 12 we will study sound waves, and in later Chapters we will encounter other forms of wave motion, including electromagnetic waves and light.

11–1 Simple Harmonic Motion

When an object **vibrates** or **oscillates** back and forth, over the same path, each vibration taking the same amount of time, the motion is **periodic**. The simplest form of periodic motion is represented by an object oscillating on the end of a uniform coil spring. Because many other types of vibrational motion closely resemble this system, we will look at it in detail. We assume that the mass of the spring can be ignored, and that the spring is mounted horizontally, as shown in Fig. 11–1a, so that the object of mass m slides without friction on the horizontal surface. Any spring has a natural length at which it exerts no force on the mass m. The position of the mass at this point is called the **equilibrium position**. If the mass is moved either to the left, which compresses the spring, or to the right, which stretches it, the spring exerts a force on the mass that acts in the direction of returning the mass to the equilibrium position; hence it is called a *restoring force*. We consider the common situation where we can assume the magnitude of the restoring force F is directly proportional to the displacement x the spring has been stretched (Fig. 11–1b) or compressed (Fig. 11–1c) from the equilibrium position:

Equilibrium position

$$F = -kx. \qquad \text{[force exerted by spring]} \quad \textbf{(11–1)}$$

Note that the equilibrium position has been chosen at $x = 0$. Equation 11–1, which is often referred to as Hooke's law (see Sections 6–4 and 9–5), is accurate as long as the spring is not compressed to the point where the coils are close to touching, or stretched beyond the elastic region (see Fig. 9–19).

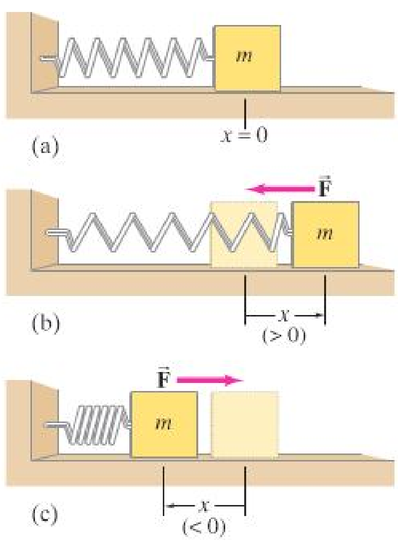


FIGURE 11–1 A mass vibrating at the end of a uniform spring.

The minus sign in Eq. 11–1 indicates that the restoring force is always in the direction opposite to the displacement x. For example, if we choose the positive direction to the right in Fig. 11–1, x is positive when the spring is stretched, but the direction of the restoring force is to the left (negative direction). If the spring is compressed, x is negative (to the left) but the force F acts toward the right (Fig. 11–1c).

The proportionality constant k in Eq. 11–1 is called the *spring constant* or *spring stiffness constant*. To stretch the spring a distance x, one has to exert an (external) force on the free end of the spring at least equal to

$$F = +kx. \qquad \text{[external force on spring]}$$

The greater the value of k, the greater the force needed to stretch a spring a given distance. That is, the stiffer the spring, the greater the spring constant k.

Note that the force F in Eq. 11–1 is *not* a constant, but varies with position. Therefore the acceleration of the mass m is not constant, so we *cannot* use the equations for constant acceleration developed in Chapter 2.

CAUTION
Force and acceleration are not *constant; Eqs. 2–11 are not useful here*

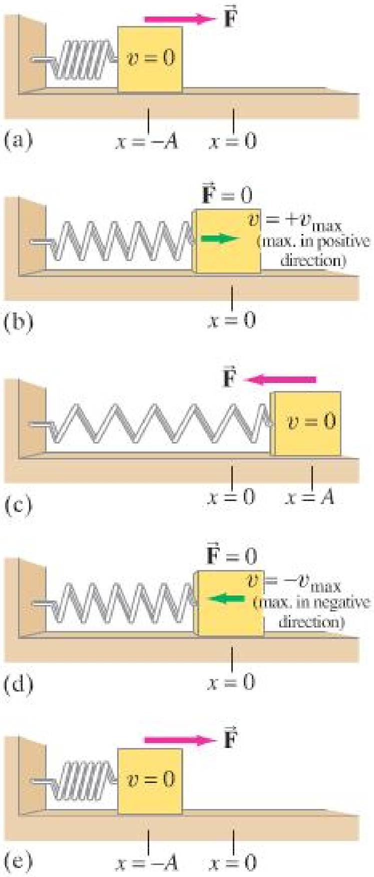


FIGURE 11–2 Force on, and velocity of, a mass at different positions of its oscillation cycle on a frictionless surface.

Let us examine what happens when our uniform spring is initially compressed a distance $x = -A$, as shown in Fig. 11–2a, and then released. The spring exerts a force on the mass that pushes it toward the equilibrium position. But because the mass has been accelerated by the force, it passes the equilibrium position with considerable speed. Indeed, as the mass reaches the equilibrium position, the force on it decreases to zero, but its speed at this point is a maximum, v_{max}, Fig. 11–2b. As the mass moves farther to the right, the force on it acts to slow it down, and it stops momentarily at $x = A$, Fig. 11–2c. It then begins moving back in the opposite direction, accelerating until it passes the equilibrium point, Fig. 11–2d, and then slows down until it reaches zero speed at the original starting point, $x = -A$, Fig. 11–2e. It then repeats the motion, moving back and forth symmetrically between $x = A$ and $x = -A$.

EXERCISE A An object is oscillating back and forth. Which of the following statements are true at some time during the course of the motion? (*a*) The object can have zero velocity and, simultaneously, nonzero acceleration. (*b*) The object can have zero velocity and, simultaneously, zero acceleration. (*c*) The object can have zero acceleration and, simultaneously, nonzero velocity. (*d*) The object can have nonzero velocity and nonzero acceleration simultaneously.

To discuss vibrational motion, we need to define a few terms. The distance x of the mass from the equilibrium point at any moment is called the **displacement**. The maximum displacement—the greatest distance from the equilibrium point—is called the **amplitude**, A. One **cycle** refers to the complete to-and-fro motion from some initial point back to that same point—say, from $x = -A$ to $x = A$ and back to $x = -A$. The **period**, T, is defined as the time required to complete one cycle. Finally, the **frequency**, f, is the number of complete cycles per second. Frequency is generally specified in hertz (Hz), where $1\text{ Hz} = 1$ cycle per second (s^{-1}). It is easy to see, from their definitions, that frequency and period are inversely related, as we saw earlier (Eqs. 5–2 and 8–8):

$$f = \frac{1}{T} \quad \text{and} \quad T = \frac{1}{f}; \tag{11–2}$$

for example, if the frequency is 5 cycles per second, then each cycle takes $\frac{1}{5}$ s.

The oscillation of a spring hung vertically is essentially the same as that of a horizontal spring. Because of gravity, the length of a vertical spring with a mass m on the end will be longer at equilibrium than when that same spring is horizontal, as shown in Fig. 11–3. The spring is in equilibrium when $\Sigma F = 0 = mg - kx_0$, so the spring stretches an extra amount $x_0 = mg/k$ to be in equilibrium. If x is measured from this new equilibrium position, Eq. 11–1 can be used directly with the same value of k.

CAUTION

For vertical spring, measure displacement (x or y) from the vertical equilibrium position

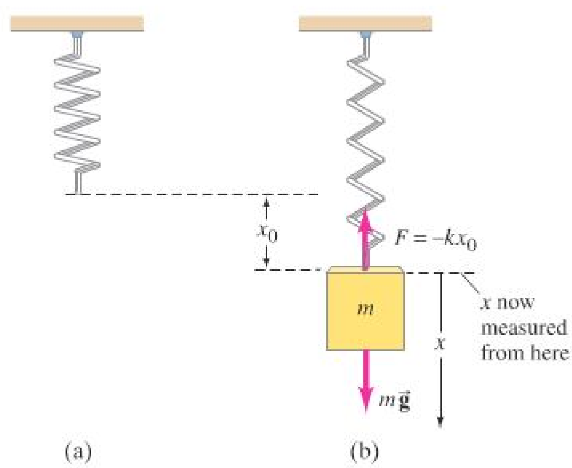


FIGURE 11–3
(a) Free spring, hung vertically.
(b) Mass m attached to spring in new equilibrium position, which occurs when $\Sigma F = 0 = mg - kx_0$.

EXAMPLE 11–1 **Car springs.** When a family of four with a total mass of 200 kg step into their 1200-kg car, the car's springs compress 3.0 cm. (*a*) What is the spring constant of the car's springs (Fig. 11–4), assuming they act as a single spring? (*b*) How far will the car lower if loaded with 300 kg rather than 200 kg?

APPROACH We use Hooke's law. The extra force equal to the weight of the people, mg, causes a 3.0-cm displacement.

SOLUTION (*a*) The added force of $(200\,\text{kg})(9.8\,\text{m/s}^2) = 1960\,\text{N}$ causes the springs to compress 3.0×10^{-2} m. Therefore (Eq. 11–1), the spring constant is

$$k = \frac{F}{x} = \frac{1960\,\text{N}}{3.0 \times 10^{-2}\,\text{m}} = 6.5 \times 10^4\,\text{N/m}.$$

(*b*) If the car is loaded with 300 kg, Hooke's law gives

$$x = \frac{F}{k} = \frac{(300\,\text{kg})(9.8\,\text{m/s}^2)}{(6.5 \times 10^4\,\text{N/m})} = 4.5 \times 10^{-2}\,\text{m},$$

or 4.5 cm.

NOTE We could have obtained x without solving for k: since x is proportional to F, if 200 kg compresses the spring 3.0 cm, then 1.5 times the force will compress the spring 1.5 times as much, or 4.5 cm.

FIGURE 11–4 Photo of a car's spring. (Also visible is the shock absorber, in red—see Section 11–5.)

Any vibrating system for which the restoring force is directly proportional to the negative of the displacement (as in Eq. 11–1, $F = -kx$) is said to exhibit **simple harmonic motion** (SHM).† Such a system is often called a **simple harmonic oscillator** (SHO). We saw in Section 9–5 that most solid materials stretch or compress according to Eq. 11–1 as long as the displacement is not too great. Because of this, many natural vibrations are simple harmonic, or sufficiently close to it that they can be treated using this SHM model.

SHM

SHO

CONCEPTUAL EXAMPLE 11–2 **Is the motion simple harmonic?** Which of the following represent a simple harmonic oscillator: (*a*) $F = -0.5x^2$, (*b*) $F = -2.3y$, (*c*) $F = 8.6x$, (*d*) $F = -4\theta$?

RESPONSE Both (*b*) and (*d*) represent simple harmonic oscillators because they give the force as minus a constant times a displacement. The displacement need not be x, but the minus sign is required to restore the system to equilibrium, which is why (*c*) is not a SHO.

11–2 Energy in the Simple Harmonic Oscillator

With forces that are not constant, such as here with simple harmonic motion, it is often convenient and useful to use the energy approach, as we saw in Chapter 6.

To stretch or compress a spring, work has to be done. Hence potential energy is stored in a stretched or compressed spring. Indeed, we have already seen in Section 6–4 that elastic potential energy is given by

$$\text{PE} = \tfrac{1}{2}kx^2.$$

The total mechanical energy E of a mass–spring system is the sum of the kinetic and potential energies,

$$E = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2, \qquad \textbf{(11–3)}$$

Total energy of SHO

where v is the velocity of the mass m when it is a distance x from the equilibrium position. As long as there is no friction, the total mechanical energy E

† The word "harmonic" refers to the motion being *sinusoidal*, which we discuss in Section 11–3. It is "simple" when there is sinusoidal motion of a single frequency.

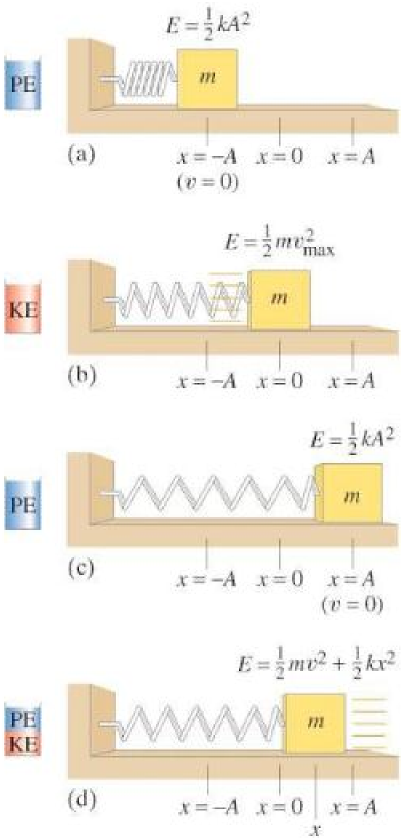


FIGURE 11–5 Energy changes from potential energy to kinetic energy and back again as the spring oscillates. Energy "buckets" (on the left) are described in Section 6–7.

remains constant. As the mass oscillates back and forth, the energy continuously changes from potential energy to kinetic energy, and back again (Fig. 11–5). At the extreme points, $x = -A$ and $x = A$ (Fig. 11–5a, c), all the energy is stored in the spring as potential energy (and is the same whether the spring is compressed or stretched to the full amplitude). At these extreme points, the mass stops momentarily as it changes direction, so $v = 0$ and

$$E = \tfrac{1}{2}m(0)^2 + \tfrac{1}{2}kA^2 = \tfrac{1}{2}kA^2. \tag{11–4a}$$

Thus, the **total mechanical energy of a simple harmonic oscillator is proportional to the square of the amplitude**. At the equilibrium point, $x = 0$ (Fig. 11–5b), all the energy is kinetic:

$$E = \tfrac{1}{2}mv^2_{max} + \tfrac{1}{2}k(0)^2 = \tfrac{1}{2}mv^2_{max}, \tag{11–4b}$$

where v_{max} represents the *maximum* velocity during the motion (which occurs at $x = 0$). At intermediate points (Fig. 11–5d), the energy is part kinetic and part potential; because energy is conserved (we use Eqs. 11–3 and 11–4a),

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}kA^2. \tag{11–4c}$$

From this conservation of energy equation, we can obtain the velocity as a function of position. Solving for v^2, we have

$$v^2 = \frac{k}{m}\left(A^2 - x^2\right) = \frac{k}{m}A^2\left(1 - \frac{x^2}{A^2}\right).$$

From Eqs. 11–4a and 11–4b, we have $\frac{1}{2}mv^2_{max} = \frac{1}{2}kA^2$, so $v^2_{max} = (k/m)A^2$. Inserting this into the equation above and taking the square root, we have

$$v = \pm\, v_{max}\sqrt{1 - \frac{x^2}{A^2}}. \tag{11–5}$$

This gives the velocity of the object at any position x. The object moves back and forth, so its velocity can be either in the + or − direction, but its magnitude depends only on the magnitude of x.

CONCEPTUAL EXAMPLE 11–3 **Doubling the amplitude.** Suppose the spring in Fig. 11–5 is stretched twice as far (to $x = 2A$). What happens to (*a*) the energy of the system, (*b*) the maximum velocity of the oscillating mass, (*c*) the maximum acceleration of the mass?

RESPONSE (*a*) From Eq. 11–4a, the total energy is proportional to the square of the amplitude A, so stretching it twice as far quadruples the energy ($2^2 = 4$). You may protest, "I did work stretching the spring from $x = 0$ to $x = A$. Don't I do the same work stretching it from A to $2A$?" No. The force you exert is proportional to the displacement x, so for the second displacement, from $x = A$ to $2A$, you do more work than for the first displacement ($x = 0$ to A).
(*b*) From Eq. 11–4b, we can see that since the energy is quadrupled, the maximum velocity must be doubled. $\left[v_{max} \propto \sqrt{E} \propto A.\right]$
(*c*) Since the force is twice as great when we stretch the spring twice as far, the acceleration is also twice as great: $a \propto F \propto x$.

EXERCISE B Suppose the spring in Fig. 11–5 is compressed to $x = -A$, but is given a push to the right so that the initial speed of the mass m is v_0. What effect does this push have on (*a*) the energy of the system, (*b*) the maximum velocity, (*c*) the maximum acceleration?

EXAMPLE 11–4 **Spring calculations.** A spring stretches 0.150 m when a 0.300-kg mass is gently lowered on it as in Fig. 11–3b. The spring is then set up horizontally with the 0.300-kg mass resting on a frictionless table as in Fig. 11–5. The mass is pulled so that the spring is stretched 0.100 m from the equilibrium point, and released from rest. Determine (*a*) the spring stiffness constant k, (*b*) the amplitude of the horizontal oscillation A, (*c*) the magnitude of the maximum velocity v_{max}, (*d*) the magnitude of the velocity v when the mass is 0.050 m from equilibrium, and (*e*) the magnitude of the maximum acceleration a_{max} of the mass.

APPROACH When the 0.300-kg mass hangs at rest from the spring as in Fig. 11–3b, we apply Newton's second law for the vertical forces: $\Sigma F = 0 = mg - kx_0$, so $k = mg/x_0$. For the horizontal oscillations, the amplitude is given, the velocities are found using conservation of energy, and the acceleration from $F = ma$.

SOLUTION (*a*) The spring stretches 0.150 m due to the 0.300-kg load, so

$$k = \frac{F}{x_0} = \frac{mg}{x_0} = \frac{(0.300\ \text{kg})(9.80\ \text{m/s}^2)}{0.150\ \text{m}} = 19.6\ \text{N/m}.$$

(*b*) The spring is now horizontal (on a table). It is stretched 0.100 m from equilibrium and is given no initial speed, so $A = 0.100$ m.

(*c*) The maximum velocity v_{max} is attained as the mass passes through the equilibrium point where all the energy is kinetic. By comparing the total energy (see Eq. 11–3) at equilibrium with that at full extension, conservation of energy tells us that

$$\tfrac{1}{2}mv_{max}^2 + 0 = 0 + \tfrac{1}{2}kA^2,$$

where $A = 0.100$ m. (Or, compare Eqs. 11–4a and b.) Solving for v_{max}, we have

$$v_{max} = A\sqrt{\frac{k}{m}} = (0.100\ \text{m})\sqrt{\frac{19.6\ \text{N/m}}{0.300\ \text{kg}}} = 0.808\ \text{m/s}.$$

(*d*) We use conservation of energy, or Eq. 11–5 derived from it, and find that

$$v = v_{max}\sqrt{1 - \frac{x^2}{A^2}} = (0.808\ \text{m/s})\sqrt{1 - \frac{(0.050\ \text{m})^2}{(0.100\ \text{m})^2}} = 0.70\ \text{m/s}.$$

(*e*) By Newton's second law, $F = ma$. So the maximum acceleration occurs where the force is greatest—that is, when $x = A = 0.100$ m. Thus

$$a_{max} = \frac{F_{max}}{m} = \frac{kA}{m} = \frac{(19.6\ \text{N/m})(0.100\ \text{m})}{0.300\ \text{kg}} = 6.53\ \text{m/s}^2.$$

NOTE We cannot use the kinematic equations, Eqs. 2–11, because the acceleration is not constant in SHM.

EXAMPLE 11–5 **More spring calculations—energy.** For the simple harmonic oscillator of Example 11–4, determine (*a*) the total energy, and (*b*) the kinetic and potential energies at half amplitude ($x = \pm A/2$).

APPROACH We use conservation of energy for a mass–spring system, Eqs. 11–3 and 11–4.

SOLUTION (*a*) With $k = 19.6$ N/m and $A = 0.100$ m, the total energy E from Eq. 11–4a is

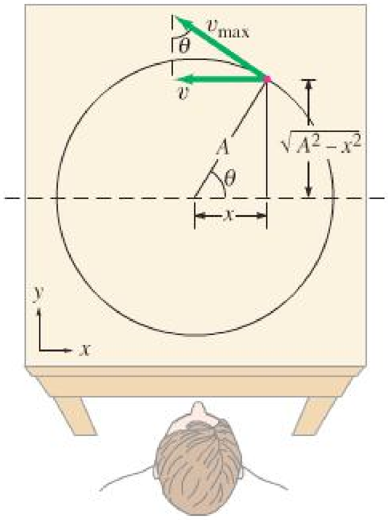
$$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}(19.6\ \text{N/m})(0.100\ \text{m})^2 = 9.80 \times 10^{-2}\ \text{J}.$$

(*b*) At $x = A/2 = 0.050$ m, we have

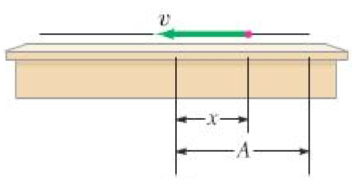
$$\text{PE} = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(19.6\ \text{N/m})(0.050\ \text{m})^2 = 2.5 \times 10^{-2}\ \text{J}.$$

By conservation of energy, the kinetic energy must be

$$\text{KE} = E - \text{PE} = 7.3 \times 10^{-2}\ \text{J}.$$



(a)



(b)

FIGURE 11–6 (a) Circular motion of a small (red) object. (b) Side view of circular motion (x component) is simple harmonic motion.

11–3 The Period and Sinusoidal Nature of SHM

The period of a simple harmonic oscillator is found to depend on the stiffness of the spring and also on the mass m that is oscillating. But—strange as it may seem—the *period does not depend on the amplitude*. You can find this out for yourself by using a watch and timing 10 or 20 cycles of an oscillating spring for a small amplitude and then for a large amplitude.

We can derive a formula for the period of simple harmonic motion (SHM) by comparing SHM to an object rotating in a circle. From this same "reference circle" we can obtain a second useful result—a formula for the position of an oscillating mass as a function of time. There is nothing actually rotating in a circle when a spring oscillates linearly, but it is the mathematical similarity that we find useful.

Period and Frequency

Consider a small object of mass m revolving counterclockwise in a circle of radius A, with constant speed v_{max}, on top of a table as shown in Fig. 11–6. As viewed from above, the motion is a circle in the xy plane. But a person who looks at the motion from the edge of the table sees an oscillatory motion back and forth, and this one-dimensional motion corresponds precisely to simple harmonic motion, as we shall now see.

What the person sees, and what we are interested in, is the projection of the circular motion onto the x axis (Fig. 11–6b). To see that this x-motion is analogous to SHM, let us calculate the magnitude of the x component of the velocity v_{max}, which is labeled v in Fig. 11–6. The two triangles involving θ in Fig. 11–6a are similar, so

$$\frac{v}{v_{max}} = \frac{\sqrt{A^2 - x^2}}{A}$$

or

$$v = v_{max}\sqrt{1 - \frac{x^2}{A^2}}.$$

This is exactly the equation for the speed of a mass oscillating with SHM, as we saw in Eq. 11–5. Thus the projection on the x axis of an object revolving in a circle has the same motion as a mass at the end of a spring.

We can now determine the period of SHM because it is equal to that of the revolving object making one complete revolution. First we note that the velocity v_{max} is equal to the circumference of the circle (distance) divided by the period T:

$$v_{max} = \frac{2\pi A}{T} = 2\pi A f. \tag{11–6}$$

We solve for the period T:

$$T = \frac{2\pi A}{v_{max}}.$$

From energy conservation, Eqs. 11–4a and b, we have $\frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2$, so $A/v_{max} = \sqrt{m/k}$. Thus

Period T of SHM

$$T = 2\pi\sqrt{\frac{m}{k}}. \tag{11–7a}$$

Period and frequency of SHM don't depend on amplitude

This is the formula we were looking for. The period depends on the mass m and the spring stiffness constant k, but not on the amplitude A. We see from Eq. 11–7a that the larger the mass, the longer the period; and the stiffer the spring (larger k), the shorter the period. This makes sense since a larger mass means more inertia and therefore slower response (smaller acceleration). And larger k means greater force and therefore quicker response (larger acceleration). Notice that Eq. 11–7a is not a direct proportion: the period varies as the *square root* of m/k. For example, the mass must be quadrupled to double the period.

Equation 11–7a is fully in accord with experiment and is valid not only for a spring, but for all kinds of simple harmonic motion—that is, for motion subject to a restoring force proportional to displacement, Eq. 11–1.

We can write the frequency using $f = 1/T$ (Eq. 11–2):

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}. \quad \textbf{(11–7b)}$$

Frequency f of SHM

EXERCISE C Does a car bounce faster on its springs when empty or fully loaded?

EXAMPLE 11–6 ESTIMATE Spider web. A spider of mass 0.30 g waits in its web of negligible mass (Fig. 11–7). A slight movement causes the web to vibrate with a frequency of about 15 Hz. (*a*) Estimate the value of the spring stiffness constant k for the web. (*b*) At what frequency would you expect the web to vibrate if an insect of mass 0.10 g were trapped in addition to the spider?

APPROACH We can only make a rough estimate because a spider's web is fairly complicated and may vibrate with a mixture of frequencies. We use SHM as an approximate model.

SOLUTION (*a*) The frequency of SHM is given by Eq. 11–7b,

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}.$$

We solve for k:

$$\begin{aligned} k &= (2\pi f)^2 m \\ &= (6.28 \times 15\,\mathrm{s}^{-1})^2(3.0 \times 10^{-4}\,\mathrm{kg}) = 2.7\,\mathrm{N/m}. \end{aligned}$$

(*b*) The total mass is now $0.10\,\mathrm{g} + 0.30\,\mathrm{g} = 4.0 \times 10^{-4}\,\mathrm{kg}$. We could substitute $m = 4.0 \times 10^{-4}\,\mathrm{kg}$ into Eq. 11–7b. Instead, we notice that the frequency decreases with the square root of the mass. Since the new mass is 4/3 times the first mass, the frequency changes by a factor of $1/\sqrt{4/3} = \sqrt{3/4}$. Thus $f = (15\,\mathrm{Hz})(\sqrt{3/4}) = 13\,\mathrm{Hz}$.

NOTE Check this result by direct substitution of k, found in part (*a*), and the new mass m into Eq. 11–7b.

FIGURE 11–7 A spider waits for its prey (Example 11–6).

Position as a Function of Time

We now use the reference circle to find the position of a mass undergoing simple harmonic motion as a function of time. From Fig. 11–6, we see that $\cos\theta = x/A$, so the projection of the object's position on the x axis is

$$x = A\cos\theta.$$

Because the mass is rotating with angular velocity ω, we can write $\theta = \omega t$, where θ is in radians (Section 8–1). Thus

$$x = A\cos\omega t. \quad \textbf{(11–8a)}$$

Position as a function of time (SHM)

Furthermore, since the angular velocity ω (specified in radians per second) can be written as $\omega = 2\pi f$, where f is the frequency (Eq. 8–7), we then write

$$x = A\cos(2\pi f t), \quad \textbf{(11–8b)}$$

or in terms of the period T,

$$x = A\cos(2\pi t/T). \quad \textbf{(11–8c)}$$

Notice in Eq. 11–8c that when $t = T$ (that is, after a time equal to one period), we have the cosine of 2π, which is the same as the cosine of zero. This makes sense since the motion repeats itself after a time $t = T$.

CAUTION
t is a variable (time); T is a constant for a given situation

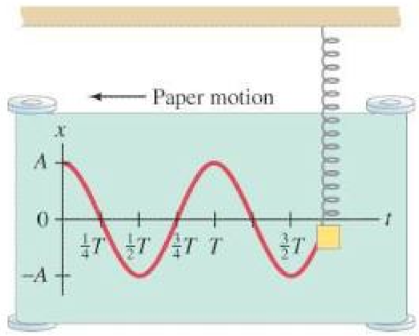


FIGURE 11–8 Position as a function of time $x = A\cos(2\pi t/T)$.

As we have seen, the x component of a uniformly rotating object's motion corresponds precisely to the motion of a simple harmonic oscillator. Thus Eqs. 11–8 give the position of an object undergoing simple harmonic motion. Since the cosine function varies between 1 and -1, x varies between A and $-A$, as it must. If a pen is attached to a vibrating mass as a sheet of paper is moved at a steady rate beneath it (Fig. 11–8), a curve will be drawn that accurately follows Eqs. 11–8.

EXAMPLE 11–7 **Starting with $x = A\cos\omega t$.** The displacement of an object is described by the following equation, where x is in meters and t is in seconds:

$$x = (0.30\text{ m})\cos(8.0\,t).$$

Determine the oscillating object's (*a*) amplitude, (*b*) frequency, (*c*) period, (*d*) maximum speed, and (*e*) maximum acceleration.

APPROACH We start by comparing the given equation for x with Eq. 11–8b, $x = A\cos(2\pi f t)$.

SOLUTION From $x = A\cos(2\pi f t)$, we see by inspection that (*a*) the amplitude $A = 0.30\text{ m}$, and (*b*) $2\pi f = 8.0\text{ s}^{-1}$; so $f = (8.0\text{ s}^{-1}/2\pi) = 1.27\text{ Hz}$. (*c*) Then $T = 1/f = 0.79\text{ s}$. (*d*) The maximum speed (see Eq. 11–6) is

$$v_{max} = 2\pi A f = (2\pi)(0.30\text{ m})(1.27\text{ s}^{-1}) = 2.4\text{ m/s}.$$

(*e*) The maximum acceleration, by Newton's second law, is $a_{max} = F_{max}/m = kA/m$, because $F(= kx)$ is greatest when x is greatest. From Eq. 11–7b we see that $k/m = (2\pi f)^2$. Hence

$$a_{max} = \frac{k}{m}A = (2\pi f)^2 A = (2\pi)^2(1.27\text{ s}^{-1})^2(0.30\text{ m}) = 19\text{ m/s}^2.$$

Sinusoidal Motion

Equation 11–8, $x = A\cos\omega t$, assumes that the oscillating object starts from rest ($v = 0$) at its maximum displacement ($x = A$) at $t = 0$. Other equations for simple harmonic motion are also possible, depending on the initial conditions (when you choose t to be zero). For example, if at $t = 0$ the object is at the equilibrium position and the oscillations are begun by giving the object a push to the right ($+x$), the equation would be

$$x = A\sin\omega t = A\sin(2\pi t/T).$$

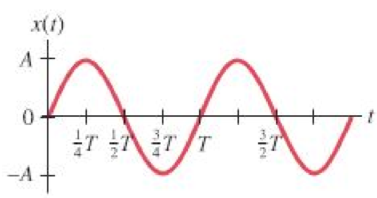


FIGURE 11–9 Sinusoidal nature of SHM as a function of time; in this case, $x = A\sin(2\pi t/T)$ because at $t = 0$ the mass is at the equilibrium position $x = 0$, but it also has (or is given) an initial speed at $t = 0$ that carries it to $x = A$ at $t = \frac{1}{4}T$.

This curve (Fig. 11–9) has the same shape as the cosine curve shown in Fig. 11–8, except it is shifted to the right by a quarter cycle. Hence at $t = 0$ it starts out at $x = 0$ instead of at $x = A$.

SHM is sinusoidal

Both sine and cosine curves are referred to as being **sinusoidal** (having the shape of a sine function). Thus simple harmonic motion† is said to be sinusoidal because the position varies as a sinusoidal function of time.

†Simple harmonic motion can be *defined* as motion that is sinusoidal. This definition is fully consistent with our earlier definition in Section 11–1.

* Velocity and Acceleration as Functions of Time

Figure 11–10a, like Fig. 11–8, shows a graph of displacement x vs. time t, as given by Eqs. 11–8. We can also find the velocity v as a function of time from Fig. 11–6a. For the position shown (red dot in Fig. 11–6a), we see that the magnitude of v is $v_{max} \sin\theta$, but $\vec{v}$ points to the left, so $v = -v_{max}\sin\theta$. Again setting $\theta = \omega t = 2\pi f t = 2\pi t/T$, we have

$$v = -v_{max}\sin\omega t = -v_{max}\sin(2\pi f t) = -v_{max}\sin(2\pi t/T). \quad \textbf{(11–9)}$$

Just after $t = 0$, the velocity is negative (points to the left) and remains so until $t = \frac{1}{2}T$ (corresponding to $\theta = 180° = \pi$ radians). After $t = \frac{1}{2}T$ until $t = T$ the velocity is positive. The velocity as a function of time (Eq. 11–9) is plotted in Fig. 11–10b. From Eqs. 11–6 and 11–7b,

$$v_{max} = 2\pi A f = A\sqrt{\frac{k}{m}}.$$

For a given spring–mass system, the maximum speed v_{max} is higher if the amplitude is larger, and always occurs as the mass passes the equilibrium point.

The acceleration as a function of time is found from Newton's second law:

$$a = \frac{F}{m} = \frac{-kx}{m} = -\left(\frac{kA}{m}\right)\cos\omega t = -a_{max}\cos(2\pi t/T) \quad \textbf{(11–10)}$$

where the maximum acceleration is

$$a_{max} = kA/m.$$

Equation 11–10 is plotted in Fig. 11–10c. Because the acceleration of a SHO is *not* constant, the equations for uniformly accelerated motion do *not* apply to SHM.

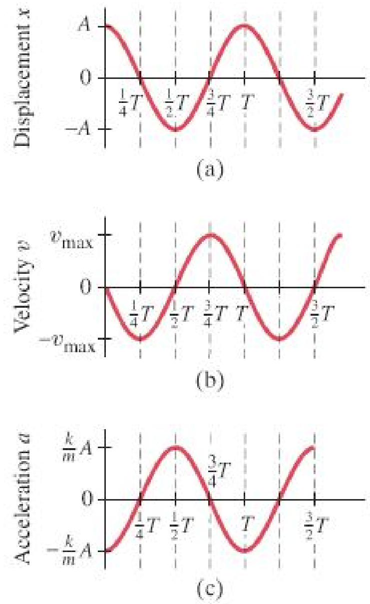


FIGURE 11–10 Graphs showing (a) displacement x as a function of time t: $x = A\cos(2\pi t/T)$; (b) velocity as a function of time: $v = -v_{max}\sin(2\pi t/T)$; (c) acceleration as a function of time: $a = -(kA/m)\cos(2\pi t/T)$.

EXAMPLE 11–8 Loudspeaker. The cone of a loudspeaker vibrates in SHM at a frequency of 262 Hz ("middle C"). The amplitude at the center of the cone is $A = 1.5 \times 10^{-4}$ m, and at $t = 0$, $x = A$. (*a*) What equation describes the motion of the center of the cone? (*b*) What are the velocity and acceleration as a funcion of time? (*c*) What is the position of the cone at $t = 1.00$ ms ($= 1.00 \times 10^{-3}$ s)?

APPROACH The motion begins ($t = 0$) with the cone at its maximum displacement ($x = A$ at $t = 0$). So we use the cosine function, $x = A\cos\omega t$, to describe this SHM.

SOLUTION (*a*) Here

$$\omega = 2\pi f = (6.28\text{ rad})(262\text{ s}^{-1}) = 1650\text{ rad/s}.$$

The motion is described as

$$x = A\cos(2\pi f t) = (1.5 \times 10^{-4}\text{ m})\cos(1650t).$$

(*b*) The maximum velocity, from Eq. 11–6, is $v_{max} = 2\pi A f = 2\pi(1.5 \times 10^{-4}\text{ m})(262\text{ s}^{-1}) = 0.25\text{ m/s}$. Then by Eq. 11–9,

$$v = -(0.25\text{ m/s})\sin(1650t).$$

From Eqs. 11–10 and 11–7b, the maximum acceleration is $a_{max} = (k/m)A = (2\pi f)^2 A = 4\pi^2(262\text{ s}^{-1})^2(1.5 \times 10^{-4}\text{ m}) = 410\text{ m/s}^2$, which is more than 40 g's. So

$$a = -(410\text{ m/s}^2)\cos(1650t).$$

(*c*) At $t = 1.00 \times 10^{-3}$ s, Eq. 11–8a gives us

$$x = A\cos\omega t = (1.5 \times 10^{-4}\text{ m})\cos[(1650\text{ rad/s})(1.00 \times 10^{-3}\text{ s})]$$
$$= (1.5 \times 10^{-4}\text{ m})\cos(1.65\text{ rad}) = -1.2 \times 10^{-5}\text{ m}.$$

NOTE Be sure your calculator is set in RAD mode, not DEG mode, for these $\cos\omega t$ calculations.

CAUTION
Always be sure your calculator is in the correct mode for angles

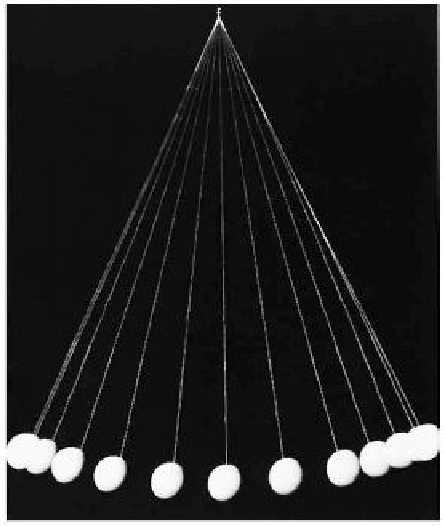


FIGURE 11–11 Strobe-light photo of an oscillating simple pendulum.

11–4 The Simple Pendulum

A **simple pendulum** consists of a small object (the pendulum bob) suspended from the end of a lightweight cord, Fig. 11–11. We assume that the cord doesn't stretch and that its mass can be ignored relative to that of the bob. The motion of a simple pendulum swinging back and forth with negligible friction resembles simple harmonic motion: the pendulum bob oscillates along the arc of a circle with equal amplitude on either side of its equilibrium point, and as it passes through the equilibrium point (where it would hang vertically) it has its maximum speed. But is it really undergoing SHM? That is, is the restoring force proportional to its displacement? Let us find out.

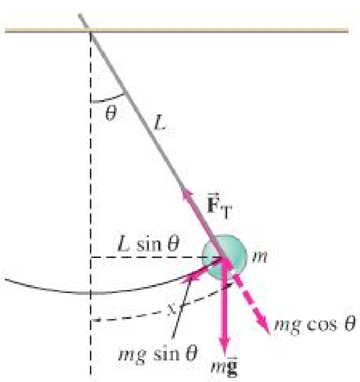


FIGURE 11–12 Simple pendulum, and a free-body diagram.

The displacement of the pendulum along the arc is given by $x = L\theta$, where θ is the angle the cord makes with the vertical and L is the length of the cord (Fig. 11–12). If the restoring force is proportional to x or to θ, the motion will be simple harmonic. The restoring force is the net force on the bob, equal to the component of the weight, mg, tangent to the arc:

$$F = -mg\sin\theta,$$

where g is the acceleration of gravity. The minus sign here, as in Eq. 11–1, means the force is in the direction opposite to the angular displacement θ. Since F is proportional to the sine of θ and not to θ itself, the motion is *not* SHM. However, if θ is small, then $\sin\theta$ is very nearly equal to θ when the latter is specified in radians. This can be seen by noting in Fig. 11–12 that the arc length $x\ (= L\theta)$ is nearly the same length as the chord $(= L\sin\theta)$ indicated by the horizontal straight dashed line, *if θ is small*. For angles less than 15°, the difference between θ (in radians) and $\sin\theta$ is less than 1%—see Table 11–1. Thus, to a very good approximation for small angles,

TABLE 11–1
Sin θ at Small Angles

θ (degrees)	θ (radians)	sin θ	% Difference
0	0	0	0
1°	0.01745	0.01745	0.005%
5°	0.08727	0.08716	0.1%
10°	0.17453	0.17365	0.5%
15°	0.26180	0.25882	1.1%
20°	0.34907	0.34202	2.0%
30°	0.52360	0.50000	4.7%

$$F = -mg\sin\theta \approx -mg\theta.$$

Substituting $x = L\theta$, or $\theta = x/L$, we have

$$F \approx -\frac{mg}{L}x.$$

Thus, for small displacements, the motion is essentially simple harmonic, since this equation fits Hooke's law, $F = -kx$. The effective force constant is $k = mg/L$.

If we substitute $k = mg/L$ into Eq. 11–7a, we obtain the period of a simple pendulum:

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{mg/L}}$$

or

$$T = 2\pi\sqrt{\frac{L}{g}}. \qquad [\theta \text{ small}] \quad \textbf{(11–11a)}$$

Period, simple pendulum

The frequency is $f = 1/T$, so

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}. \qquad [\theta \text{ small}] \quad \textbf{(11–11b)}$$

Frequency, simple pendulum

The mass m of the pendulum bob does not appear in these formulas for T and f. Thus we have the surprising result that the period and frequency of a simple pendulum do not depend on the mass of the pendulum bob. You may have noticed this if you pushed a small child and a large one on the same swing.

We also see from Eq. 11–11a that the period of a pendulum does not depend on the amplitude (like any SHM, Section 11–3), as long as the amplitude θ is small. Galileo is said to have first noted this fact while watching a swinging lamp in the cathedral at Pisa (Fig. 11–13). This discovery led to the invention of the pendulum clock, the first really precise timepiece, which became the standard for centuries.

PHYSICS APPLIED

Pendulum clock

Because a pendulum does not undergo *precisely* SHM, the period does depend slightly on the amplitude—the more so for large amplitudes. The accuracy of a pendulum clock would be affected, after many swings, by the decrease in amplitude due to friction. But the mainspring in a pendulum clock (or the falling weight in a grandfather clock) supplies energy to compensate for the friction and to maintain the amplitude constant, so that the timing remains precise.

EXAMPLE 11–9 **Measuring g.** A geologist uses a simple pendulum that has a length of 37.10 cm and a frequency of 0.8190 Hz at a particular location on the Earth. What is the acceleration of gravity at this location?

APPROACH We can use the length L and frequency f of the pendulum in Eq. 11–11b, which contains our unknown, g.

SOLUTION We solve Eq. 11–11b for g and obtain

$$g = (2\pi f)^2 L = (6.283 \times 0.8190\,\text{s}^{-1})^2(0.3710\,\text{m}) = 9.824\,\text{m/s}^2.$$

EXERCISE D (*a*) Estimate the length of the pendulum in a grandfather clock that ticks once per second. (*b*) What would be the period of a clock with a 1.0-m-long pendulum?

Equations 11–11 apply to a simple pendulum—a concentrated mass at the end of a string of negligible mass—but not to the oscillation of, say, a baseball bat suspended from one end.

FIGURE 11–13 The swinging motion of this lamp, hanging by a very long cord from the ceiling of the cathedral at Pisa, is said to have been observed by Galileo and to have inspired him to the conclusion that the period of a pendulum does not depend on amplitude.

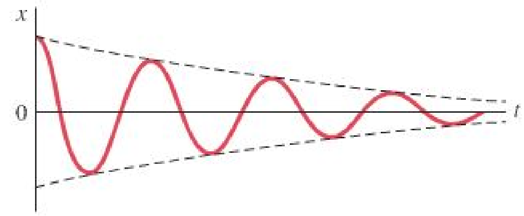


FIGURE 11–14 Damped harmonic motion.

11–5 Damped Harmonic Motion

The amplitude of any real oscillating spring or swinging pendulum will slowly decrease in time until the oscillations stop altogether. Figure 11–14 shows a typical graph of the displacement as a function of time. This is called **damped harmonic motion**. The damping† is generally due to the resistance of air and to internal friction within the oscillating system. The energy that is dissipated to thermal energy results in a decreased amplitude of oscillation.

Since natural oscillating systems are damped in general, why do we even talk about (undamped) simple harmonic motion? The answer is that SHM is much easier to deal with mathematically. And if the damping is not large, the oscillations can be thought of as simple harmonic motion on which the damping is superposed. The decrease in amplitude shown by the dashed curves in Fig. 11–14 represents the damping. Although frictional damping does alter the frequency of vibration, the effect is usually small unless the damping is large; thus Eqs. 11–7 can still be used in most cases.

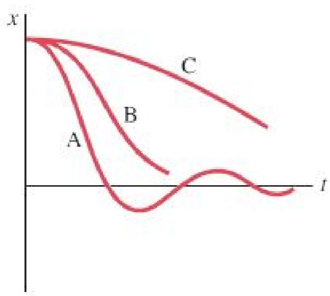


FIGURE 11–15 Graphs that represent (A) underdamped, (B) critically damped, and (C) overdamped oscillatory motion.

Sometimes the damping is so large, however, that the motion no longer resembles simple harmonic motion. Three common cases of heavily damped systems are shown in Fig. 11–15. Curve A represents an **underdamped** situation, in which the system makes several swings before coming to rest, and corresponds to a more heavily damped version of Fig. 11–14. Curve C represents the **overdamped** situation, for which the damping is so large that it takes a long time to reach equilibrium. Curve B represents **critical damping**: in this case equilibrium is reached in the shortest time. These terms all derive from the use of practical damped systems such as door-closing mechanisms and shock absorbers in a car (Fig. 11–16). Such devices are usually designed to give critical damping. But as they wear out, underdamping occurs: a door slams or a car bounces up and down several times each time it hits a bump.

PHYSICS APPLIED

Shock absorbers and building dampers

In many systems, the oscillatory motion is what counts, as in clocks and watches, and damping needs to be minimized. In other systems, oscillations are the problem, such as a car's springs, so a proper amount of damping (i.e., critical) is desired. Well-designed damping is needed for all kinds of applications. Large buildings, especially in California, are now built (or retrofitted) with huge dampers to reduce earthquake damage (Fig. 11–17).

† To "damp" means to diminish, restrain, or extinguish, as to "dampen one's spirits."

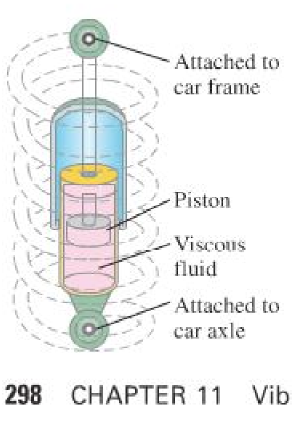


FIGURE 11–16 Automobile spring and shock absorber to provide damping so that a car won't bounce up and down so much.

FIGURE 11–17 These huge dampers placed in a building look a lot like huge automobile shock absorbers, and they serve a similar purpose—to reduce the amplitude and the acceleration of movement when the shock of an earthquake hits.

11–6 Forced Vibrations; Resonance

When a vibrating system is set into motion, it vibrates at its natural frequency (Eqs. 11–7b and 11–11b). However, a system may have an external force applied to it that has its own particular frequency. Then we have a **forced vibration**. For example, we might pull the mass on the spring of Fig. 11–1 back and forth at an externally applied frequency f. The mass then vibrates at the external frequency f of the external force, even if this frequency is different from the **natural frequency** of the spring, which we will now denote by f_0, where (see Eq. 11–7b)

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}.$$

For a forced vibration, the amplitude of vibration is found to depend on the difference between f and f_0, and is a maximum when the frequency of the external force equals the natural frequency of the system—that is, when $f = f_0$. The amplitude is plotted in Fig. 11–18 as a function of the external frequency f. Curve A represents light damping and curve B heavy damping. The amplitude can become large when the external driving frequency f is near the natural frequency, $f \approx f_0$, as long as the damping is not too large. When the damping is small, the increase in amplitude near $f = f_0$ is very large (and often dramatic). This effect is known as **resonance**. The natural vibrating frequency f_0 of a system is also called its **resonant frequency**.

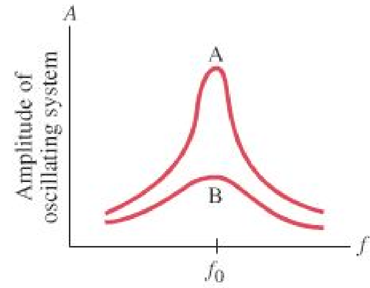


FIGURE 11–18 Resonance for lightly damped (A) and heavily damped (B) systems.

A simple illustration of resonance is pushing a child on a swing. A swing, like any pendulum, has a natural frequency of oscillation. If you push on the swing at a random frequency, the swing bounces around and reaches no great amplitude. But if you push with a frequency equal to the natural frequency of the swing, the amplitude increases greatly. At resonance, relatively little effort is required to obtain a large amplitude.

PHYSICS APPLIED
Swinging

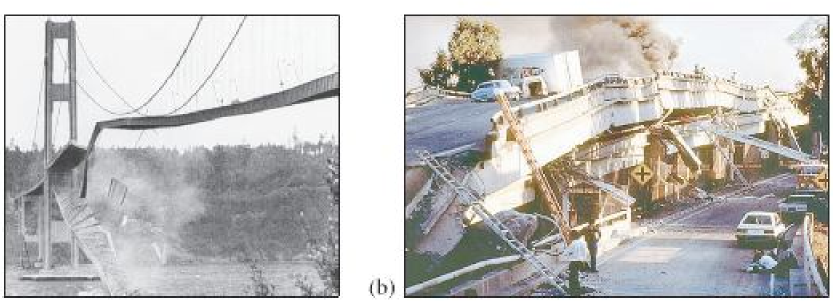
The great tenor Enrico Caruso was said to be able to shatter a crystal goblet by singing a note of just the right frequency at full voice. This is an example of resonance, for the sound waves emitted by the voice act as a forced vibration on the glass. At resonance, the resulting vibration of the goblet may be large enough in amplitude that the glass exceeds its elastic limit and breaks.

PHYSICS APPLIED
Shattering glass via resonance

Since material objects are, in general, elastic, resonance is an important phenomenon in a variety of situations. It is particularly important in building, although the effects are not always foreseen. For example, it has been reported that a railway bridge collapsed because a nick in one of the wheels of a crossing train set up a resonant vibration in the bridge. Marching soldiers break step when crossing a bridge to avoid the possibility that their rhythmic march might match a resonant frequency of the bridge. The collapse of the Tacoma Narrows Bridge (Fig. 11–19a) in 1940 occurred as a result of gusting winds whose approximate frequency matched that of a natural frequency of the bridge, thus driving the span into large-amplitude oscillatory motion. Bridges and tall buildings are now designed with more inherent damping. The Oakland freeway collapse in the 1989 California earthquake (Fig. 11–19b) involved resonant oscillation of a section built on mudfill.

PHYSICS APPLIED
Resonant collapse

Resonance can be very useful, too, and we will meet important examples later, such as in musical instruments and tuning a radio. We will also see that vibrating objects often have not one, but many resonant frequencies.



(a) (b)

FIGURE 11–19 (a) Large-amplitude oscillations of the Tacoma Narrows Bridge, due to gusty winds, led to its collapse (November 7, 1940). (b) Collapse of a freeway in California, due to the 1989 earthquake, in which resonance played a part.

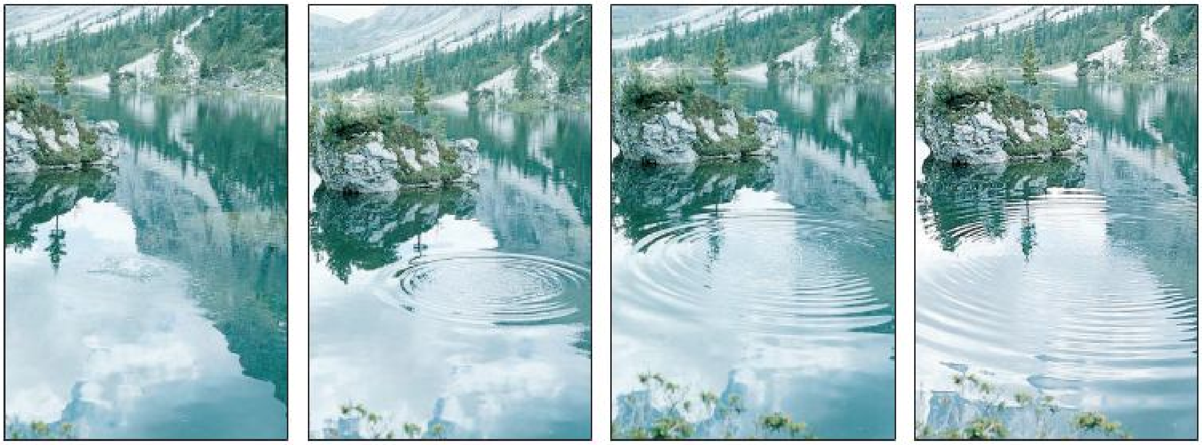


FIGURE 11–20 Water waves spreading outward from a source.

11–7 Wave Motion

When you throw a stone into a lake or pool of water, circular waves form and move outward, Fig. 11–20. Waves will also travel along a cord that is stretched out straight on a table if you vibrate one end back and forth as shown in Fig. 11–21. Water waves and waves on a cord are two common examples of wave motion. We will discuss other kinds of waves later, but for now we will concentrate on these **mechanical waves**.

If you have ever watched ocean waves moving toward shore before they break†, you may have wondered if the waves were carrying water from far out at sea into the beach. They don't. Water waves move with a recognizable velocity. But each particle (or molecule) of the water itself merely oscillates about an equilibrium point. This is clearly demonstrated by observing leaves on a pond as waves move by. The leaves (or a cork) are not carried forward by the waves, but simply oscillate about an equilibrium point because this is the motion of the water itself.

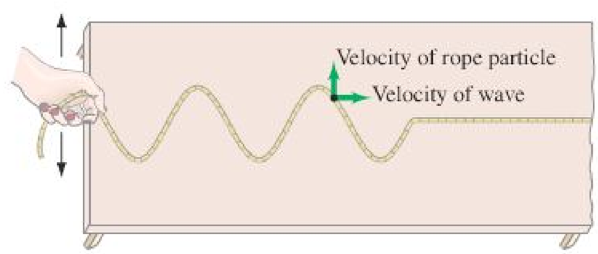


FIGURE 11–21 Wave traveling on a cord. The wave travels to the right along the cord. Particles of the cord oscillate back and forth on the tabletop.

CONCEPTUAL EXAMPLE 11–10 **Wave vs. particle velocity.** Is the velocity of a wave moving along a cord the same as the velocity of a particle of the cord? See Fig. 11–21.

RESPONSE No. The two velocities are different, both in magnitude and direction. The wave on the rope of Fig. 11–21 moves to the right along the tabletop, but each piece of the rope only vibrates to and fro. (The rope clearly does not travel in the direction that the wave on it does.)

Waves can move over large distances, but the medium (the water or the rope) itself has only a limited movement, oscillating about an equilibrium point

†Do not be confused by the "breaking" of ocean waves, which occurs when a wave interacts with the ground in shallow water and hence is no longer a simple wave.

as in simple harmonic motion. Thus, although a wave is not matter, the wave pattern can travel in matter. A wave consists of oscillations that move without carrying matter with them.

Waves are moving oscillations, not carrying matter along

Waves carry energy from one place to another. Energy is given to a water wave, for example, by a rock thrown into the water, or by wind far out at sea. The energy is transported by waves to the shore. The oscillating hand in Fig. 11–21 transfers energy to the rope, and that energy is transported down the rope and can be transferred to an object at the other end. All forms of traveling waves transport energy.

Let us look a little more closely at how a wave is formed and how it comes to "travel." We first look at a single wave bump, or **pulse**. A single pulse can be formed on a rope by a quick up-and-down motion of the hand, Fig. 11–22. The hand pulls up on one end of the rope. Because the end section is attached to adjacent sections, these also feel an upward force and they too begin to move upward. As each succeeding section of rope moves upward, the wave crest moves outward along the rope. Meanwhile, the end section of rope has been returned to its original position by the hand. As each succeeding section of rope reaches its peak position, it too is pulled back down again by the adjacent section of rope. Thus the source of a traveling wave pulse is a disturbance, and cohesive forces between adjacent sections of rope cause the pulse to travel outward. Waves in other media are created and propagate outward in a similar fashion.

Wave pulse

A **continuous** or **periodic wave**, such as that shown in Fig. 11–21, has as its source a disturbance that is continuous and oscillating; that is, the source is a *vibration* or *oscillation*. In Fig. 11–21, a hand oscillates one end of the rope. Water waves may be produced by any vibrating object at the surface, such as your hand; or the water itself is made to vibrate when wind blows across it or a rock is thrown into it. A vibrating tuning fork or drum membrane gives rise to sound waves in air. And we will see later that oscillating electric charges give rise to light waves. Indeed, almost any vibrating object sends out waves.

Periodic wave

The source of any wave, then, is a vibration. And it is a *vibration* that propagates outward and thus constitutes the wave. If the source vibrates sinusoidally in SHM, then the wave itself—if the medium is perfectly elastic—will have a sinusoidal shape both in space and in time. (1) In space: if you take a picture of the wave in space at a given instant of time, the wave will have the shape of a sine or cosine as a function of position. (2) In time: if you look at the motion of the medium at one place over a long period of time—for example, if you look between two closely spaced posts of a pier or out of a ship's porthole as water waves pass by—the up-and-down motion of that small segment of water will be simple harmonic motion. The water moves up and down sinusoidally in time.

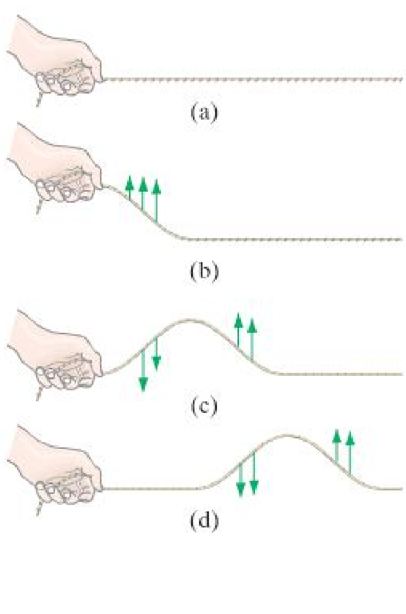


FIGURE 11–22 Motion of a wave pulse to the right. Arrows indicate velocity of cord particles.

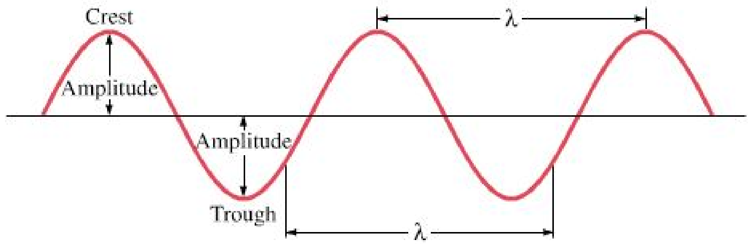


FIGURE 11–23 Characteristics of a single-frequency continuous wave.

Amplitude, A

Wavelength, λ

Frequency, f
Period, T

Wave velocity

Some of the important quantities used to describe a periodic sinusoidal wave are shown in Fig. 11–23. The high points on a wave are called *crests*; the low points, *troughs*. The **amplitude**, A, is the maximum height of a crest, or depth of a trough, relative to the normal (or equilibrium) level. The total swing from a crest to a trough is twice the amplitude. The distance between two successive crests is called the **wavelength**, λ (the Greek letter lambda). The wavelength is also equal to the distance between *any* two successive identical points on the wave. The **frequency**, f, is the number of crests—or complete cycles—that pass a given point per unit time. The **period**, T, equals $1/f$ and is the time elapsed between two successive crests passing by the same point in space.

The **wave velocity**, v, is the velocity at which wave crests (or any other part of the waveform) move. The wave velocity must be distinguished from the velocity of a particle of the medium itself as we saw in Example 11–10.

A wave crest travels a distance of one wavelength, λ, in a time equal to one period, T. Thus the wave velocity is $v = \lambda/T$. Then, since $1/T = f$,

$v = \lambda f$ (sinusoidal waves)

$$v = \lambda f. \tag{11–12}$$

For example, suppose a wave has a wavelength of 5 m and a frequency of 3 Hz. Since three crests pass a given point per second, and the crests are 5 m apart, the first crest (or any other part of the wave) must travel a distance of 15 m during the 1 s. So its speed is 15 m/s.

The magnitude of the velocity of a wave, or its speed, depends on the properties of the medium in which it travels. The speed of a wave on a stretched string or cord, for example, depends on the tension in the cord, F_T, and on the cord's mass per unit length, m/L. For waves of small amplitude, the relationship is

Speed of wave on a cord

$$v = \sqrt{\frac{F_T}{m/L}}. \tag{11–13}$$

This formula makes sense qualitatively on the basis of Newtonian mechanics. That is, we expect the tension to be in the numerator and the mass per unit length in the denominator. Why? Because when the tension is greater, we expect the velocity to be greater since each segment of cord is in tighter contact with its neighbor; and the greater the mass per unit length, the more inertia the cord has and the more slowly the wave would be expected to propagate.

EXAMPLE 11–11 **Wave on a wire.** A wave whose wavelength is 0.30 m is traveling down a 300-m-long wire whose total mass is 15 kg. If the wire is under a tension of 1000 N, what are the speed and frequency of this wave?

APPROACH We assume the velocity of this wave on a wire is given by Eq. 11–13. We get the frequency from Eq. 11–12, $f = v/\lambda$.

SOLUTION From Eq. 11–13, the velocity is

$$v = \sqrt{\frac{1000\text{ N}}{(15\text{ kg})/(300\text{ m})}} = \sqrt{\frac{1000\text{ N}}{(0.050\text{ kg/m})}} = 140\text{ m/s}.$$

The frequency is

$$f = \frac{v}{\lambda} = \frac{140\text{ m/s}}{0.30\text{ m}} = 470\text{ Hz}.$$

NOTE A higher tension would increase both v and f, whereas a thicker, denser wire would reduce v and f.

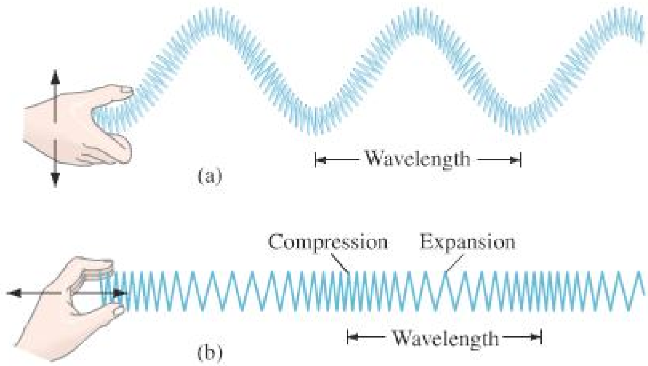


FIGURE 11–24
(a) Transverse wave;
(b) longitudinal wave.

11–8 Types of Waves: Transverse and Longitudinal

Transverse and longitudinal waves

When a wave travels down a rope—say, from left to right as in Fig. 11–21—the particles of the rope vibrate up and down in a direction transverse (that is, perpendicular) to the motion of the wave itself. Such a wave is called a **transverse wave** (Fig. 11–24a). There exists another type of wave known as a **longitudinal wave**. In a longitudinal wave, the vibration of the particles of the medium is *along* the direction of the wave's motion. Longitudinal waves are readily formed on a stretched spring or Slinky by alternately compressing and expanding one end. This is shown in Fig. 11–24b, and can be compared to the transverse wave in Fig. 11–24a. A series of compressions and expansions propagate along the spring. The *compressions* are those areas where the coils are momentarily close together. *Expansions* (sometimes called *rarefactions*) are regions where the coils are momentarily far apart. Compressions and expansions correspond to the crests and troughs of a transverse wave.

An important example of a longitudinal wave is a sound wave in air. A vibrating drumhead, for instance, alternately compresses and rarefies the air in contact with it, producing a longitudinal wave that travels outward in the air, as shown in Fig. 11–25.

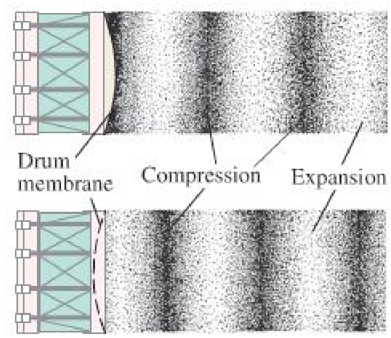


FIGURE 11–25 Production of a sound wave, which is longitudinal, shown at two moments in time about a half period $(\frac{1}{2}T)$ apart.

As in the case of transverse waves, each section of the medium in which a longitudinal wave passes oscillates over a very small distance, whereas the wave itself can travel large distances. Wavelength, frequency, and wave velocity all have meaning for a longitudinal wave. The wavelength is the distance between successive compressions (or between successive expansions), and frequency is the number of compressions that pass a given point per second. The wave velocity is the velocity with which each compression appears to move; it is equal to the product of wavelength and frequency, $v = \lambda f$ (Eq. 11–12).

A longitudinal wave can be represented graphically by plotting the density of air molecules (or coils of a Slinky) versus position at a given instant, as shown in Fig. 11–26. Such a graphical representation makes it easy to illustrate what is happening. Note that the graph looks much like a transverse wave.

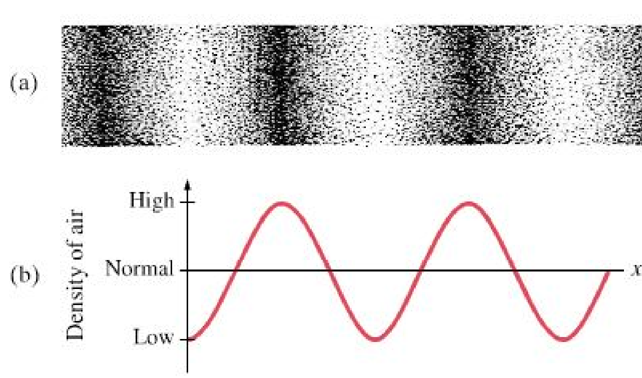


FIGURE 11–26
(a) A longitudinal wave with
(b) its graphical representation at a particular instant in time.

* Speed of Longitudinal Waves

The speed of a longitudinal wave has a form similar to that for a transverse wave on a cord (Eq. 11–13):

$$v = \sqrt{\frac{\text{elastic force factor}}{\text{inertia factor}}}.$$

In particular, for a longitudinal wave traveling down a long solid rod,

Longitudinal wave speed in a long solid rod

$$v = \sqrt{\frac{E}{\rho}}, \tag{11–14a}$$

where E is the elastic modulus (Section 9–5) of the material and ρ is its density. For a longitudinal wave traveling in a liquid or gas,

Longitudinal wave speed in a fluid

$$v = \sqrt{\frac{B}{\rho}}, \tag{11–14b}$$

where B is the bulk modulus (Section 9–5) and ρ is the density.

PHYSICS APPLIED

Space perception by animals using sound waves

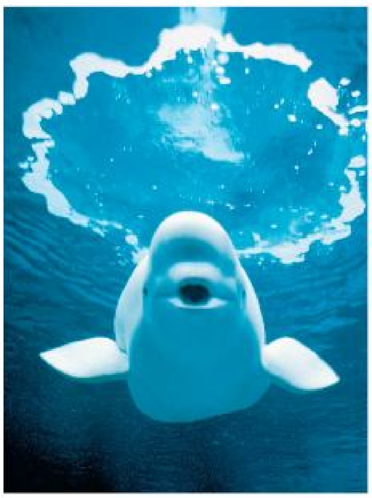


FIGURE 11–27 A toothed whale (Example 11–12).

EXAMPLE 11–12 **Echolocation.** Echolocation is a form of sensory perception used by animals such as bats, toothed whales, and porpoises. The animal emits a pulse of sound (a longitudinal wave) which, after reflection from objects, is detected by the animal. Echolocation waves emitted by whales (Fig. 11–27) have frequencies of about 200,000 Hz. (*a*) What is the wavelength of the whale's echolocation wave? (*b*) If an obstacle is 100 m from the whale, how long after the whale emits a wave is its reflection detected?

APPROACH We first compute the speed of longitudinal (sound) waves in sea water, using Eq. 11–14b and Tables 9–1 and 10–1. The wavelength is $\lambda = v/f$.

SOLUTION (*a*) The speed of longitudinal waves in sea water, which is slightly more dense than pure water, is

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2.0 \times 10^9\,\mathrm{N/m^2}}{1.025 \times 10^3\,\mathrm{kg/m^3}}} = 1.40 \times 10^3\,\mathrm{m/s}.$$

Then, using Eq. 11–12, we find

$$\lambda = \frac{v}{f} = \frac{(1.40 \times 10^3\,\mathrm{m/s})}{(2.0 \times 10^5\,\mathrm{Hz})} = 7.0\,\mathrm{mm}.$$

(*b*) The time required for the round-trip between the whale and the object is

$$t = \frac{\text{distance}}{\text{speed}} = \frac{2(100\,\mathrm{m})}{1.40 \times 10^3\,\mathrm{m/s}} = 0.14\,\mathrm{s}.$$

NOTE We shall see later that waves can "resolve" (or detect) objects only if the wavelength is comparable to or smaller than the object. Thus, a whale can resolve objects on the order of a centimeter or larger in size.

Other Waves

PHYSICS APPLIED

Earthquake waves

Both transverse and longitudinal waves are produced when an **earthquake** occurs. The transverse waves that travel through the body of the Earth are called S waves (S for shear), and the longitudinal waves are called P waves (P for pressure) or *compression* waves. Both longitudinal and transverse waves can travel through a solid since the atoms or molecules can vibrate about their relatively fixed positions in any direction. But in a fluid, only longitudinal waves can propagate, because any transverse motion would experience no restoring force since a fluid is readily deformable. This fact was used by geophysicists to infer that a portion of the Earth's core must be liquid: after an earthquake, longitudinal waves are detected diametrically across the Earth, but not transverse waves.

Besides these two types of waves, *surface waves* can travel along the boundary between two materials. A wave on water is actually a surface wave that moves on the boundary between water and air. The motion of each particle of water at the surface is circular or elliptical (Fig. 11–28), so it is a combination of transverse and longitudinal motions. Below the surface, there is also transverse plus longitudinal wave motion, as shown. At the bottom, the motion is only longitudinal. When a wave approaches shore, the water drags at the bottom and is slowed down, while the crests move ahead at higher speed (Fig. 11–29) and "spill" over the top.

Surface waves are also set up on the Earth when an earthquake occurs. The waves that travel along the surface are mainly responsible for the damage caused by earthquakes.

Waves traveling along a line, as on a stretched string, are *one-dimensional waves*. Surface waves, such as the water waves of Fig. 11–20, are *two-dimensional waves*. Waves that move out from a source in all directions in a medium, such as sound from a speaker or earthquake waves through the Earth, are *three-dimensional waves*.

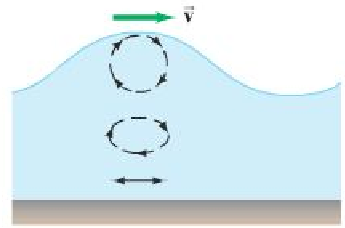
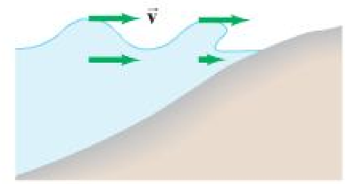


FIGURE 11–28 A water wave is an example of a *surface wave*, which is a combination of transverse and longitudinal wave motions.

FIGURE 11–29 How a wave breaks. The green arrows represent the local velocity of water molecules.



11–9 Energy Transported by Waves

Waves transport energy from one place to another. As waves travel through a medium, the energy is transferred as vibrational energy from particle to particle of the medium. For a sinusoidal wave of frequency f, the particles move in SHM as a wave passes, so each particle has an energy $E = \frac{1}{2}kA^2$, where A is the amplitude of its motion, either transversely or longitudinally. (See Eq. 11–4a.)

Thus, we have the important result that the **energy transported by a wave is proportional to the square of the amplitude**. The **intensity** I of a wave is defined as the power (energy per unit time) transported across unit area perpendicular to the direction of energy flow:

Wave energy $\propto$ *(amplitude)*^{2}

$$I = \frac{\text{energy/time}}{\text{area}} = \frac{\text{power}}{\text{area}}.$$

Intensity (defined)

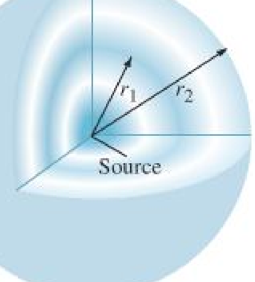
The SI unit of intensity is watts per square meter (W/m^2). Since the energy is proportional to the wave amplitude squared, so too is the intensity:

$$I \propto A^2. \qquad \textbf{(11–15)}$$

Intensity $\propto$ *(amplitude)*^{2}

If a wave flows out from the source in all directions, it is a three-dimensional wave. Examples are sound traveling in open air, earthquake waves, and light waves. If the medium is isotropic (same in all directions), the wave is a *spherical wave* (Fig. 11–30). As the wave moves outward, the energy it carries is spread over a larger and larger area since the surface area of a sphere of radius r is $4\pi r^2$. Thus the intensity of a spherical wave is

FIGURE 11–30 A wave traveling outward in three dimensions from a source is spherical. Two crests (or compressions) are shown, of radii r_1 and r_2.



$$I = \frac{\text{power}}{\text{area}} = \frac{P}{4\pi r^2}. \qquad \text{[spherical wave]} \quad \textbf{(11–16a)}$$

If the power output P of the source is constant, then the intensity decreases as the inverse square of the distance from the source:

$$I \propto \frac{1}{r^2}. \qquad \textbf{(11–16b)}$$

If we consider two points at distances r_1 and r_2 from the source, as in Fig. 11–30, then $I_1 = P/4\pi r_1^2$ and $I_2 = P/4\pi r_2^2$, so

$$\frac{I_2}{I_1} = \frac{r_1^2}{r_2^2}. \qquad \textbf{(11–16c)}$$

$I \propto \frac{1}{r^2}$

Thus, for example, when the distance doubles $(r_2/r_1 = 2)$, the intensity is reduced to $\frac{1}{4}$ its earlier value: $I_2/I_1 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$.

Sounds are quieter farther from the source

The amplitude of a wave also decreases with distance. Since the intensity is proportional to the square of the amplitude (Eq. 11–15), the amplitude A must decrease as $1/r$ so that $I \propto A^2$ will be proportional to $1/r^2$ (as in Eq. 11–16b). Hence

$$A \propto \frac{1}{r}.$$

If we consider again two distances from the source, r_1 and r_2, then

$$\frac{A_2}{A_1} = \frac{r_1}{r_2}.$$

When the wave is twice as far from the source, the amplitude is half as large, and so on (ignoring damping due to friction).

EXAMPLE 11–13 **Earthquake intensity.** The intensity of an earthquake P wave traveling through the Earth and detected 100 km from the source is $1.0 \times 10^6\ \mathrm{W/m^2}$. What is the intensity of that wave if detected 400 km from the source?

APPROACH We assume the wave is spherical, so the intensity decreases as the square of the distance from the source.

SOLUTION At 400 km the distance is 4 times greater than at 100 km, so the intensity will be $\left(\frac{1}{4}\right)^2 = \frac{1}{16}$ of its value at 100 km, or $(1.0 \times 10^6\ \mathrm{W/m^2})/16 = 6.3 \times 10^4\ \mathrm{W/m^2}$.

NOTE Using Eq. 11–16c directly gives:

$$I_2 = I_1 r_1^2/r_2^2 = (1.0 \times 10^6\ \mathrm{W/m^2})(100\ \mathrm{km})^2/(400\ \mathrm{km})^2 = 6.3 \times 10^4\ \mathrm{W/m^2}.$$

The situation is different for a one-dimensional wave, such as a transverse wave on a string or a longitudinal wave pulse traveling down a thin uniform metal rod. The area remains constant, so the amplitude A also remains constant (ignoring friction). Thus the amplitude and the intensity do not decrease with distance.

In practice, frictional damping is generally present, and some of the energy is transformed into thermal energy. Thus the amplitude and intensity of a one-dimensional wave will decrease with distance from the source. For a three-dimensional wave, the decrease will be greater than that discussed above, although the effect may often be small.

* 11–10 Intensity Related to Amplitude and Frequency

We can obtain an explicit relation between the energy carried by a wave, or the wave's intensity I, and the amplitude and frequency of the wave. For a sinusoidal wave of frequency f, the particles move in SHM as a wave passes, so each particle has an energy $E = \frac{1}{2}kA^2$, where A is the amplitude of its motion, either transversely or longitudinally. Using Eq. 11–7b, we can write k in terms of the frequency: $k = 4\pi^2 m f^2$, where m is the mass of a particle (or small volume) of the medium. Then

$$E = \tfrac{1}{2}kA^2 = 2\pi^2 m f^2 A^2.$$

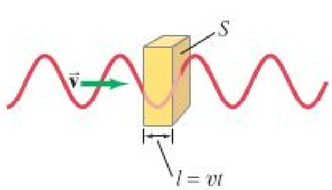


FIGURE 11–31 Calculating the energy carried by a wave moving with velocity v.

The mass $m = \rho V$, where ρ is the density of the medium and V the volume of a small slice of the medium as shown in Fig. 11–31. The volume $V = Sl$, where S is the cross-sectional surface area through which the wave travels. (We use S instead of A for area because we are using A for amplitude.) We can write l as the distance the wave travels in a time t as $l = vt$, where v is the speed of the wave. Thus $m = \rho V = \rho S l = \rho S v t$, and

$$E = 2\pi^2 \rho S v t f^2 A^2. \qquad \textbf{(11–17a)}$$

From this equation, we see again the important result that the energy transported by a wave is proportional to the square of the amplitude. The power transported, $P = E/t$, is

$$P = \frac{E}{t} = 2\pi^2 \rho S v f^2 A^2. \quad \textbf{(11–17b)}$$

Finally, the **intensity** I of a wave is the power transported across unit area perpendicular to the direction of energy flow:

$$I = \frac{P}{S} = 2\pi^2 v \rho f^2 A^2. \quad \textbf{(11–18)}$$

This relation shows explicitly that the intensity of a wave is proportional both to the square of the wave amplitude A at any point and to the square of the frequency f.

11–11 Reflection and Transmission of Waves

When a wave strikes an obstacle, or comes to the end of the medium it is traveling in, at least a part of the wave is reflected. You have probably seen water waves reflect off a rock or the side of a swimming pool. And you may have heard a shout reflected from a distant cliff—which we call an "echo."

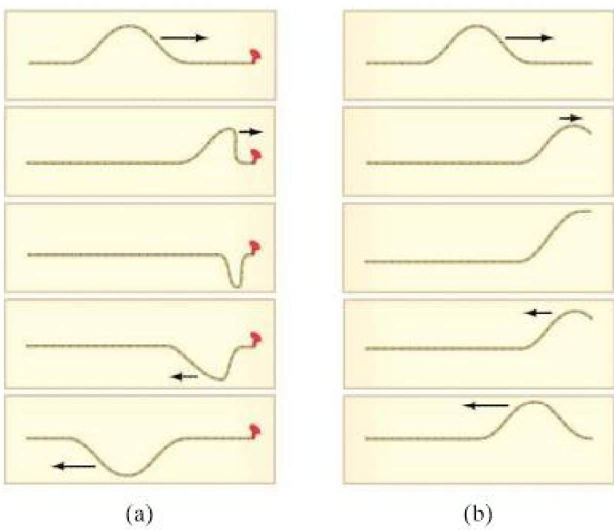
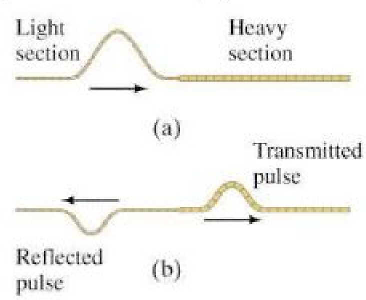


FIGURE 11–32 Reflection of a wave pulse on a rope lying on a table top. (a) The end of the rope is fixed to a peg. (b) The end of the rope is free to move.

A wave pulse traveling down a rope is reflected as shown in Fig. 11–32. The reflected pulse returns inverted as in Fig. 11–32a if the end of the rope is fixed; it returns right side up if the end is free as in Fig. 11–32b. When the end is fixed to a support, as in Fig. 11–32a, the pulse reaching that fixed end exerts a force (upward) on the support. The support exerts an equal but opposite force downward on the rope (Newton's third law). This downward force on the rope is what "generates" the inverted reflected pulse.

Consider next a pulse that travels down a rope which consists of a light section and a heavy section, as shown in Fig. 11–33. When the wave pulse reaches the boundary between the two sections, part of the pulse is reflected and part is transmitted, as shown. The heavier the second section of rope, the less the energy that is transmitted. (When the second section is a wall or rigid support, very little is transmitted and most is reflected, as in Fig. 11–32a.) For a periodic wave, the frequency of the transmitted wave does not change across the boundary because the boundary point oscillates at that frequency. Thus if the transmitted wave has a lower speed, its wavelength is also shorter ($\lambda = v/f$).

FIGURE 11–33 When a wave pulse traveling to the right along a thin cord (a) reaches a discontinuity where the rope becomes thicker and heavier, then part is reflected and part is transmitted (b).



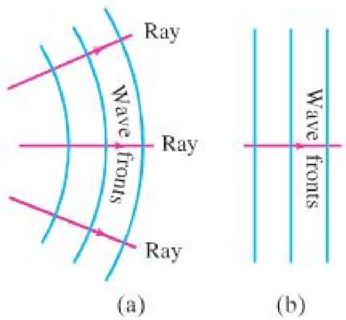
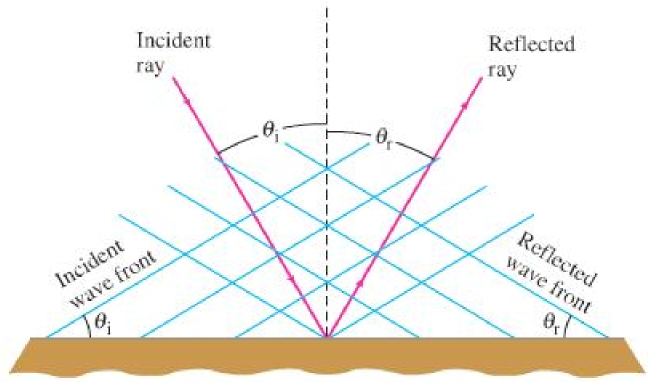


FIGURE 11–34 Rays, signifying the direction of wave motion, are always perpendicular to the wave fronts (wave crests). (a) Circular or spherical waves near the source. (b) Far from the source, the wave fronts are nearly straight or flat, and are called plane waves.

For a two- or three-dimensional wave, such as a water wave, we are concerned with **wave fronts**, by which we mean all the points along the wave forming the wave crest (what we usually refer to simply as a "wave" at the seashore). A line drawn in the direction of wave motion, perpendicular to the wave front, is called a **ray**, as shown in Fig. 11–34. Wave fronts far from the source have lost almost all their curvature (Fig. 11–34b) and are nearly straight, as ocean waves often are; they are then called **plane waves**.

For reflection of a two- or three-dimensional plane wave, as shown in Fig. 11–35, the angle that the incoming or *incident wave* makes with the reflecting surface is equal to the angle made by the reflected wave. This is the **law of reflection: the angle of reflection equals the angle of incidence**. The "angle of incidence" is defined as the angle the incident ray makes with the perpendicular to the reflecting surface (or the wave front makes with a tangent to the surface). The "angle of reflection" is the corresponding angle for the reflected wave.

FIGURE 11–35 Law of reflection.



11–12 Interference; Principle of Superposition

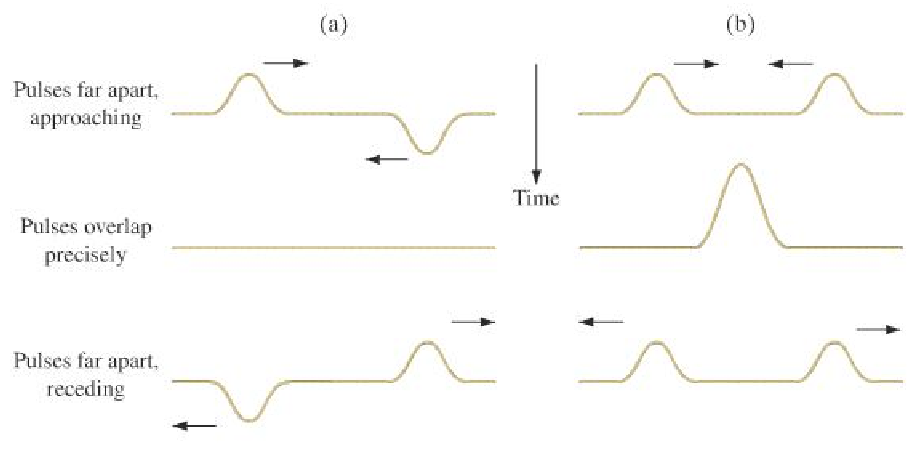
Interference refers to what happens when two waves pass through the same region of space at the same time. Consider, for example, the two wave pulses on a string traveling toward each other as shown in Fig. 11–36. In Fig. 11–36a the two pulses have the same amplitude, but one is a crest and the other a trough; in Fig. 11–36b they are both crests. In both cases, the waves meet and pass right by each other. However, in the region where they overlap, the resultant displacement is the *algebraic sum of their separate displacements* (a crest is considered positive and a trough negative). This is called the **principle of superposition**. In Fig. 11–36a, the two waves have opposite displacements at the instant they pass one another, and they add to zero. The result is called **destructive interference**. In Fig. 11–36b, at the instant the two pulses overlap, they produce a resultant displacement that is greater than the displacement of either separate pulse, and the result is **constructive interference**.

Superposition principle

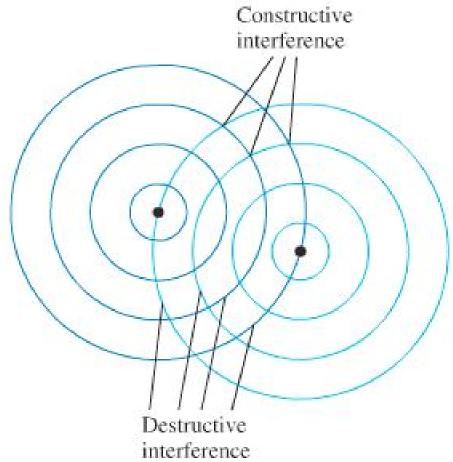
Destructive interference

Constructive interference

FIGURE 11–36 Two wave pulses pass each other. Where they overlap, interference occurs: (a) destructive, and (b) constructive.



(a)



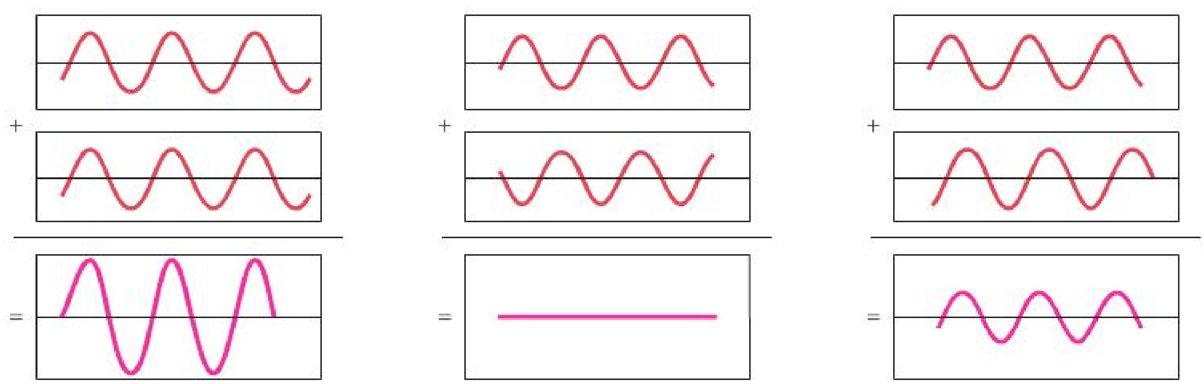
(b)

FIGURE 11–37 Interference of water waves.

When two rocks are thrown into a pond simultaneously, the two sets of circular waves interfere with one another as shown in Fig. 11–37a. In some areas of overlap, crests of one wave repeatedly meet crests of the other (and troughs meet troughs); see Fig. 11–37b. Constructive interference is occurring at these points, and the water continuously oscillates up and down with greater amplitude than either wave separately. In other areas, destructive interference occurs where the water does not move up and down at all over time. This is where crests of one wave meet troughs of the other, and vice versa. Figure 11–38a shows the displacement of two waves graphically as a function of time, as well as their sum, for the case of constructive interference. For any two such waves, we use the term **phase** to describe the relative positions of their crests. When the crests and troughs are aligned as in Fig. 11–38a, for constructive interference, the two waves are **in phase**. At points where destructive interference occurs—see Fig. 11–38b—crests of one wave repeatedly meet troughs of the other wave and the two waves are said to be completely **out of phase** or, more precisely, out of phase by one-half wavelength. That is, the crests of one wave occur a half wavelength behind the crests of the other wave. The relative phase of the two water waves in Fig. 11–37 in most areas is intermediate between these two extremes, resulting in *partially* destructive interference, as illustrated in Fig. 11–38c. If the amplitudes of two interfering waves are not equal, fully destructive interference (as in Fig. 11–38b) does not occur.

Phase

FIGURE 11–38 Graphs showing two waves, and their sum, as a function of time at three locations. In (a) the two waves interfere constructively, in (b) destructively, and in (c) partially destructively.



(a) (b) (c)

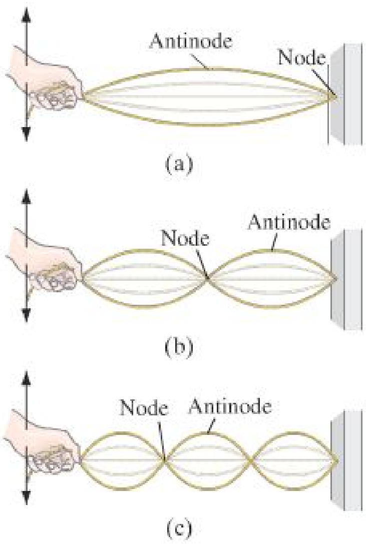


FIGURE 11–39 Standing waves corresponding to three resonant frequencies.

11–13 Standing Waves; Resonance

If you shake one end of a cord and the other end is kept fixed, a continuous wave will travel down to the fixed end and be reflected back, inverted, as we saw in Fig. 11–32a. As you continue to vibrate the cord, waves will travel in both directions, and the wave traveling along the cord, away from your hand, will interfere with the reflected wave coming back. Usually there will be quite a jumble. But if you vibrate the cord at just the right frequency, the two traveling waves will interfere in such a way that a large-amplitude **standing wave** will be produced, Fig. 11–39. It is called a "standing wave" because it doesn't appear to be traveling. The cord simply appears to have segments that oscillate up and down in a fixed pattern. The points of destructive interference, where the cord remains still at all times, are called **nodes**. Points of constructive interference, where the cord oscillates with maximum amplitude, are called **antinodes**. The nodes and antinodes remain in fixed positions for a particular frequency.

Standing waves can occur at more than one frequency. The lowest frequency of vibration that produces a standing wave gives rise to the pattern shown in Fig. 11–39a. The standing waves shown in Figs. 11–39b and 11–39c are produced at precisely twice and three times the lowest frequency, respectively, assuming the tension in the cord is the same. The cord can also vibrate with four loops (four antinodes) at four times the lowest frequency, and so on.

Resonant frequencies

The frequencies at which standing waves are produced are the **natural frequencies** or **resonant frequencies** of the cord, and the different standing wave patterns shown in Fig. 11–39 are different "resonant modes of vibration." A standing wave on a cord is the result of the interference of two waves traveling in opposite directions. A standing wave is also a vibrating object at resonance. Standing waves represent the same phenomenon as the resonance of a vibrating spring or pendulum, which we discussed in Section 11–6. The only difference is that a spring or pendulum has only one resonant frequency, whereas the cord has an infinite number of resonant frequencies, each of which is a whole-number multiple of the lowest resonant frequency.

Consider a string stretched between two supports that is plucked like a guitar or violin string, Fig. 11–40a. Waves of a great variety of frequencies will

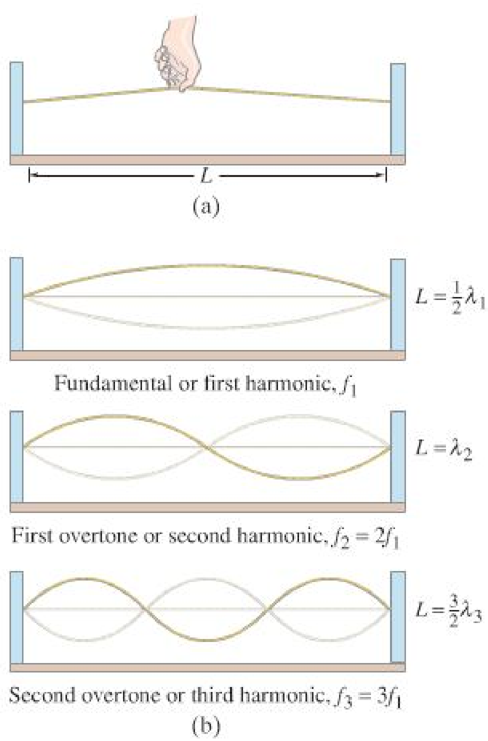


FIGURE 11–40 (a) A string is plucked. (b) Only standing waves corresponding to resonant frequencies persist for long.

travel in both directions along the string, will be reflected at the ends, and will travel back in the opposite direction. Most of these waves interfere with each other and quickly die out. However, those waves that correspond to the resonant frequencies of the string will persist. The ends of the string, since they are fixed, will be nodes. There may be other nodes as well. Some of the possible resonant modes of vibration (standing waves) are shown in Fig. 11–40b. Generally, the motion will be a combination of these different resonant modes, but only those frequencies that correspond to a resonant frequency will be present.

To determine the resonant frequencies, we first note that the wavelengths of the standing waves bear a simple relationship to the length L of the string. The lowest frequency, called the **fundamental frequency**, corresponds to one antinode (or loop). And as can be seen in Fig. 11–40b, the whole length corresponds to one-half wavelength. Thus $L = \frac{1}{2}\lambda_1$, where λ_1 stands for the wavelength of the fundamental frequency. The other natural frequencies are called **overtones**; for a vibrating string they are whole-number (integral) multiples of the fundamental, and then are also called **harmonics**, with the fundamental being referred to as the **first harmonic**.† The next mode of vibration after the fundamental has two loops and is called the **second harmonic** (or first overtone), Fig. 11–40b. The length of the string L at the second harmonic corresponds to one complete wavelength: $L = \lambda_2$. For the third and fourth harmonics, $L = \frac{3}{2}\lambda_3$, and $L = 2\lambda_4$, respectively, and so on. In general, we can write

Fundamental frequency

Overtones and harmonics

$$L = \frac{n\lambda_n}{2}, \qquad \text{where } n = 1, 2, 3, \cdots.$$

The integer n labels the number of the harmonic: $n = 1$ for the fundamental, $n = 2$ for the second harmonic, and so on. We solve for λ_n and find

$$\lambda_n = \frac{2L}{n}, \qquad n = 1, 2, 3, \cdots. \qquad \textbf{(11–19a)}$$

To find the frequency f of each vibration we use Eq. 11–12, $f = v/\lambda$, and we see that

$$f_n = \frac{v}{\lambda_n} = n\frac{v}{2L} = nf_1, \qquad n = 1, 2, 3, \cdots, \qquad \textbf{(11–19b)}$$

where $f_1 = v/\lambda_1 = v/2L$ is the fundamental frequency. We see that each resonant frequency is an integer multiple of the fundamental frequency.

Because a standing wave is equivalent to two traveling waves moving in opposite directions, the concept of wave velocity still makes sense and is given by Eq. 11–13 in terms of the tension F_T in the string and its mass per unit length (m/L). That is, $v = \sqrt{F_T/(m/L)}$ for waves traveling in both directions.

EXAMPLE 11–14 **Piano string.** A piano string is 1.10 m long and has a mass of 9.00 g. (*a*) How much tension must the string be under if it is to vibrate at a fundamental frequency of 131 Hz? (*b*) What are the frequencies of the first four harmonics?

APPROACH To determine the tension, we need to find the wave speed using Eq. 11–12 $(v = \lambda f)$, and then use Eq. 11–13, solving it for F_T.

SOLUTION (*a*) The wavelength of the fundamental is $\lambda = 2L = 2.20\text{ m}$ (Eq. 11–19a with $n = 1$). The speed of the wave on the string is $v = \lambda f = (2.20\text{ m})(131\text{ s}^{-1}) = 288\text{ m/s}$. Then we have (Eq. 11–13)

$$F_T = \frac{m}{L}v^2 = \left(\frac{9.00 \times 10^{-3}\text{ kg}}{1.10\text{ m}}\right)(288\text{ m/s})^2 = 679\text{ N}.$$

(*b*) The frequencies of the second, third, and fourth harmonics are two, three, and four times the fundamental frequency: 262, 393, and 524 Hz.

NOTE The speed of the wave on the string is *not* the same as the speed of the sound that is produced in the air (as we shall see in Chapter 12).

† The term "harmonic" comes from music, because such integral multiples of frequencies "harmonize."

Standing waves are produced not only on strings, but on any object that is struck, such as a drum membrane or an object made of metal or wood. The resonant frequencies depend on the dimensions of the object, just as for a string they depend on its length. Large objects have lower resonant frequencies than small objects. All musical instruments, from stringed instruments to wind instruments (in which a column of air vibrates as a standing wave) to drums and other percussion instruments, depend on standing waves to produce their musical sounds, as we shall see in Chapter 12.

* 11–14 Refraction[†]

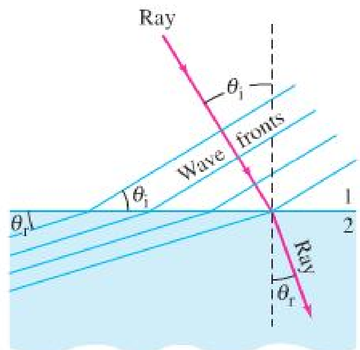


FIGURE 11–41 Refraction of waves passing a boundary.

When any wave strikes a boundary, some of the energy is reflected and some is transmitted or absorbed. When a two- or three-dimensional wave traveling in one medium crosses a boundary into a medium where its speed is different, the transmitted wave may move in a different direction than the incident wave, as shown in Fig. 11–41. This phenomenon is known as **refraction**. One example is a water wave; the velocity decreases in shallow water and the waves refract, as shown in Fig. 11–42 below. [When the wave velocity changes gradually, as in Fig. 11–42, without a sharp boundary, the waves change direction (refract) gradually.]

In Fig. 11–41, the velocity of the wave in medium 2 is less than in medium 1. In this case, the wave front bends so it travels more nearly parallel to the boundary. That is, the *angle of refraction*, θ_r, is less than the *angle of incidence*, θ_i. To see why this is so, and to help us get a quantitative relation between θ_r and θ_i, let us think of each wave front as a row of soldiers. The soldiers are marching from firm ground (medium 1) into mud (medium 2) and hence are slowed down after the boundary. The soldiers that reach the mud first are slowed down first, and the row bends as shown in Fig. 11–43a. Let us consider the wave front (or row of soldiers) labeled A in Fig. 11–43b. In the same time t that A_1 moves a distance $l_1 = v_1 t$, we see that A_2 moves a distance $l_2 = v_2 t$. The two right triangles in Fig. 11–43b, shaded yellow and green, have the side labeled a in common. Thus

$$\sin\theta_1 = \frac{l_1}{a} = \frac{v_1 t}{a}$$

since a is the hypotenuse, and

$$\sin\theta_2 = \frac{l_2}{a} = \frac{v_2 t}{a}.$$

Dividing these two equations, we obtain the *law of refraction*:

Law of refraction

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1}. \qquad \textbf{(11–20)}$$

Since θ_1 is the angle of incidence (θ_i), and θ_2 is the angle of refraction (θ_r), Eq. 11–20 gives the quantitative relation between the two. If the wave were

[†]This Section and the next are covered in more detail in Chapters 23 to 25, on optics.

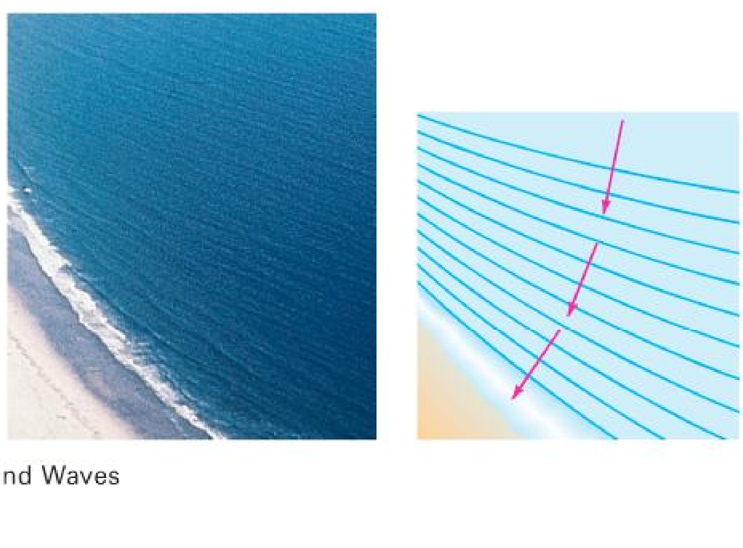


FIGURE 11–42 Water waves refract gradually as they approach the shore, as their velocity decreases. There is no distinct boundary, as in Fig. 11–41, because the wave velocity changes gradually.

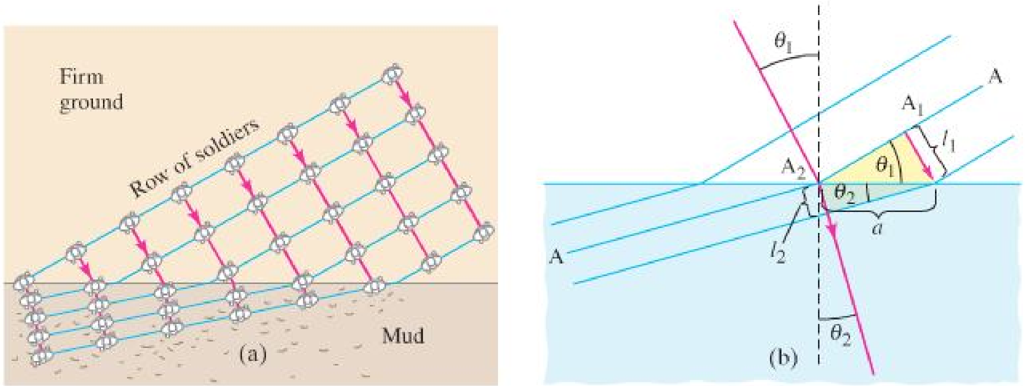


FIGURE 11–43 (a) Soldier analogy to derive (b) law of refraction for waves.

going in the opposite direction, the geometry would not change; only θ_1 and θ_2 would change roles: θ_1 would be the angle of refraction and θ_2 the angle of incidence. Clearly then, if the wave travels into a medium where it can move faster, it will bend the opposite way, $\theta_r > \theta_i$. We see from Eq. 11–20 that if the velocity increases, the angle increases, and vice versa.

Earthquake waves refract within the Earth as they travel through rock layers of different densities (and therefore the velocity is different) just as water waves do. Light waves refract as well, and when we discuss light, we shall find Eq. 11–20 very useful.

PHYSICS APPLIED
Earthquake wave refraction

EXAMPLE 11–15 **Refraction of an earthquake wave.** An earthquake P wave passes across a boundary in rock where its velocity increases from 6.5 km/s to 8.0 km/s. If it strikes this boundary at 30°, what is the angle of refraction?

APPROACH We apply the law of refraction, Eq. 11–20.

SOLUTION Since $\sin 30° = 0.50$, Eq. 11–20 yields

$$\sin\theta_2 = \frac{(8.0\ \text{m/s})}{(6.5\ \text{m/s})}(0.50) = 0.62.$$

So $\theta_2 = \sin^{-1}(0.62) = 38°$.

NOTE Be careful with angles of incidence and refraction. As we discussed in Section 11–11 (Fig. 11–35), these angles are between the wave front and the boundary line, or—equivalently—between the ray (direction of wave motion) and the line perpendicular to the boundary. Inspect Fig. 11–43b carefully.

* 11–15 Diffraction

Waves spread as they travel. When they encounter an obstacle, they bend around it somewhat and pass into the region behind as shown in Fig. 11–44 for water waves. This phenomenon is called **diffraction**.

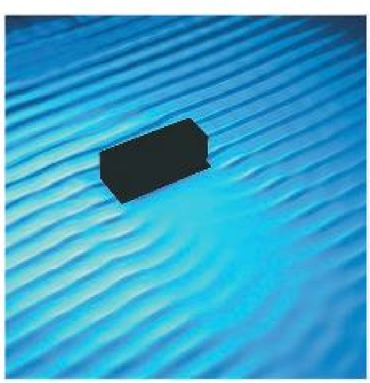
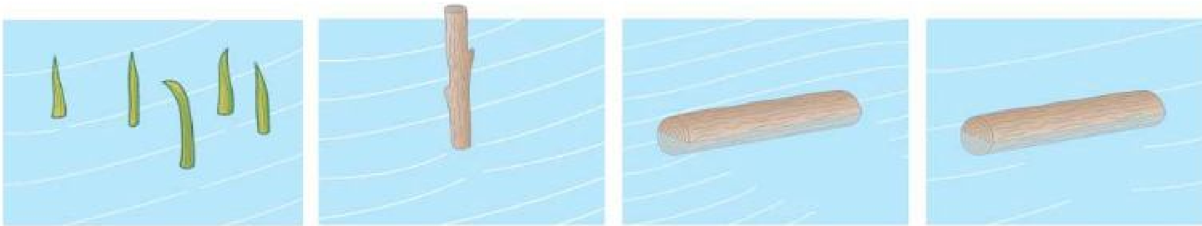


FIGURE 11–44 Wave diffraction. The waves are coming from the upper left. Note how the waves, as they pass the obstacle, bend around it, into the "shadow region" behind it.



(a) Water waves passing blades of grass | (b) Stick in water | (c) Short-wavelength waves passing log | (d) Long-wavelength waves passing log

FIGURE 11–45 Water waves passing objects of various sizes. Note that the longer the wavelength compared to the size of the object, the more diffraction there is into the "shadow region."

The amount of diffraction depends on the wavelength of the wave and on the size of the obstacle, as shown in Fig. 11–45. If the wavelength is much larger than the object, as with the grass blades of Fig. 11–45a, the wave bends around them almost as if they are not there. For larger objects, parts (b) and (c), there is more of a "shadow" region behind the obstacle where we might not expect the waves to penetrate—but they do, at least a little. Then notice in part (d), where the obstacle is the same as in part (c) but the wavelength is longer, that there is more diffraction into the shadow region. As a rule of thumb, *only if the wavelength is smaller than the size of the object will there be a significant shadow region.* This rule applies to *reflection* from an obstacle as well. Very little of a wave is reflected unless the wavelength is smaller than the size of the obstacle.

A rough guide to the amount of diffraction is

$$\theta(\text{radians}) \approx \frac{\lambda}{L},$$

where θ is roughly the angular spread of waves after they have passed through an opening of width L or around an obstacle of width L.

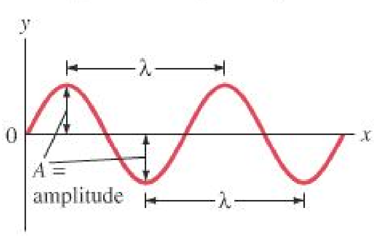
That waves can bend around obstacles, and thus can carry energy to areas behind obstacles, is very different from energy carried by material particles. A clear example is the following: if you are standing around a corner on one side of a building, you can't be hit by a baseball thrown from the other side, but you can hear a shout or other sound because the sound waves diffract around the edges.

CONCEPTUAL EXAMPLE 11–16 **Cell phones.** Cellular phones operate by radio waves with frequencies of about 1 or 2 GHz ($1 \text{ gigahertz} = 10^9 \text{ Hz}$). These waves cannot penetrate objects that conduct electricity, such as a tree trunk or a sheet of metal. The connection is best if the transmitting antenna is within clear view of the handset. Yet it is possible to carry on a phone conversation even if the tower is blocked by trees, or if the handset is inside a car. Why?

RESPONSE If the radio waves have a frequency of about 2 GHz, and the speed of propagation is equal to the speed of light, 3×10^8 m/s (Section 1–5), then the wavelength is $\lambda = v/f = (3 \times 10^8 \text{ m/s})/(2 \times 10^9 \text{ Hz}) = 0.15 \text{ m}$. The waves can diffract readily around objects 15 cm in diameter or smaller.

* 11–16 Mathematical Representation of a Traveling Wave

FIGURE 11–46 The characteristics of a single-frequency wave at $t = 0$ (just as in Fig. 11–23).



A simple wave with a single frequency, as in Fig. 11–46, is sinusoidal. To express such a wave mathematically, we assume it has a particular wavelength λ and frequency f. At $t = 0$, the wave shape shown is

$$y = A \sin \frac{2\pi}{\lambda} x, \qquad \textbf{(11–21)}$$

where y is the **displacement** of the wave (be it a longitudinal or transverse wave) at position x; A is the **amplitude** of the wave, and λ is the wavelength. [Equation 11–21 works because it repeats itself every wavelength: when $x = \lambda$, $y = \sin 2\pi = \sin 0$.]

Suppose the wave is moving to the right with velocity v. After a time t, each part of the wave (indeed, the whole wave "shape") has moved to the right a distance vt. Figure 11–47 shows the wave at $t = 0$ as a solid curve, and at a later time t as a dashed curve. Consider any point on the wave at $t = 0$: say, a crest at some position x. After a time t, that crest will have traveled a distance vt, so its new position is a distance vt greater than its old position. To describe this same point on the wave shape, the argument of the sine function must have the same numerical value, so we replace x in Eq. 11–21 by $(x - vt)$:

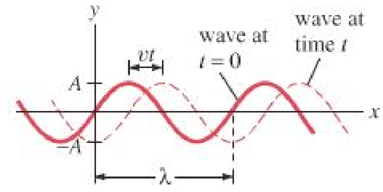


FIGURE 11–47 A traveling wave. In time t, the wave moves a distance vt.

$$y = A\sin\left[\frac{2\pi}{\lambda}(x - vt)\right]. \qquad \textbf{(11–22)}$$

1-D wave, moving in positive x direction

Said another way, if you are on a crest, as t increases, x must increase at the same rate so that $(x - vt)$ remains constant.

For a wave traveling along the x axis to the left, toward decreasing values of x, v becomes $-v$, so

$$y = A\sin\left[\frac{2\pi}{\lambda}(x + vt)\right].$$

1-D wave, traveling in negative x direction (to the left)

Summary

A vibrating object undergoes **simple harmonic motion** (SHM) if the restoring force is proportional to the displacement,

$$F = -kx. \qquad \textbf{(11–1)}$$

The maximum displacement is called the **amplitude**.

The **period**, T, is the time required for one complete cycle (back and forth), and the **frequency**, f, is the number of cycles per second; they are related by

$$f = \frac{1}{T}. \qquad \textbf{(11–2)}$$

The period of vibration for a mass m on the end of a spring is given by

$$T = 2\pi\sqrt{\frac{m}{k}}. \qquad \textbf{(11–7a)}$$

SHM is **sinusoidal**, which means that the displacement as a function of time follows a sine or cosine curve.

During SHM, the total energy

$$E = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 \qquad \textbf{(11–3)}$$

is continually changing from potential to kinetic and back again.

A **simple pendulum** of length L approximates SHM if its amplitude is small and friction can be ignored. For small amplitudes, its period is then given by

$$T = 2\pi\sqrt{\frac{L}{g}}, \qquad \textbf{(11–11a)}$$

where g is the acceleration of gravity.

When friction is present (for all real springs and pendulums), the motion is said to be **damped**. The maximum displacement decreases in time, and the energy is eventually all transformed to thermal energy.

If an oscillating force is applied to a system capable of vibrating, the system's amplitude of vibration can be very large if the frequency of the applied force matches the **natural** (or **resonant**) **frequency** of the oscillator. This effect is called **resonance**.

Vibrating objects act as sources of **waves** that travel outward from the source. Waves on water and on a string are examples. The wave may be a **pulse** (a single crest), or it may be continuous (many crests and troughs).

The **wavelength** of a continuous sinusoidal wave is the distance between two successive crests.

The **frequency** is the number of wavelengths (or crests) that pass a given point per unit time.

The **amplitude** of a wave is the maximum height of a crest, or depth of a trough, relative to the normal (or equilibrium) level.

The **wave velocity** (how fast a crest moves) is equal to the product of wavelength and frequency,

$$v = \lambda f. \qquad \textbf{(11–12)}$$

In a **transverse wave**, the oscillations are perpendicular to the direction in which the wave travels. An example is a wave on a string.

In a **longitudinal wave**, the oscillations are along (parallel to) the line of travel; sound is an example.

The **intensity** of a wave is the energy per unit time carried across unit area (in watts/m^2). For three-dimensional waves traveling in open space, the intensity decreases inversely as the distance from the source squared:

$$I \propto \frac{1}{r^2}. \qquad \textbf{(11–16b)}$$

[*Wave intensity is proportional to the amplitude squared and to the frequency squared.]

Waves reflect off objects in their path. When the **wave front** (of a two- or three-dimensional wave) strikes an object, the *angle of reflection* is equal to the *angle of incidence*. When a wave strikes a boundary between two materials in which it can travel, part of the wave is reflected and part is transmitted.

When two waves pass through the same region of space at the same time, they **interfere**. The resultant displacement at any point and time is the sum of their separate displacements; this can result in **constructive interference, destructive interference**, or something in between, depending on the amplitudes and relative phases of the waves.

Waves traveling on a string of fixed length interfere with waves that have reflected off the end and are traveling back in the opposite direction. At certain frequencies, **standing waves** can be produced in which the waves seem to be standing still rather than traveling. The string (or other medium) is vibrating as a whole. This is a resonance phenomenon, and the frequencies at which standing waves occur are called **resonant frequencies**. Points of destructive interference (no vibration) are called **nodes**. Points of constructive interference (maximum amplitude of vibration) are called **antinodes**.

[*Waves change direction, or **refract**, when traveling from one medium into a second medium where their speed is different. Waves spread, or **diffract**, as they travel and encounter obstacles. A rough guide to the amount of diffraction is $\theta \approx \lambda/L$, where λ is the wavelength and L the width of an opening or obstacle. There is a significant "shadow region" only if the wavelength λ is smaller than the size of the obstacle.]

[*A traveling wave can be represented mathematically as $y = A\sin\{(2\pi/\lambda)(x - vt)\}$.]

Questions

1. Give some examples of everyday vibrating objects. Which exhibit SHM, at least approximately?
2. Is the acceleration of a simple harmonic oscillator ever zero? If so, where?
3. Explain why the motion of a piston in an automobile engine is approximately simple harmonic.
4. Real springs have mass. Will the true period and frequency be larger or smaller than given by the equations for a mass oscillating on the end of an idealized massless spring? Explain.
5. How could you double the maximum speed of a simple harmonic oscillator (SHO)?
6. A 5.0-kg trout is attached to the hook of a vertical spring scale, and then is released. Describe the scale reading as a function of time.
7. If a pendulum clock is accurate at sea level, will it gain or lose time when taken to high altitude? Why?
8. A tire swing hanging from a branch reaches nearly to the ground (Fig. 11–48). How could you estimate the height of the branch using only a stopwatch?

FIGURE 11–48 Question 8.

9. Why can you make water slosh back and forth in a pan only if you shake the pan at a certain frequency?
10. Give several everyday examples of resonance.
11. Is a rattle in a car ever a resonance phenomenon? Explain.
12. Is the frequency of a simple periodic wave equal to the frequency of its source? Why or why not?
13. Explain the difference between the speed of a transverse wave traveling down a cord and the speed of a tiny piece of the cord.
14. Why do the strings used for the lowest-frequency notes on a piano normally have wire wrapped around them?
15. What kind of waves do you think will travel down a horizontal metal rod if you strike its end (*a*) vertically from above and (*b*) horizontally parallel to its length?

* 16. Since the density of air decreases with an increase in temperature, but the bulk modulus B is nearly independent of temperature, how would you expect the speed of sound waves in air to vary with temperature?

17. Give two reasons why circular water waves decrease in amplitude as they travel away from the source.

* 18. Two linear waves have the same amplitude and speed, and otherwise are identical, except one has half the wavelength of the other. Which transmits more energy? By what factor?

19. When a sinusoidal wave crosses the boundary between two sections of cord as in Fig. 11–33, the frequency does not change (although the wavelength and velocity do change). Explain why.
20. If a string is vibrating in three segments, are there any places you could touch it with a knife blade without disturbing the motion?
21. When a standing wave exists on a string, the vibrations of incident and reflected waves cancel at the nodes. Does this mean that energy was destroyed? Explain.

* 22. If we knew that energy was being transmitted from one place to another, how might we determine whether the energy was being carried by particles (material bodies) or by waves?

Problems

11–1 to 11–3 Simple Harmonic Motion

1. (I) If a particle undergoes SHM with amplitude 0.18 m, what is the total distance it travels in one period?

2. (I) An elastic cord is 65 cm long when a weight of 75 N hangs from it but is 85 cm long when a weight of 180 N hangs from it. What is the "spring" constant k of this elastic cord?

3. (I) The springs of a 1500-kg car compress 5.0 mm when its 68-kg driver gets into the driver's seat. If the car goes over a bump, what will be the frequency of vibrations?

4. (II) A fisherman's scale stretches 3.6 cm when a 2.7-kg fish hangs from it. (*a*) What is the spring stiffness constant and (*b*) what will be the amplitude and frequency of vibration if the fish is pulled down 2.5 cm more and released so that it vibrates up and down?

5. (II) An elastic cord vibrates with a frequency of 3.0 Hz when a mass of 0.60 kg is hung from it. What is its frequency if only 0.38 kg hangs from it?

6. (II) Construct a Table indicating the position x of the mass in Fig. 11–2 at times $t = 0, \frac{1}{4}T, \frac{1}{2}T, \frac{3}{4}T, T,$ and $\frac{5}{4}T$, where T is the period of oscillation. On a graph of x vs. t, plot these six points. Now connect these points with a smooth curve. Based on these simple considerations, does your curve resemble that of a cosine or sine wave (Fig. 11–8a or 11–9)?

7. (II) A small fly of mass 0.25 g is caught in a spider's web. The web vibrates predominately with a frequency of 4.0 Hz. (*a*) What is the value of the effective spring stiffness constant k for the web? (*b*) At what frequency would you expect the web to vibrate if an insect of mass 0.50 g were trapped?

8. (II) A mass m at the end of a spring vibrates with a frequency of 0.88 Hz. When an additional 680-g mass is added to m, the frequency is 0.60 Hz. What is the value of m?

9. (II) A 0.60-kg mass at the end of a spring vibrates 3.0 times per second with an amplitude of 0.13 m. Determine (*a*) the velocity when it passes the equilibrium point, (*b*) the velocity when it is 0.10 m from equilibrium, (*c*) the total energy of the system, and (*d*) the equation describing the motion of the mass, assuming that x was a maximum at $t = 0$.

10. (II) At what displacement from equilibrium is the speed of a SHO half the maximum value?

11. (II) A mass attached to the end of a spring is stretched a distance x_0 from equilibrium and released. At what distance from equilibrium will it have acceleration equal to half its maximum acceleration?

12. (II) A mass of 2.62 kg stretches a vertical spring 0.315 m. If the spring is stretched an additional 0.130 m and released, how long does it take to reach the (new) equilibrium position again?

13. (II) An object with mass 3.0 kg is attached to a spring with spring stiffness constant $k = 280\,\mathrm{N/m}$ and is executing simple harmonic motion. When the object is 0.020 m from its equilibrium position, it is moving with a speed of 0.55 m/s. (*a*) Calculate the amplitude of the motion. (*b*) Calculate the maximum velocity attained by the object. [*Hint*: Use conservation of energy.]

14. (II) It takes a force of 80.0 N to compress the spring of a toy popgun 0.200 m to "load" a 0.180-kg ball. With what speed will the ball leave the gun?

15. (II) A mass sitting on a horizontal, frictionless surface is attached to one end of a spring; the other end is fixed to a wall. 3.0 J of work is required to compress the spring by 0.12 m. If the mass is released from rest with the spring compressed, the mass experiences a maximum acceleration of $15\,\mathrm{m/s^2}$. Find the value of (*a*) the spring stiffness constant and (*b*) the mass.

16. (II) A 0.60-kg mass vibrates according to the equation $x = 0.45\cos 6.40t$, where x is in meters and t is in seconds. Determine (*a*) the amplitude, (*b*) the frequency, (*c*) the total energy, and (*d*) the kinetic energy and potential energies when $x = 0.30$ m.

17. (II) At what displacement from equilibrium is the energy of a SHO half KE and half PE?

18. (II) If one vibration has 7.0 times the energy of a second, but their frequencies and masses are the same, what is the ratio of their amplitudes?

19. (II) A 2.00-kg pumpkin oscillates from a vertically hanging light spring once every 0.65 s. (*a*) Write down the equation giving the pumpkin's position y (+ upward) as a function of time t, assuming it started by being compressed 18 cm from the equilibrium position (where $y = 0$), and released. (*b*) How long will it take to get to the equilibrium position for the first time? (*c*) What will be the pumpkin's maximum speed? (*d*) What will be its maximum acceleration, and where will that first be attained?

20. (II) A block of mass m is supported by two identical parallel vertical springs, each with spring stiffness constant k (Fig. 11–49). What will be the frequency of vibration?

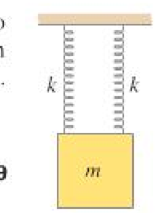


FIGURE 11–49 Problem 20.

21. (II) A 300-g mass vibrates according to the equation $x = 0.38\sin 6.50t$, where x is in meters and t is in seconds. Determine (*a*) the amplitude, (*b*) the frequency, (*c*) the period, (*d*) the total energy, and (*e*) the KE and PE when x is 9.0 cm. (*f*) Draw a careful graph of x vs. t showing the correct amplitude and period.

22. (II) Figure 11–50 shows two examples of SHM, labeled A and B. For each, what is (*a*) the amplitude, (*b*) the frequency, and (*c*) the period? (*d*) Write the equations for both A and B in the form of a sine or cosine.

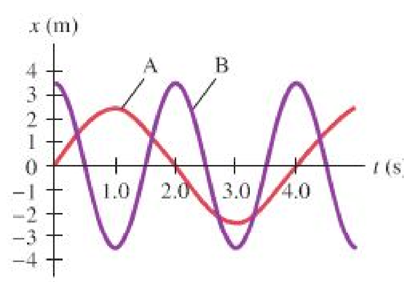


FIGURE 11–50 Problem 22.

23. (II) At $t = 0$, a 755-g mass at rest on the end of a horizontal spring ($k = 124\,\mathrm{N/m}$) is struck by a hammer, which gives the mass an initial speed of 2.96 m/s. Determine (*a*) the period and frequency of the motion, (*b*) the amplitude, (*c*) the maximum acceleration, (*d*) the position as a function of time, and (*e*) the total energy.

24. (II) A vertical spring with spring stiffness constant 305 N/m vibrates with an amplitude of 28.0 cm when 0.260 kg hangs from it. The mass passes through the equilibrium point ($y = 0$) with positive velocity at $t = 0$. (*a*) What equation describes this motion as a function of time? (*b*) At what times will the spring have its maximum and minimum extensions?

25. (II) A mass m is connected to two springs, with spring stiffness constants k_1 and k_2, as shown in Fig. 11–51. Ignore friction. Show that the period is given by

$$T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}.$$

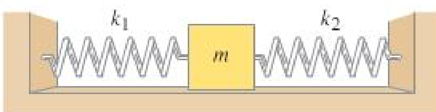


FIGURE 11–51 Problem 25.

26. (III) A 25.0-g bullet strikes a 0.600-kg block attached to a fixed horizontal spring whose spring stiffness constant is $7.70 \times 10^3\,\mathrm{N/m}$. The block is set into vibration with an amplitude of 21.5 cm. What was the speed of the bullet before impact if the bullet and block move together after impact?

27. (III) A bungee jumper with mass 65.0 kg jumps from a high bridge. After reaching his lowest point, he oscillates up and down, hitting a low point eight more times in 38.0 s. He finally comes to rest 25.0 m below the level of the bridge. Calculate the spring stiffness constant and the unstretched length of the bungee cord.

11–4 Simple Pendulum

28. (I) A pendulum makes 36 vibrations in exactly 60 s. What is its (*a*) period, and (*b*) frequency?

29. (I) How long must a simple pendulum be if it is to make exactly one swing per second? (That is, one complete vibration takes exactly 2.0 s.)

30. (I) A pendulum has a period of 0.80 s on Earth. What is its period on Mars, where the acceleration of gravity is about 0.37 that on Earth?

31. (II) What is the period of a simple pendulum 80 cm long (*a*) on the Earth, and (*b*) when it is in a freely falling elevator?

32. (II) The length of a simple pendulum is 0.760 m, the pendulum bob has a mass of 365 grams, and it is released at an angle of 12.0° to the vertical. (*a*) With what frequency does it vibrate? Assume SHM. (*b*) What is the pendulum bob's speed when it passes through the lowest point of the swing? (*c*) What is the total energy stored in this oscillation, assuming no losses?

33. (II) Your grandfather clock's pendulum has a length of 0.9930 m. If the clock loses half a minute per day, how should you adjust the length of the pendulum?

34. (II) Derive a formula for the maximum speed $v_{\max}$ of a simple pendulum bob in terms of g, the length L, and the angle of swing θ_0.

35. (III) A clock pendulum oscillates at a frequency of 2.5 Hz. At $t = 0$, it is released from rest starting at an angle of 15° to the vertical. Ignoring friction, what will be the position (angle) of the pendulum at (*a*) $t = 0.25$ s, (*b*) $t = 1.60$ s, and (*c*) $t = 500$ s? [*Hint*: Do not confuse the angle of swing θ of the pendulum with the angle that appears as the argument of the cosine.]

11–7 and 11–8 Waves

36. (I) A fisherman notices that wave crests pass the bow of his anchored boat every 3.0 s. He measures the distance between two crests to be 6.5 m. How fast are the waves traveling?

37. (I) A sound wave in air has a frequency of 262 Hz and travels with a speed of 343 m/s. How far apart are the wave crests (compressions)?

38. (I) (*a*) AM radio signals have frequencies between 550 kHz and 1600 kHz (kilohertz) and travel with a speed of $3.00 \times 10^8\,\mathrm{m/s}$. What are the wavelengths of these signals? (*b*) On FM, the frequencies range from 88.0 MHz to 108 MHz (megahertz) and travel at the same speed; what are their wavelengths?

* 39. (I) Calculate the speed of longitudinal waves in (*a*) water, (*b*) granite, and (*c*) steel.

* 40. (II) Two solid rods have the same elastic modulus, but one is twice as dense as the other. In which rod will the speed of longitudinal waves be greater, and by what factor?

41. (II) A cord of mass 0.65 kg is stretched between two supports 28 m apart. If the tension in the cord is 150 N, how long will it take a pulse to travel from one support to the other?

42. (II) A ski gondola is connected to the top of a hill by a steel cable of length 620 m and diameter 1.5 cm. As the gondola comes to the end of its run, it bumps into the terminal and sends a wave pulse along the cable. It is observed that it took 16 s for the pulse to return. (*a*) What is the speed of the pulse? (*b*) What is the tension in the cable?

* 43. (II) A sailor strikes the side of his ship just below the surface of the sea. He hears the echo of the wave reflected from the ocean floor directly below 3.0 s later. How deep is the ocean at this point?

44. (II) P and S waves from an earthquake travel at different speeds, and this difference helps in locating the earthquake "epicenter" (where the disturbance took place). (*a*) Assuming typical speeds of 8.5 km/s and 5.5 km/s for P and S waves, respectively, how far away did the earthquake occur if a particular seismic station detects the arrival of these two types of waves 2.0 min apart? (*b*) Is one seismic station sufficient to determine the position of the epicenter? Explain.

45. (III) An earthquake-produced surface wave can be approximated by a sinusoidal transverse wave. Assuming a frequency of 0.50 Hz (typical of earthquakes, which actually include a mixture of frequencies), what amplitude is needed so that objects begin to leave contact with the ground? [*Hint*: Set the acceleration $a > g$.]

11–9 Wave Energy

46. (II) What is the ratio of (*a*) the intensities, and (*b*) the amplitudes, of an earthquake P wave passing through the Earth and detected at two points 10 km and 20 km from the source.

47. (II) The intensity of an earthquake wave passing through the Earth is measured to be $2.0 \times 10^6\,\mathrm{J/m^2 \cdot s}$ at a distance of 48 km from the source. (*a*) What was its intensity when it passed a point only 1.0 km from the source? (*b*) At what rate did energy pass through an area of $5.0\,\mathrm{m^2}$ at 1.0 km?

* 11–10 Intensity Related to A and f

* 48. (I) Two earthquake waves of the same frequency travel through the same portion of the Earth, but one is carrying twice the energy. What is the ratio of the amplitudes of the two waves?

* 49. (I) Two waves traveling along a stretched string have the same frequency, but one transports three times the power of the other. What is the ratio of the amplitudes of the two waves?

* 50. (II) A bug on the surface of a pond is observed to move up and down a total vertical distance of 6.0 cm, from the lowest to the highest point, as a wave passes. If the ripples decrease to 4.5 cm, by what factor does the bug's maximum KE change?

11–12 Interference

51. (I) The two pulses shown in Fig. 11–52 are moving toward each other. (*a*) Sketch the shape of the string at the moment they directly overlap. (*b*) Sketch the shape of the string a few moments later. (*c*) In Fig. 11–36a, at the moment the pulses pass each other, the string is straight. What has happened to the energy at this moment?

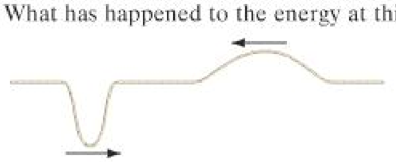


FIGURE 11–52 Problem 51.

11–13 Standing Waves; Resonance

52. (I) If a violin string vibrates at 440 Hz as its fundamental frequency, what are the frequencies of the first four harmonics?

53. (I) A violin string vibrates at 294 Hz when unfingered. At what frequency will it vibrate if it is fingered one-third of the way down from the end? (That is, only two-thirds of the string vibrates as a standing wave.)

54. (I) A particular string resonates in four loops at a frequency of 280 Hz. Name at least three other frequencies at which it will resonate.

55. (II) The velocity of waves on a string is 92 m/s. If the frequency of standing waves is 475 Hz, how far apart are two adjacent nodes?

56. (II) If two successive overtones of a vibrating string are 280 Hz and 350 Hz, what is the frequency of the fundamental?

57. (II) A guitar string is 90 cm long and has a mass of 3.6 g. The distance from the bridge to the support post is $L = 62$ cm, and the string is under a tension of 520 N. What are the frequencies of the fundamental and first two overtones?

58. (II) A particular guitar string is supposed to vibrate at 200 Hz, but it is measured to vibrate at 205 Hz. By what percent should the tension in the string be changed to correct the frequency?

59. (II) One end of a horizontal string is attached to a small-amplitude mechanical 60-Hz vibrator. The string's mass per unit length is $3.9 \times 10^{-4}\,\mathrm{kg/m}$. The string passes over a pulley, a distance $L = 1.50$ m away, and weights are hung from this end, Fig. 11–53. What mass m must be hung from this end of the string to produce (*a*) one loop, (*b*) two loops, and (*c*) five loops of a standing wave? Assume the string at the vibrator is a node, which is nearly true.

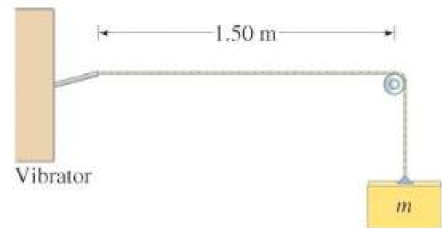


FIGURE 11–53 Problems 59 and 60.

60. (II) In Problem 59, the length of the string may be adjusted by moving the pulley. If the hanging mass m is fixed at 0.080 kg, how many different standing wave patterns may be achieved by varying L between 10 cm and 1.5 m?

61. (II) When you slosh the water back and forth in a tub at just the right frequency, the water alternately rises and falls at each end, remaining relatively calm at the center. Suppose the frequency to produce such a standing wave in a 65-cm-wide tub is 0.85 Hz. What is the speed of the water wave?

* 11–14 Refraction

* 62. (I) An earthquake P wave traveling at 8.0 km/s strikes a boundary within the Earth between two kinds of material. If it approaches the boundary at an incident angle of 47° and the angle of refraction is 35°, what is the speed in the second medium?

* 63. (I) Water waves approach an underwater "shelf" where the velocity changes from 2.8 m/s to 2.1 m/s. If the incident wave crests make a 34° angle with the shelf, what will be the angle of refraction?

* 64. (II) A sound wave is traveling in warm air when it hits a layer of cold, dense air. If the sound wave hits the cold air interface at an angle of 25°, what is the angle of refraction? Assume that the cold air temperature is −10°C and the warm air temperature is +10°C. The speed of sound as a function of temperature can be approximated by $v = (331 + 0.60\,T)$ m/s, where T is in °C.

* 65. (III) A longitudinal earthquake wave strikes a boundary between two types of rock at a 38° angle. As the wave crosses the boundary, the specific gravity of the rock changes from 3.6 to 2.8. Assuming that the elastic modulus is the same for both types of rock, determine the angle of refraction.

* 11–15 Diffraction

* 66. (II) A satellite dish is about 0.5 m in diameter. According to the user's manual, the dish has to be pointed in the direction of the satellite, but an error of about 2° is allowed without loss of reception. Estimate the wavelength of the electromagnetic waves received by the dish.

General Problems

67. A tsunami of wavelength 250 km and velocity 750 km/h travels across the Pacific Ocean. As it approaches Hawaii, people observe an unusual decrease of sea level in the harbors. Approximately how much time do they have to run to safety? (In the absence of knowledge and warning, people have died during tsunamis, some of them attracted to the shore to see stranded fishes and boats.)

68. An energy-absorbing car bumper has a spring stiffness constant of 550 kN/m. Find the maximum compression of the bumper if the car, with mass 1500 kg, collides with a wall at a speed of 2.2 m/s (approximately 5 mi/h). [*Hint*: Use conservation of energy.]

69. A 65-kg person jumps from a window to a fire net 18 m below, which stretches the net 1.1 m. Assume that the net behaves like a simple spring, and (*a*) calculate how much it would stretch if the same person were lying in it. (*b*) How much would it stretch if the person jumped from 35 m?

70. A mass m is gently placed on the end of a freely hanging spring. The mass then falls 33 cm before it stops and begins to rise. What is the frequency of the oscillation?

71. A 950-kg car strikes a huge spring at a speed of 22 m/s (Fig. 11–54), compressing the spring 5.0 m. (*a*) What is the spring stiffness constant of the spring? (*b*) How long is the car in contact with the spring before it bounces off in the opposite direction?

FIGURE 11–54 Problem 71.

72. When you walk with a cup of coffee (diameter 8 cm) at just the right pace of about 1 step per second, the coffee sloshes more and more until eventually it starts to spill over the top (Fig. 11–55). Estimate the speed of waves in the coffee.

FIGURE 11–55 Problem 72.

73. The ripples in a certain groove 10.8 cm from the center of a 33-rpm phonograph record have a wavelength of 1.70 mm. What will be the frequency of the sound emitted?

74. A 2.00-kg mass vibrates according to the equation $x = 0.650 \cos 7.40t$, where x is in meters and t in seconds. Determine (*a*) the amplitude, (*b*) the frequency, (*c*) the total energy, and (*d*) the kinetic energy and potential energy when $x = 0.260$ m.

75. A simple pendulum oscillates with frequency f. What is its frequency if it accelerates at $0.50g$ (*a*) upward, and (*b*) downward?

76. A 220-kg wooden raft floats on a lake. When a 75-kg man stands on the raft, it sinks 4.0 cm deeper into the water. When he steps off, the raft vibrates for a while. (*a*) What is the frequency of vibration? (*b*) What is the total energy of vibration (ignoring damping)?

77. Two strings on a musical instrument are tuned to play at 392 Hz (G) and 440 Hz (A). (*a*) What are the frequencies of the first two overtones for each string? (*b*) If the two strings have the same length and are under the same tension, what is the ratio of their masses (m_G/m_A)? (*c*) If the strings instead have the same mass per unit length and are under the same tension, what is the ratio of their lengths (L_G/L_A)? (*d*) If their masses and lengths are the same, what must be the ratio of the tensions in the two strings?

78. Consider a sine wave traveling down the stretched two-part cord of Fig. 11–33. Determine a formula (*a*) for the ratio of the speeds of the wave in the heavy section versus that in the lighter section, v_H/v_L, and (*b*) for the ratio of the wavelengths in the two sections. (The frequency is the same in both sections. Why?) (*c*) Is the wavelength greater in the heavier section of cord or the lighter?

79. A tuning fork vibrates at a frequency of 264 Hz, and the tip of each prong moves 1.8 mm to either side of center. Calculate (*a*) the maximum speed and (*b*) the maximum acceleration of the tip of a prong.

80. A diving board oscillates with simple harmonic motion of frequency 1.5 cycles per second. What is the maximum amplitude with which the end of the board can vibrate in order that a pebble placed there (Fig. 11–56) will not lose contact with the board during the oscillation?

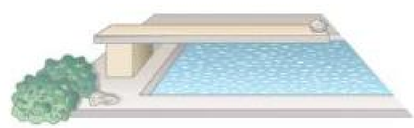


FIGURE 11–56 Problem 80.

81. A string can have a "free" end if that end is attached to a ring that can slide without friction on a vertical pole (Fig. 11–57). Determine the wavelengths of the resonant vibrations of such a string with one end fixed and the other free.

FIGURE 11–57 Problem 81.

82. A "seconds" pendulum has a period of exactly 2.000 s—each one-way swing takes 1.000 s. (*a*) What is the length of a seconds pendulum in Austin, Texas, where $g = 9.793\ \mathrm{m/s^2}$? (*b*) If the pendulum is moved to Paris, where $g = 9.809\ \mathrm{m/s^2}$, by how many millimeters must we lengthen the pendulum? (*c*) What would be the length of a seconds pendulum on the Moon, where $g = 1.62\ \mathrm{m/s^2}$?

83. A mass hanging from a spring can oscillate in the vertical direction or can swing as a pendulum of small amplitude, but not both at the same time. Which one is longer, the period of the vertical oscillations or the period of the horizontal swings, and by what amount? [*Hint*: Let l_0 be the length of the unstretched spring, and L be its length with the mass attached at rest.]

84. A block with mass $M = 5.0\ \mathrm{kg}$ rests on a frictionless table and is attached by a horizontal spring ($k = 130\ \mathrm{N/m}$) to a wall. A second block, of mass $m = 1.25\ \mathrm{kg}$, rests on top of M. The coefficient of static friction between the two blocks is 0.30. What is the maximum possible amplitude of oscillation such that m will not slip off M?

85. A 10.0-m-long wire of mass 123 g is stretched under a tension of 255 N. A pulse is generated at one end, and 20.0 ms later a second pulse is generated at the opposite end. Where will the two pulses first meet?

86. A block of mass M is suspended from a ceiling by a spring with spring stiffness constant k. A penny of mass m is placed on top of the block. What is the maximum amplitude of oscillations that will allow the penny to just stay on top of the block? (Assume $m \ll M$.)

* 87. A crane has hoisted a 1200-kg car at the junkyard. The steel crane cable is 22 m long and has a diameter of 6.4 mm. A breeze starts the car bouncing at the end of the cable. What is the period of the bouncing? [*Hint*: Refer to Table 9–1.]

* 88. A block of jello rests on a plate as shown in Fig. 11–58 (which also gives the dimensions of the block). You push it sideways as shown, and then you let go. The jello springs back and begins to vibrate. In analogy to a mass vibrating on a spring, estimate the frequency of this vibration, given that the shear modulus (Section 9–5) of jello is $520\ \mathrm{N/m^2}$ and its density is $1300\ \mathrm{kg/m^3}$.

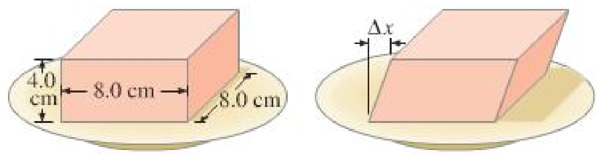


FIGURE 11–58 Problem 88.

Answers to Exercises

A: (*a*), (*c*), (*d*).
B: (*a*) Increases; (*b*) increases; (*c*) increases.
C: Empty.
D: (*a*) 25 cm; (*b*) 2.0 s.